COMMENTS ON PRESSURE BUMP INSTABILITY IN THE ENERGY DOUBLER

1. Stability Criteria

Use the criterion given by Benvenuti,\textsuperscript{1} the derivation of which is summarized on the attached sheets.

\[(\Theta I)_\text{crit} = 0.64 \, R\]

where \(\Theta\) is the fraction of a monolayer of hydrogen on the walls, \(I\) is the beam current, and \(R\) is the radius in cm of the vacuum pipe.

The expression is conservative in several respects. It assumes that the desorption coefficient for hydrogen is \(5 \times 10^4 \Theta\) independent of ion energy. If this is the value for 1 keV ions, the coefficient will surely be less for the <100 eV ions accelerated by the Doubler beam. It also assumes that the desorbed hydrogen molecules are traversing the vacuum chamber with speeds characteristic of a Boltzmann distribution at 4.5\(^\circ\)K - the speed can hardly be less.

If we take \(\Theta = 1\) so that the desorption coefficient has its maximum value (it gradually decreases for \(\Theta > 1\)), and use \(R = 3\) cm, then

\[I_\text{crit} = 1.9\text{ amperes}\]

We must look at this criterion as it relates to three modes of Doubler operation.

2. Operation as an Accelerator

A beam of \(2 \times 10^{13}\) protons corresponds to a current of 0.15 amperes, more than an order of magnitude less than the \(I_\text{crit}\) above. But the beam is present only \(\sim 50\%\) of the time, and the walls are pumping in the absence of beam. Even if the stability criterion is in error by an order of magnitude, it is hard to see how the instability can build up.
3. Proton-Proton Collisions
   In this case, one wishes to store the 0.15 ampere proton beam for the better part of an hour. The stability criterion still alleges that no problem should arise. However, note that one can abort the proton beam and reinject in a short time if necessary.

4. Proton-Antiproton Collisions
   Here, long storage times are desired. But the currents are much lower. If there are ten proton bunches each containing \(10^{11}\) protons, the proton current is 5\% of that in the two preceding cases. Moreover, in the 20 µsec between successive passages of the bunches, hydrogen molecules can cross the beam path without encountering the beam even at speeds corresponding to 4.5 °K (\(\bar{v} = 2 \times 10^4\) cm/sec).

Reference
Remarks on Pressure Stability in
Cold Bore Vacuum Systems

In proton accelerators and storage rings constructed with superconducting
magnets, it is natural to consider a cold bore vacuum system, for the continuously
distributed cryopumping may result in reduced aperture requirements, an improved
packing factor in the machine lattice, and lower costs as compared with a
conventional vacuum system.

Two recent reports by CERN authors, Cris Benvenuti\textsuperscript{1} and Roger Calder\textsuperscript{2}, have
been encouraging as to the potential benefits of cold bore systems. There are
quantitative differences in their conclusions, and the purpose of these remarks
is to indicate where those differences arise.

We begin by deriving a condition for stability in a cold bore system.
Assume that the conductance along the bore tube is negligible. Let $N$ be the
number of molecules per unit length in the pipe, and $n(r,\theta)$ be the distribution
of number density in the cross section of the (circular) pipe of radius $R$.
That is,

$$ N = \int n(r,\theta) \, r \, dr \, d\theta $$

For each positive ion created, and driven into the wall by the beam
electrostatic field, the net number of neutrals appearing in the beam will be
denoted by $\eta$ (i.e., if one neutral particle is returned to the beam pipe for each
ion created, $\eta = 0$).

The "luminosity" per unit length for ion creation is

\textsuperscript{1} C. Benvenuti, Fermilab 74/109, December 1974.

\textsuperscript{2} R. S. Calder, CERN/ISR-AS/74-73, October 1974.
\[ \frac{dL}{dz} = \int |v_{\text{rel}}| n(r, \theta) n_{\text{beam}} \, dA \]

\[ = c \times n(0) \lambda \]

where \( \lambda \) is the line density of particles in the proton beam; \( \lambda = I/eC \). For an ionization cross-section, \( \sigma \), the rate of ion production per unit length is

\[ \frac{dN_i}{dt} = \lambda c n(0) \sigma \]

and so the contribution to the rate of neutral production from this source is

\[ \lambda c n(0) \sigma \eta \]

beam production per unit time of neutrals

The rate of absorption at the walls will be

\[ \int n(R, \theta) \bar{v}_\perp (R, \theta) \alpha R \, d\theta \]

where \( \bar{v}_\perp \) is the average of the perpendicular component of the speed at the wall, and \( \alpha \) is a sticking probability. Let \( Q \) represent the production rate of neutrals per unit length from other processes. Then

\[ \frac{dN}{dt} = \eta \lambda c n(0) \sigma - \int n(R, \theta) \bar{v}_\perp (R, \theta) \alpha R \, d\theta + Q \]

To obtain the expressions used by Benvenuti and Calder, take \( n(R, \theta) = n - \) a constant independent of \( r \) and \( \theta \). Then in equilibrium

\[ \eta \lambda c n \sigma - \alpha n \bar{v}_\perp 2\pi R Q = 0 \]
or
\[
n = \frac{Q}{\alpha \nu \cdot 2\pi R - \eta \lambda \sigma} = \frac{Q(\alpha \nu, 2\pi R)}{1 - \frac{\eta \lambda \sigma}{\alpha \nu \cdot 2\pi R}}
\]

If we next use the Boltzmann distribution relation between \( \overline{v} \) and the average speed:

\[
\overline{v}_\perp = \frac{\overline{v}}{4}
\]

then we arrive at the criterion for stability used by both authors:

\[
\frac{2\eta \lambda \sigma}{\alpha \nu \pi R} < 1
\]

or converting to current

\[
(\eta I)_{\text{crit}} = \frac{\alpha \nu \pi R e}{2\sigma}
\]

If we insert the ionization cross section for \( N_2 \)

\[
\sigma_{N_2} = 1.2 \times 10^{-18} \text{ cm}^2
\]

\[
(\eta I)_{\text{crit}} = \frac{\alpha \nu R}{2} \cdot \frac{1.6 \times 10^{-19}}{1.2 \times 10^{-18}}
\]

\[
= \frac{\pi}{15} \alpha \nu R \left( \overline{v} \text{ in cm/sec} , \ R \text{ in cm}, I \text{ in amperes} \right)
\]

Now the authors diverge. Consider Benvenuti first. He takes \( \sigma \) for \( H_2 \) as 1/7 that of \( N_2 \):
\[(\eta I)_{\text{crit}} = \frac{7\pi}{15} \alpha \bar{v} R\]

and

\[\eta = 5 \times 10^4 \theta\]

where \(\theta\) is the fraction of a monolayer of \(H_2\) present. Then

\[(\theta I)_{\text{crit}} = \frac{7\pi}{15 \times 5 \times 10^4} \alpha \bar{v} R\]

He also sets \(\alpha = 1\), and using

\[\bar{v} = 14551 \sqrt{\frac{\mu}{M}} \text{ cm/sec} = 2.19 \times 10^4 \text{ cm/sec for } H_2 \text{ at } 4.5K\]

\[(\theta I)_{\text{crit}} = \frac{7\pi}{15 \times 5 \times 10^4} 2.19 \times 10^4 R\]

\[= 0.64R \quad \text{R in cm}\]

He further argues that \(\theta\) will not be greater than 0.1; then

\[I_{\text{crit}} = 6.4 \text{ R}\]

Next, consider Calder. He stays with nitrogen cross sections, but uses the hydrogen speed and uses \(\eta = 100\) for 0.1 monolayer. Then for \(\alpha = 1\)

\[I_{\text{crit}} = \frac{\pi}{15} \times \frac{2.19 \times 10^4}{10^2} R = 46R\]

Thus Calder result is a factor of \(\frac{50}{7}\) larger.