The Effect of Beam Centering on the Energy Calibration
LU-232

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The beam energy measurement line is shown in Fig.1. As known, when the energy is measured, beam centering to the design axis is very important. The beam centering is checked by two BPM, W_2 and W_3. Since the distance between W_2 and W_3 is short, $d \sim 1.26\,\text{m}$, a small error Δx in the beam position will result in a considerable error in injection angle δ,

$$\delta_{max} \sim 2\Delta x/d. \tag{1}$$

In this paper the effect of beam centering on the energy calibation is more accurately calculated in the analytic way.

Assume that: (1) There is a uniform magnetic field within two hard effective edges, as shown in Fig.2, where L_{in} and L_{out} are the effective edge, point N is the position of measuring filament center. (2) The z-axis of the coordinate system xoz is the beam injection direction, the origin O is the cross point of the effective edge and the injection direction. (3) The trajectory from point O to the exit point M at the output effective edge is a circular curve with radius R, and the trajectory of MN is a linear line. Obviously, there is only one radius R , corresponding to a definite deflection angle θ, along which the central ion can move from point O to point N. If the equation of the output effective edge in the system xoz is:

$$z = ax + b, \tag{2}$$

the coordinate of point $M(x_m, z_m)$ should satisfy the following equation:

$$\begin{cases} z_m = ax_m + b, \\ z_m = R\sin\theta, \\ x_m = R(1-\cos\theta). \end{cases} \tag{3}$$

The function of the coordinates of point $N(x_n, z_n)$ with the point $M(x_m, z_m)$ is:

$$x_n = x_m + (z_n - z_m)\tan\theta. \tag{4}$$

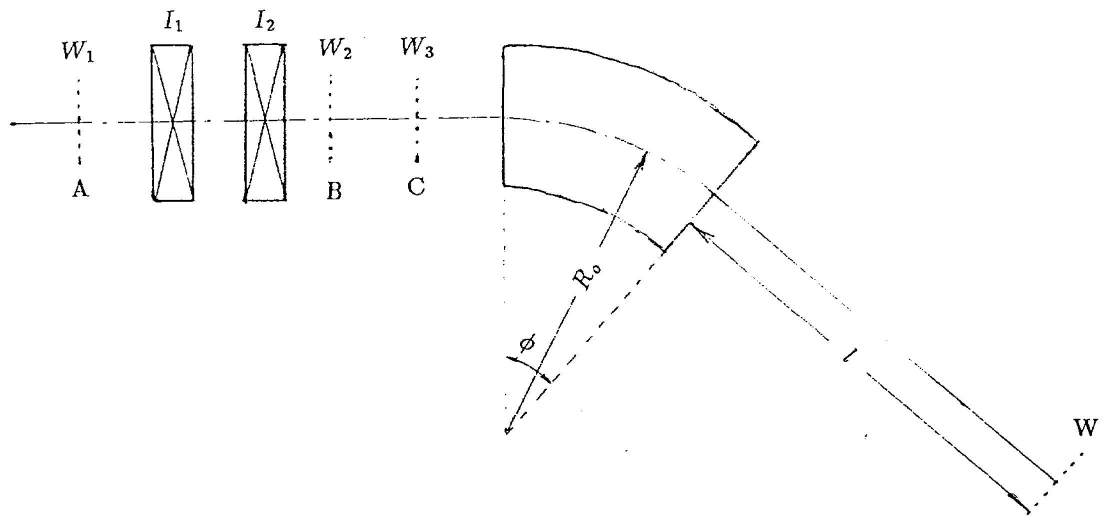

Figure 1: The energy measurement line

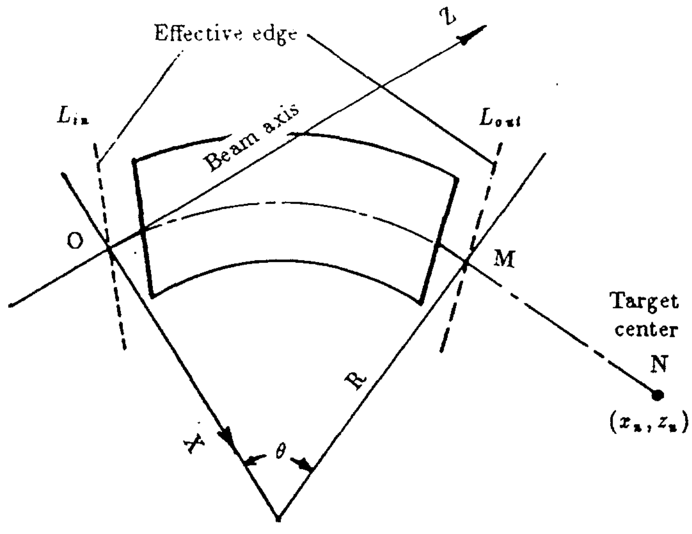

Figure 2: The scheme of a uniform magnetic analyzer

From Eq.(3), we have

$$R = \frac{b}{\sin\theta - a(1-\cos\theta)}. \tag{5}$$

Substitute Eq.(5) to Eq.(3), we have

$$z_m = \frac{b\sin\theta}{\sin\theta - a(1-\cos\theta)}, \tag{6}$$

and

$$x_m = \frac{b(1-\cos\theta)}{\sin\theta - a(1-\cos\theta)}. \tag{7}$$

Substitute Eqs.(6),(7) to Eq.(4), we have

$$x_n = \frac{b(1-\cos\theta)}{\sin\theta - a(1-\cos\theta)} + \{z_n - \frac{b\sin\theta}{\sin\theta - a(1-\cos\theta)}\}\tan\theta. \tag{8}$$

Thus , we can obtain the deflection angle θ from Eq.(8). Then using the value of θ, the trajectory radius R can be solved by Eq.(5). The beam energy can be determined by the following equation:

$$W = \sqrt{W_o^2 + 9\times 10^4 B^2 R^2} - W_o, \tag{9}$$

where W_o=939.3 MeV, B(T), R(m).

As shown in Fig.2, for the centering beam, the equation of output effective edge in coordinate xoz is known as:

$$\begin{aligned} z &= -\tan\theta_o \cdot x + R_o\tan\theta_o = ax + b, & (10)\\ a &= -\tan\theta_o, & (11)\\ b &= R_o\tan\theta_o, & (12) \end{aligned}$$

here $\theta_o = 40°$, R_o=4.328 m, corresponding to a magnetic field B_o=0.7369 T for beam energy of 401.5 MeV. Assuming l=6.1 m, we have $x_n = 4.93356, z_n = 7.45486$. If the beam deviates from the ideal axis with a deviation angle δ, in the new coordinate system x'oz', the output effective edge has the equation:

$$\begin{aligned} z' &= -\frac{\sin\delta + \cos\delta\tan\theta_o}{\cos\delta - \sin\delta\tan\theta_o}x' + \frac{R_o\tan\theta_o}{\cos\delta - \sin\delta\tan\theta_o}, & (13)\\ a' &= -\frac{\sin\delta + \cos\delta\tan\theta_o}{\cos\delta - \sin\delta\tan\theta_o}, & (14)\\ b' &= \frac{R_o\tan\theta_o}{\cos\delta - \sin\delta\tan\theta_o}. & (15) \end{aligned}$$

And the center of target filaments has the coordinate:

$$x'_n = x_n \cos\delta + z_n \sin\delta, \tag{16}$$
$$z'_n = -x_n \sin\delta + z_n \cos\delta. \tag{17}$$

Thus we can determined the new deflection angle θ' by the following equation:

$$x_n \cos\delta + z_n \sin\delta = \frac{b'(1-\cos\theta')}{\sin\theta' - a'(1-\cos\theta')} + \{(-x_n \sin\delta + z_n \cos\delta) - \frac{b' \sin\theta'}{\sin\theta' - a'(1-\cos\theta')}\} \tan\theta'. \tag{18}$$

Then the new radius of curvature R' can be determined by:

$$R' = \frac{b'}{\sin\theta' - a'(1-\cos\theta')}. \tag{19}$$

The requisite magnetic field B' for the beam with same energy can be calculated by:

$$(B'R')^2 = (B_o R_o)^2. \tag{20}$$

A wrong energy is calculated by the equation:

$$W' = \sqrt{W_o^2 + 9 \times 10^4 B'^2 R_o^2}. \tag{21}$$

Thus the error caused by the deviation of beam injection angle can be estimated by:

$$\Delta W = W' - W. \tag{22}$$

The calculated results for different errors in beam position Δx are shown in Tab.1.

Tab.1 The energy calibration error by injection angle deviation

Δx(mm)	θ'(°)	R(m)	W(MeV)	ΔW(MeV)
0	39.9946	4.328	401.506	0.0
1	40.1106	4.31686	403.263	1.758
2	40.2292	4.30552	405.069	3.564
3	40.3312	4.29618	406.567	5.061
4	40.4515	4.28477	408.406	6.90
5	40.5538	4.27551	409.908	8.403

Roughly we have:

$$\Delta W/\Delta x \doteq 1.73\,\text{MeV/mm, or } \Delta W/\delta \doteq 1.09\,\text{MeV/mrad}; \tag{23}$$
$$\frac{\Delta W}{W}/\Delta x \doteq 4.3 \times 10^{-3}/\text{mm, or } \frac{\Delta W}{W}/\delta \doteq 2.71; \tag{24}$$
$$\frac{\Delta P}{P}/\Delta x \doteq 2.5 \times 10^{-3}/\text{mm, or } \frac{\Delta P}{P}/\delta \doteq 1.58. \tag{25}$$

Similarly, for an error in parallel displacement Δx, we have:

$$z' = -\tan\theta_o \cdot x' + (R_o + \Delta x)\tan\theta_o, \tag{26}$$

$$a' = -\tan\theta_o, \tag{27}$$

$$b' = (R_o + \Delta x)\tan\theta_o. \tag{28}$$

And the coordinate of point N is:

$$x_n' = x_n + \Delta x, \tag{29}$$

$$z_n' = z_n \tag{30}$$

The deflection angle θ' and radius of curvature can be calculated by:

$$x_n + \Delta x = \frac{b'(1-\cos\theta')}{\sin\theta' - a'(1-\cos\theta')} + \left[z_n - \frac{b'\sin\theta'}{\sin\theta' - a'(1-\cos\theta')}\right]\tan\theta', \tag{31}$$

$$R' = \frac{b'}{\sin\theta' - a'(1-\cos\theta')}. \tag{32}$$

The calculated results for parallel displacement are shown in following table.

Tab.2 The energy calibration error by parallel displacement

Δx(mm)	θ'(°)	R(m)	W(MeV)	ΔW(MeV)
0	39.9946	4.328	401.506	0.0
1	39.99712	4.328663	401.3958	0.11
2	39.99855	4.329494	401.2647	0.241
3	30.00227	4.330056	401.1761	0.33
4	40.00227	4.331057	401.0186	0.487
5	40.00457	4.331787	400.9035	0.602

Roughly we have:

$$\Delta W/\Delta x \doteq 0.117\,\text{MeV/mm}. \tag{33}$$

Conclusion

(1) The main error in energy calibration caused by the error of beam centering is the error of beam injection angle. Thus when the beam energy is calibrated, the beam position must be centered by two beam position monitors.

(2) A very high accuracy of measurement of beam center is required. For an error $\Delta P/P = \pm 0.1\%$, the requisite accuracy in BPM is $\approx \pm 0.4$ mm. Also the magnet must be accurately positioned within an arror of angle deviation of less than 5'.

(3) It is required to define the output energy accurately in future. In order to calibrate the energy accurately, two slits, which are accurately positioned, may be necessary.