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ERROR ANALYSIS OF SHORT SAMPLE Jc MEASUREMENTS AT THE SHORT SAMPLE TEST FACILITY

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Abstract:

The sensitivity of the critical current density to the experimental parameters (temperature, magnetic field, strain where applicable, noise), and to the conductor's geometry is studied for NbTi and Nb_3Sn superconducting wires.

1. INTRODUCTION

The critical current density, J_c, of a superconducting wire is one of the most important parameters in the design of a superconducting magnet. A Short Sample Test Facility (SSTF) has been recently set up at the TD within the High Field Magnet project (HFM) in order to perform precise J_c measurements of binary and ternary NbTi, and Nb_3Sn samples from commercial and R&D strands. Measurements at different temperatures and different fields can be obtained in a Variable Temperature Insert (VTI) within a 15/17 T magnetic cryostat.

2. NbTi STRANDS

CRITICAL SURFACE PARAMETRIZATION

For NbTi, the critical current density, J_c, can be parametrized as a function of temperature, T, magnetic field, B, and critical current density at 4.2 K and 5 T, J_{cref}, using [3]:

$$\frac{J_c(B,T)}{J_{cref}} = \frac{C_0}{B}\left[\frac{B}{B_{c2}(T)}\right]^a \left[1-\frac{B}{B_{c2}(T)}\right]^b \left[1-\left(\frac{T}{T_{c0}}\right)^{1.7}\right]^g \qquad (1)$$

where C_0, α, β, and γ are fitting parameters, and [1]:

$$B_{c2}(T) = B_{c20}\left[1-\left(\frac{T}{T_{c0}}\right)^{1.7}\right] \qquad (2)$$

Equation (1) allows to operate over the entire temperature and field ranges of interest. Linear parametrizations [1] are more accurate, but apply to restricted ranges of temperatures and fields. The following values, which are typical for LHC strands, have been adopted [3]: C_0 = 30 T, α = 0.6, β = 1.0, γ = 2.0, and: J_{cref} = 2800 A/mm^2, T_{c0} = 9.2 K, and B_{c20} = 14.5 T.

ERROR ANALYSIS

The error analysis was based on the following differential expression:

$$dJ_c(B,T) = \left.\frac{\partial J_c}{\partial B}\right|_T dB + \left.\frac{\partial J_c}{\partial T}\right|_B dT \qquad (3)$$

Assuming a relative variation of 1% of J_c, the relative errors on the temperature, T, and on the field, B, were independently calculated as follows:

$$\left(\frac{\Delta B}{B}\right) = \frac{J_c}{B}\cdot\frac{1}{\left.\frac{\partial J_c}{\partial B}\right|_T}\cdot\left(\frac{\Delta J_c}{J_c}\right) \qquad (4)$$

$$\left(\frac{\Delta T}{T}\right) = \frac{J_c}{T}\cdot\frac{1}{\left.\frac{\partial J_c}{\partial T}\right|_B}\cdot\left(\frac{\Delta J_c}{J_c}\right) \qquad (5)$$

J_c partial derivatives were obtained analytically from equations (1) and (2). Calculations were carried out @ 4.2 K, 7 T, and @ 1.9 K, 10.5 T. The results are shown in Table 1.

T	B	J_c	ΔT/T	ΔB/B	Δ J_c / J_c
4.2 K	7 T	1750 A/mm^2	-	-0.44 % (≅0.031 T)	1 %
4.2 K	7 T	1750 A/mm^2	0.7 % (≅30mK)	-	1 %
1.9 K	10.5 T	1300 A/mm^2	-	-0.25 % (≅0.026 T)	1 %
1.9 K	10.5 T	1300 A/mm^2	1.6 % (≅30mK)	-	1 %

Table 1: Relative errors on the temperature, T, and on the field, B, corresponding to a J_c variation of 1% in NbTi wires.

COMPARISON WITH DATA

- **Temperature**

Figures 1 and 2 show the maximum temperature time variation of the sample versus the maximum current reached during I_c measurements of coil samples at 4.2K and various fields. The improved contact resistance which is observed in Figure 2 is due to indium foils that were placed between the end rings of the barrel and the copper contacts of the supporting fixture. In this latter configuration, the maximum variation in temperature is about 16mK at 4.2K. For NbTi at 4.2K and 7T, this translates into a relative error of about 0.5% on J_c. The spatial variation of temperature along the sample due to the gradient in the helium bath is less than 7mK for both 4.2K and 1.9K measurements.

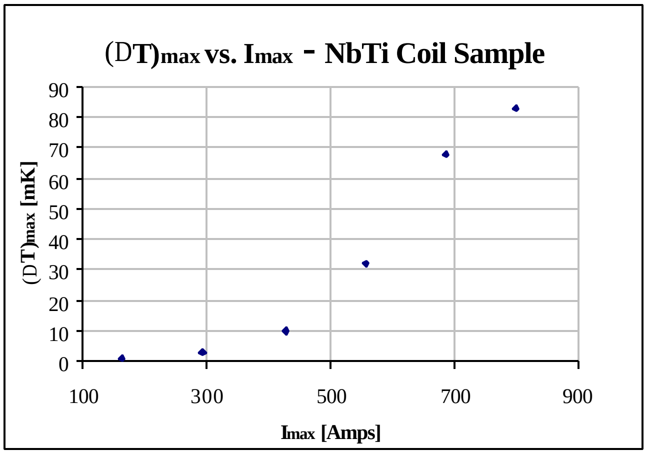


Fig.1: Maximum temperature variation in time vs. maximum current at 4.2K and different fields for a NbTi coil sample without indium foils.

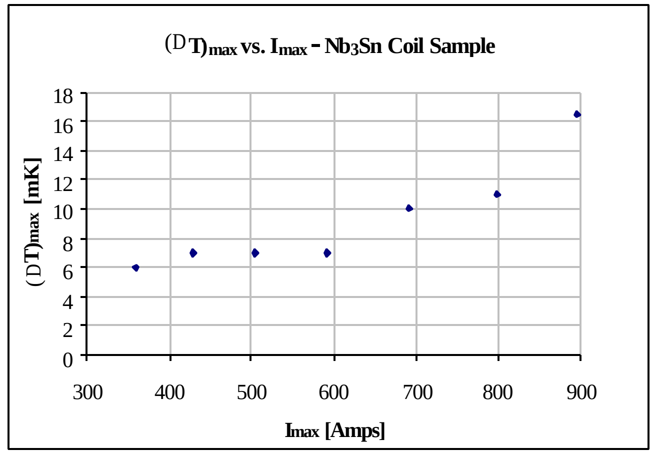


Fig.2: Maximum temperature variation in time vs. maximum current at 4.2K and different fields for a Nb_3Sn coil sample with indium foils.

- **Magnetic field**

In the coil geometry, the maximum relative inhomogeneity of the magnetic field within the sample with respect to the nominal central field is about 0.27%. The relative field stability being 10^{-4}/hr, the field variations in time are negligible. For NbTi, this leads to a relative error of about 0.6% on J_c at 4.2K and 7T, and to a relative error of about 1% at 1.9K and 10.5T.

In the hairpin geometry, the maximum relative inhomogeneity of the magnetic field within the straight section with respect to the nominal central field is about 0.25%. This leads to approximately the same relative errors as in the coil geometry.

Self-field effect: In the coil configuration, the self-field, B_{SF}, in the proximity of the solenoidal sample depends on the current, I, flowing in it as follows (see Appendix A):

$$B_{SF} = 3.264 \cdot 10^{-4}\, I$$

For a current of 500A, B_{SF} is about 0.16T. This would modify J_c by about 5% at 4.2K and 7T, and by about 6% at 1.9K and 10.5T.

3. Nb_3Sn STRANDS

CRITICAL SURFACE PARAMETRIZATION

For Nb_3Sn, the latest parametrization of the critical current density, J_c, valid over the temperature, field, and strain (ε) ranges of interest is [2]:

$$J_c(B,T,e) = \frac{C(e)}{\sqrt{B}}\left[1-\frac{B}{B_{c2}(T,e)}\right]^2\left[1-\left(\frac{T}{T_{c0}(e)}\right)^2\right]^2 \qquad (6)$$

where the following simplified expression was used for the upper critical magnetic field, $B_{c2}(T,\varepsilon)$:

$$\frac{B_{c2}(T,e)}{B_{c20}(e)} = \left[1-\left(\frac{T}{T_{c0}(e)}\right)^2\right] \qquad (7)$$

In equations (6) and (7), $B_{c20}(\varepsilon)$ is the upper critical magnetic field at zero temperature, and $T_{c0}(\varepsilon)$ is the critical temperature at zero field:

$$B_{c20}(e) = B_{c20m}(1-a|e|^{1.7}) \qquad (8.1)$$

$$T_{c0}(e) = T_{c0m}(1-a|e|^{1.7})^{\frac{1}{3}} \qquad (8.2)$$

and:

$$C(e) = C_0(1-a|e|^{1.7})^{\frac{1}{2}} \qquad (8.3)$$

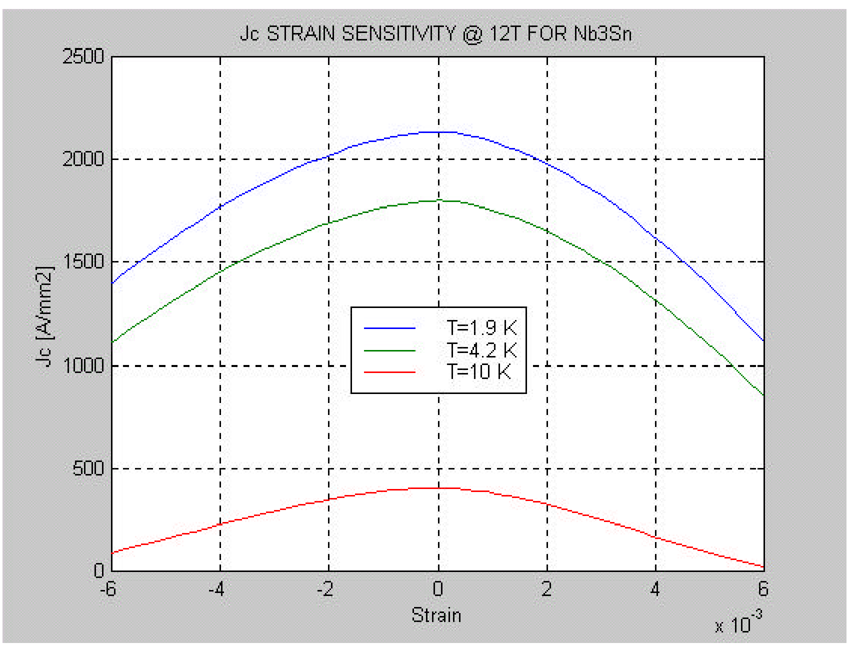


Fig.3: J_c dependence on the strain for a Nb_3Sn sample at a field of 12 T and at several temperatures.

In the above expressions, a is equal to 900 for compressive strain ($\varepsilon \leq 0$) and to 1250 for tensile strain ($\varepsilon > 0$). B_{c20m} is the upper critical magnetic field at zero temperature and zero strain, T_{c0m} is the critical temperature at zero field and zero strain, and C_0 is a fitting parameter expressed in $AT^{1/2}mm^{-2}$. The following values have been adopted: T_{c0m} = 18 K, and B_{c20m} = 26 T. C_0 = 26616 $AT^{1/2}mm^{-2}$ was chosen in order to describe a Nb_3Sn sample with a J_c of 1800 A/mm^2 @ 4.2 K, 12 T, and zero strain. In Figure 3 the J_c dependence on the strain for such a composite conductor is shown at a field of 12 T and at several temperatures.

ERROR ANALYSIS

The error analysis was based on the following differential expression:

$$dJ_c(B,T) = \left.\frac{\partial J_c}{\partial B}\right|_{T,e} dB + \left.\frac{\partial J_c}{\partial T}\right|_{B,e} dT + \left.\frac{\partial J_c}{\partial e}\right|_{B,T} de \qquad (9)$$

Assuming a relative variation of 1% of J_c, the relative errors on the temperature, T, and on the field, B, were independently calculated as follows:

$$\left(\frac{\Delta B}{B}\right) = \frac{J_c}{B} \cdot \frac{1}{\left.\frac{\partial J_c}{\partial B}\right|_{T,e}} \cdot \left(\frac{\Delta J_c}{J_c}\right) \qquad (10)$$

$$\left(\frac{\Delta T}{T}\right) = \frac{J_c}{T} \cdot \frac{1}{\left.\frac{\partial J_c}{\partial T}\right|_{B,e}} \cdot \left(\frac{\Delta J_c}{J_c}\right) \qquad (11)$$

$$\left(\frac{\Delta e}{e}\right) = \frac{J_c}{e} \cdot \frac{1}{\left.\frac{\partial J_c}{\partial e}\right|_{B,T}} \cdot \left(\frac{\Delta J_c}{J_c}\right) \qquad (12)$$

J_c partial derivatives were obtained analytically from equations (6), (7) and (8). The calculation of the partial derivative of J_c with respect to the strain, $(\partial J_c/\partial \varepsilon)_{B,T}$, is shown in Appendix B. In Table 2 the relative temperature and field errors in Nb_3Sn wires at zero strain are shown @ 4.2 K, 12 T, and @ 1.9 K, 12 T.

T	B	J_c	$\Delta T/T$	$\Delta B/B$	$\Delta J_c / J_c$
4.2 K	12 T	1800 A/mm^2	-	-0.41 % (≅0.049 T)	1 %
4.2 K	12 T	1800 A/mm^2	-2.2 % (≅92mK)	-	1 %
1.9 K	12 T	2150 A/mm^2	-	-0.44 % (≅0.053 T)	1 %
1.9 K	12 T	2150 A/mm^2	-11.7 % (≅0.22 K)	-	1 %

Table 2: Relative errors on the temperature, T, and on the field, B, corresponding to a J_c variation of 1% in Nb_3Sn wires @ zero strain.

In Table 3 the relative errors on the strain in Nb_3Sn wires are shown @ 4.2 K, 12 T for two different compressive strains, ε = -0.25 % and ε = -0.40 %.

ε	T	B	J_c	$\Delta\varepsilon/\varepsilon$	$\Delta J_c / J_c$
-0.25 %	4.2 K	12 T	1644 A/mm^2	6 % (≅0.015 %)	1 %
-0.40 %	4.2 K	12 T	1453 A/mm^2	2.5 % (≅0.010 %)	1 %

Table 3: Relative errors on the strain, ε, corresponding to a J_c variation of 1% in Nb_3Sn wires at two different compressive strains.

COMPARISON WITH DATA

- **Temperature**

As shown in Section 2, in the coil configuration with indium foils, the maximum variation in temperature is about 16mK at 4.2K. For Nb_3Sn at 4.2K and 12T, this translates into a relative error of about 0.2% on J_c. At 1.9K, the maximum variation in temperature during the transition can be held within 30mK, giving a relative error of less than 0.2% on J_c for Nb_3Sn at 1.9K and 12T.

In the hairpin configuration, the maximum variation in temperature that was reached close to 600A for a Nb3Sn sample is 54mK at 4.2K. For Nb_3Sn at 4.2K and 12T, this translates into a relative error of about 0.6% on J_c.

- **Magnetic field**

For Nb_3Sn in the coil geometry, the inhomogeneity of the magnetic field within the sample leads to a relative error of about 0.7% on J_c at 4.2K and 12T, and to a relative error of about 0.6% at 1.9K and 12T.

For Nb_3Sn in the hairpin geometry, the inhomogeneity of the magnetic field within the straight section leads to approximately the same relative errors as in the coil geometry.

Self-field effect: For a current of 1000A, B_{SF} on the solenoidal sample is about 0.3T (see Appendix A). This would modify J_c by about 6% both at 4.2K and 12T, and at 1.9K and 12T.

- **Strain**

In the coil configuration, the relative directions of the external magnetic field and of the transport current are chosen such as to have an inward Lorentz force. This allows to avoid the use of bonding agents along the length of the sample, since the strand tightens on the barrel. For a current of 1000A at 4.2K, the compressive magnetic strain of a 1mm Nb_3Sn free coil sample would yield 0.55% at 12T. Under the same Lorentz force, the Ti-alloy barrel contracts by only 0.06%*. In the case of Nb_3Sn, advantage is also taken of the 0.11% differential thermal contraction of the Nb_3Sn strand with respect to the Ti-alloy in the cooling from room temperature to 4.2K. Hence, during 12T measurements at 4.2K, the Nb_3Sn composite is subject to a total tensile strain of (0.110-0.06)% = +0.05%. This strain variation caused by the experimental procedure to the intrinsic strain of the Nb_3Sn filaments increases J_c by about 3% assuming an intrinsic compressive strain of 0.25%, and by 5% assuming an intrinsic compressive strain of 0.4%.

* For a current of 400A at 4.2K, the compressive magnetic strain of a 0.8mm NbTi free coil sample would yield 0.18% at 7T. The Ti-alloy barrel contracts by about 0.02%.

4. NOISE EFFECTS

4.1 The oscillatory behavior expected in the noise of I_c measurements has been modeled as a cosine, $V_a \cdot cos\omega t$. This was averaged in time over a quarter period ($\pi/2\omega$) in order to take into account only the effect of the noise amplitude, V_a, and get rid of the frequency dependence. Therefore in the proximity of the superconducting to normal transition, the V(I) behavior includes the contribution of the superconductor's characteristic curve and of the noise amplitude as follows:

$$V = V_0 \left(\frac{I}{I_0} \right)^n \pm \frac{2V_a}{p} \tag{13}$$

where I_0 is the observed current at voltage V_0. The exponent n reflects the abruptness of the transition from the superconducting to the normal state and usually lies between 20 and 100. Solving for the current, I:

$$I = I_0 \left(\frac{V \mp \frac{2V_a}{p}}{V_0} \right)^{\frac{1}{n}} \tag{14}$$

applying a critical current criterion, such as $I = I_c \Leftrightarrow V = V_c$, and taking the logarithmic derivatives of equation (14), one gets:

$$\frac{\Delta I_c}{I_c} = \mp \frac{2}{np} \cdot \frac{\Delta V_a}{V_c}$$

where in the denominator $2V_a/\pi$ has been neglected compared to V_c. The noise amplitude corresponding to an I_c relative error of 1% depends on n as follows*:

$$\left| \frac{V_a}{V_c} \right| = 0.0157 \cdot n \tag{15.1}$$

For example, if n were 30, $|V_a/V_c|$ should be less than 47%.

COMPARISON WITH DATA

The above requirement is easily fulfilled in the case of coil samples. For a Nb_3Sn coil sample tested at 4.2K and 12T, $|V_a/V_c|$ was 6% using a resistivity criterion of 10^{-14} $\Omega\cdot$m. This would lead to a relative error of about 0.1% on I_c with n = 30. The relative noise amplitude is much higher in the case of hairpins. For a Nb_3Sn hairpin sample tested at 4.2K and 12T, $|V_a/V_c|$ was 100% using a resistivity criterion of 10^{-14} $\Omega\cdot$m. This corresponds to a relative error of about 2% on I_c with n = 30. By increasing the resistivity criterion to 10^{-13} $\Omega\cdot$m, $|V_a/V_c|$ would reduce to 8%, giving a relative error of about 0.2% on I_c.

4.2 The errors on the current measurements given by the readback accuracy of the power supply are directly included in the overall error on I_c as follows:

$$\frac{\Delta I_c}{I_c} = \left(\frac{\Delta I}{I} \right)_{PS} = 0.1\% + 0.6A \tag{15.2}$$

* V_a has been written instead of ΔV_a since V_a is a perturbation from zero.

COMPARISON WITH DATA

For critical currents above about 70A, the relative error on I_c would be less than 1%, reducing to less than 0.2% above 600A.

4.3 As in Paragraph 4.1, the accuracy with which the critical voltage is determined can be calculated as:

$$\frac{\Delta V_c}{V_c} = n \cdot \frac{\Delta I_c}{I_c} \qquad (15.3)$$

COMPARISON WITH DATA

The distance between the voltage taps used to test a coil sample is 50cm. An accuracy of 1cm in placing the voltage taps on the strand leads to $|\Delta V_c/V_c|$ = 2%. For n = 30, the relative error on I_c would be negligible, *i.e.* less than 0.1%.

In a hairpin sample the distance between the voltage taps is 2cm and a location accuracy of 6mm is needed to get a relative error on I_c of 1%.

5. SENSITIVITY TO THE STRAND'S GEOMETRY

In the non-copper fraction of the composite cross section, J_c is calculated as:

$$J_c = \frac{I_c}{lA} = \frac{(1+y)I_c}{A} \qquad (16)$$

where A is the wire cross section, $\lambda = A_{non\text{-}copper}/A$, and y is the copper to non-copper ratio. The relative errors on J_c due to tolerances either on the copper to non-copper ratio, y, or on the wire diameter, d_w, are given respectively by:

$$\left(\frac{\Delta J_c}{J_c}\right) = \frac{y}{1+y} \cdot \left(\frac{\Delta y}{y}\right) \qquad (17)$$

$$\left(\frac{\Delta J_c}{J_c}\right) = -2 \cdot \left(\frac{\Delta d_w}{d_w}\right) \qquad (18)$$

COMPARISON WITH DATA

For an IGC Nb_3Sn strand delivered in June '98 having y = 0.59 ± 0.1/1, the relative error on J_c would be as high as 6%. The same result was found for a specified OST Nb_3Sn strand having y = 0.85 ± 0.1/1.

A $|\Delta d_w/d_w|$ of 0.5% would lead to a relative error of 1% on J_c.

6. CONCLUSIONS

For both the hairpin and coil configurations, the factors that by far (*i.e.* 5÷6%) bring the highest error on the J_c are the self-field effect and the tolerance on the copper to non-copper ratio. This suggests the following: corrections should be applied to the nominal magnetic field to take into account the self-field, and the copper to non-copper ratio should be accurately measured for each tested strand.

As a second order approximation, the hairpin configuration leads to a lower accuracy on the J_c (*i.e.* 2%), due to the higher relative noise. Chosing a $\rho_c = 10^{-13}\ \Omega\cdot m$ resistivity criterion when testing hairpin samples would lower the relative error on the J_c by one order of magnitude.

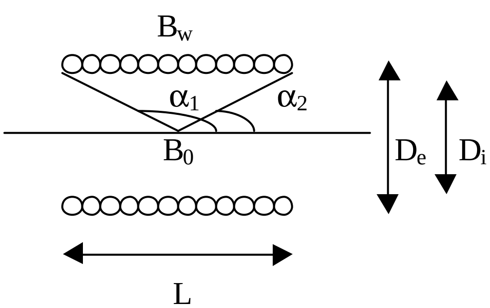
In Section 3, the instrinsic strain of the Nb_3Sn filaments had to be assumed. The only way for making unambiguous measurements on Nb_3Sn superconductors is to use a sample mounting which can be adjusted to set the strain at a known level. In the absence of such a device, the experimental procedure needs to be standardized in order to produce repeatable results on similar samples.

APPENDIX A

The following equation [4] was used to calculate the central field, B_0, of a coil sample carrying a current I, and having the geometry shown in figure:

$$B_0 = \mu_0\, n\, I \cdot (\cos\alpha_2 - \cos\alpha_1)/2 ,$$

where n is the number of coil turns per unit length.



The following values apply:

n = 315 turns/m; $\alpha_1 = (136.65)^o$; $\alpha_2 = (43.35)^o$.

The field in proximity of the coil, B_w, can be written as [5]:

$$B_w = B_0 \cdot f(\alpha, \beta)$$

where $\alpha = D_e/\ D_i \approx 1.07$, and $\beta = L/\ D_i \approx 1.1$. With these parameter values, one finds $f = 1.134$.

The overall result for the self-field is then: $B_w = B_{SF} = 3.264 \cdot 10^{-4}\ I$.

APPENDIX B

In this appendix the calculation of the partial derivative of J_c with respect to the strain, $(\partial J_c/\partial\varepsilon)_{B,T}$, is shown. In order to point out the dependence on the strain in the J_c parametrization given by equation (6), the latter can be written as:

$$J_c(B,T,e) = \frac{C(e)}{\sqrt{B}}\left[1 - \frac{B}{B_{c20}(e)\, f(T,e)}\right]^2 f^2(T,e) \qquad (B.1)$$

where:

$$f(T,e) = \left[1 - \left(\frac{T}{T_{c0}(e)}\right)^2\right] \qquad (B.2)$$

The following expressions were used:

$$\frac{dT_{c0}(e)}{de} = \mp 0.5\bar{6}T_{c0m}a|e|^{0.7}\left(1-a|e|^{1.7}\right)^{-\frac{2}{3}}$$

$$\frac{dB_{c20}(e)}{de} = \mp B_{c20m}1.7a|e|^{0.7}$$

$$\frac{dC(e)}{de} = \mp 0.85C_0 a|e|^{0.7}\left(1-a|e|^{1.7}\right)^{-\frac{1}{2}}$$

$$\frac{\partial f(T,e)}{\partial e} = \mp 1.1\bar{3}\frac{T^2}{T^2_{c0m}}a|e|^{0.7}\left(1-a|e|^{1.7}\right)^{\frac{1}{3}}$$

in order to obtain $(\partial J_c/\partial\varepsilon)_{B,T}$:

$$\left.\frac{\partial J_c}{\partial e}\right|_{B,T} = \frac{1}{\sqrt{B}}\left\{\frac{2BC(e)B_{c20m}a|e|^{0.7}}{B^2{}_{c20}(e)}\left[1.7f(T,e)+1.1\bar{3}\frac{T^2}{T^2{}_{c0m}}\left(1-a|e|^{1.7}\right)^{\frac{4}{3}}\right]\left[1-\frac{B}{B_{c20}(e)f(T,e)}\right]+\right.$$
$$\left.+\left[\frac{0.85C_0{}^2a|e|^{0.7}}{C(e)}f^2(T,e)+2.2\bar{6}C(e)f(T,e)\frac{T^2T_{c0}(e)}{T^3{}_{c0m}}a|e|^{0.7}\right]\left[1-\frac{B}{B_{c20}(e)f(T,e)}\right]^2\right\} \qquad (B.3)$$

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