Fermi National Accelerator Laboratory

FERMILAB-TM-1752

Mechanical Design and Analysis of the Horn, a Magnetic Lens, for the Main Injector Neutrino Program

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Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Summary

In order to produce a high intensity neutrino beam, it is necessary to design a magnetic lens that collects the output of secondary pions and kaons from the target and maximize the focusing of the particles into the decay region.[1]

The magnetic lens, Horn, has been designed to maximize flux while maintaining overall reliability. The Horn is an electrical conducting sheet which has the shape of two hollow cones joined at the smaller ends as shown in Fig.1 and Fig.2. The shape and dimension of the Horn have been chosen to optimize the focusing and allow only minimum absorption loss of particles that are passing through the Horn while maintaining sufficient mechanical strength to withstand the cyclic effects of the magnetic pressure and thermal stress acting on the Horn. The magnetic field generated will reach 36 kilo-Gauss

A constant wall thickness, 2 mm, has been chosen for the Horn taking into consideration the cyclic effects, since the Horn is to be loaded and unloaded every 2 seconds.

The loading condition of the Horn is quite unique, but in some way similar to a submarine in deep waters. The primary concern in analyzing the stresses acting on the Horn is the buckling of the Horn because of the cyclic magnetic pressure and thermal stress. Careful study shows that the thermal stress can be greatly reduced, because thermal stress is mainly created by the constraints which do not expand or contract along with the Horn.[2] Table.1 shows that for operating temperature of the Horn below 40 degrees Celsius, even if the constraints at ends and supports do not expand or contract at all, the thermal stress generated would not be a main factor for the failure of the Horn. The resultant or effective stress, Von Mises stress, at maximum operating condition with 200kA peak current, as calculated using finite element software, ANSYS [3] and PATRAN [4], gives a safety factor of 1.6 when compared to the mechanical fatigue stress limit at 500 million cycles for the Horn material, aluminum alloy 6061-T6 [5] as shown in Fig.6.

The conditions which should be met for this design, with 2mm wall thickness, are (1) the Horn should be manufactured without detectable cracks,and with a very smoothly finished surface, (2) the cooling system in the Horn must be able to keep the operating temperature in the range of 0~20 degree Celsius since the change in surface temperature of the Horn by one pulse of 200kA current is about 20 degree Celsius. These two conditions have been studied and suggest that the problems are workable. Mechanically, with this design the Horn would work for at least one year with confidence and reliability.

Design Criteria

basic design criteria

1. minimum wall thickness for low particle absorption loss
2. meets the specified shape and dimension
3. must withstand the dynamic fatigue for at least one year of operation, or ~ 31 million cyc.

design for fatigue

1. avoid crack initiation
    - special manufacturing process
        - to obtain defect free aluminum alloy
        - very smooth finishing surface
2. avoid stress concentration
    - avoid discontinuities, such as sudden change in cross sectional area
    - use rounded edges, or fillet to avoid sharp edges or corners

design for thermal

1. avoid over constraining
2. keep operating temperature between 0~20 C
3. constraints should be able to expand or contract along with the Horn to some degree

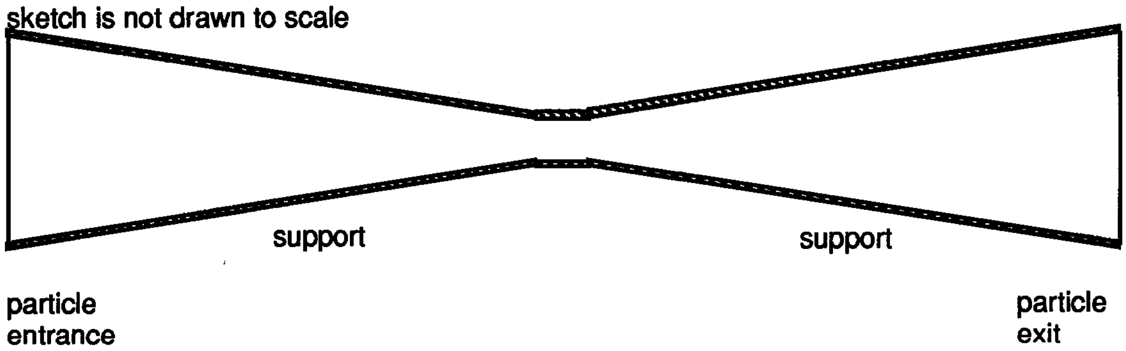
Material Selection Criteria

1. strength
2. low particle absorption rate
3. good electrical conductor with high conductivity and low resistivity
4. stiffness
5. availability
6. machinability
7. corrosion resistant
8. weldablility
9. light weight

material selected: Aluminum Alloy 6061-T6

Boundary Condition

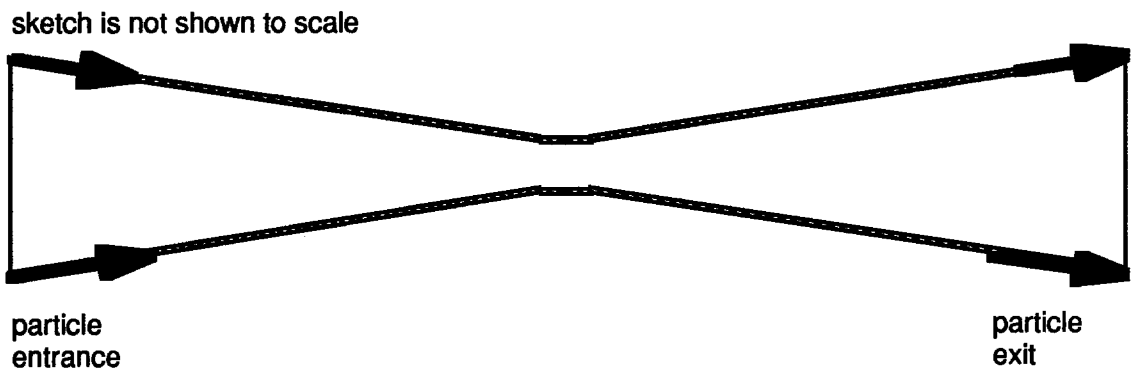
Mechanical



degree of freedom for the constraints:

	type	constraint	comment
particle entrance:	flange	Fix all degrees.	partially constrained in axial is acceptable
particle exit:	flange	fix all except in axial direction	partially constrained in axial is better if operating temp. is between 0~20 C
support:	special	fix only 3 deg. against gravity and 2 in rotation	should be able to expand and contract along with the Horn or over constrained would be fine if the operating temp is in (0~20 C)

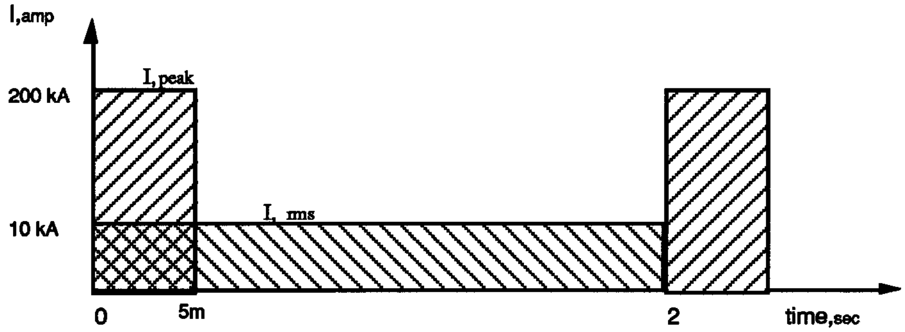
Electrical



The arrows indicate the direction of the applied current.

Loading Condition

Electrical



Peak Horn current: 200k amperes

Total Transformed: Load Inductance = 2.8 m H

Load Resistance = 0.98 O

Spill Length: 1 ms

Current deviation (peak to peak) during spill = 10 %

Period in a cycle: 2 seconds

The exact wave form has not yet been designed, but Age Visser in RD/EE [6] has already made preliminary calculations, and he suggests the following:

(1) calculations for power dissipation can assume the function of the applied current to be a square waves

(2) elapsed time for the applied current = 0.005 second

(3) calculated root-mean-square (rms) current is = 10k A

Mechanical

The mechanical loading is caused by the applied current and induced current. No direct mechanical loading is applied to the Horn except for its weight against gravity.

List of loadings

(1) impact and cyclic-hydrostatic magnetic pressure,P

(2) thermal stress

(3) gravitational force

Analysis method

Because the loading condition of the Horn is unique, a combination of methods is used to analyze and design the Horn. In order to choose the right methods for analyzing the Horn, it is necessary to identify the factors that would contribute to a failure of the Horn.

The forces that are acting on the Horn are cyclic magnetic pressure and thermal stress, reaction force from flanges at the ends, and gravitational force. The magnetic pressure acting on the Horn should be viewed as an impact or shock stress since the time of loading, 5 milli seconds is less than half the natural period of vibration, less than 1 m second. The impact loading actually is favorable because it changes the mechanical properties of material and create a delayed yielding in some metals.[2] Fig.3 shows only a small increase in strength for the two aluminum alloys in a rapid loading test. In fact, since the material of the Horn, aluminum alloy 6061-T6, has similar properties to the two aluminum alloys, and the Horn will be loaded faster than the rapid loading test for the two aluminum alloys, a small increase in strength is expected. Also since the impact stress is hydrostatic, and applied symmetrically in the radial direction, the torsion force, or the force that tends to twist the Horn would not be significant. Thus, the impact stress would be favorable for strengthening the material.

One of the concerns about the magnetic pressure is buckling of the Horn, i.e. failure due to compression or expansion of the Horn in the radial direction. Since the Horn has a conical shape, the magnetic pressure acting on the Horn increases quadratically with decreasing radius, ie. $P=\text{constant}/r^2$ [A4.2]. At the neck where the radius is smallest and the corresponding pressure is largest one might think that the buckling is important. But with careful study, one finds that the critical stress, or the stress required to buckle the Horn, also increases quadratically [A5.1]; thus buckling is important at larger radii. Fortunately, the pressure acting on larger radii is quadratically smaller, so the magnetic pressure will not cause the Horn to buckle. This is shown in Fig.4, where the critical buckling stress and the resultant stress, or Von Mises stress calculated using finite element software PATRAN, are plotted from one end of the Horn to the neck. This plot shows a minimum safety factor of 3.8 near the end.

The other concern about the magnetic pressure is the fatigue of the material from the cyclic loading. Fatigue failure denotes a failure by fracture brought about by repeated reversal or removal of the applied load. The magnetic pressure will be loaded and unloaded every 2 seconds or every cycle with approximately of 30 million cycles per year. It should be noted that the maximum applied current 200kA, in 5m second, would create radial compression and axial tension. Also the current which is induced by the applied magnetic field would create an induced magnetic pressure pushing the wall outward.[7] So, in one cycle or 2 seconds, the Horn will be loaded and unloaded in one direction by an applied magnetic pressure in 5m second and loaded and unloaded in the opposite direction by the induced magnetic pressure. This phenomenon is a typical fatigue problem studied in engineering. Fatigue tests on tubes that have similar dimensions to the Horn and are of the same material, aluminum alloy 6061-T6, have been conducted by The American Society for Testing and Material. The standard fatigue stress limit (100% failure) at 500 million cycles is given as 14,000 psi or 9,600 N/cm^2 .[8] A plot of fatigue stress vs. cycles to failure is shown in Fig.6. The fatigue stress limit 14,000 psi or 9,600 N/cm^2 can be used as a design stress limit at 30 million cycles or one year of operating time. Since the stress concentration for the Horn is at the neck, fatigue is important at the neck as shown in Fig.4,7.

Another important factor that could contribute to the failure of the Horn is the thermal stress that would tend to buckle the Horn. The change in surface temperature of the Horn from 1 pulse of 200kA current in 5m second is calculated to be about 20 degree Celsius.[A3.3] This change in temperature would cause the Horn to expand. The thermal stress should be understood as being mainly caused by the constraints placed on the subject. For example, if for a certain change in temperature, the Horn and its supports and end flanges can expand and contract freely with same amount, then there is no thermal stress. Since in reality the constraints would not expand and contract the same amount as the Horn, a thermal stress is created. The magnitude of the thermal stress is proportional to how much the constraints are able to expand or contract relative to the subject that is being constrained. The direction of the thermal stress is determined by comparing the thermal expansion coefficients. When heating, if the constraints expand more than the subject, then the subject is in tension or if the constraints expand less than the subject then the subject is in compression. For the Horn, since the flange at ends and the supports have not yet been manufactured and tested, their coefficients of thermal expansion are not certain. But a table and a plot have been created to analyze the effect of thermal stress. Table 1 shows the thermal stress as a function of the coefficient of thermal expansion at the ends and supports at various operating temperatures. The coefficients are presented in both percentages of the Horn's coefficient and their corresponding values. The buckling stresses at the ends and supports and the fatigue stress limit are identified in the table with different fonts for clarity, and similarly, Fig.5 shows the thermal stress vs. coefficients presented in percentages at an operating temperature of 100 C. This figure also shows the minimum amount of expansion needed at the ends and supports. Table 1 should be referenced when designing the cooling system for the Horn.

The gravitational force, or simply the weight of the Horn, is just the mass times the acceleration of gravity. The deflection due to the weight of the Horn is accurately calculated using finite element software. The effect of the reaction forces at the end of the Horn is analyzed using finite element software as well.

It is necessary to consider possible harmful effects of vibration on Horn and its members and assemblies. The natural frequencies of the parts, and of the assembly as a whole, should lie outside a range from half to twice the frequency of the regularly repeated impulses; if not, the deflections of the part or assembly can build-up, and the stresses can be amplified. The natural frequencies of the Horn are determined using PATRAN.[9]

Results

Material for the Horn : Aluminum alloy 6061-T6

Thickness of the wall: 2 mm constant thickness

Properties of the Horn: refer to Appendix 1.

Electrical

Given: I,peak = 200 kA

period = 2 second

Result:

time of applied current per cycle = 5 mili second

assumed square wave function for the current

I,rms = 10 kA

at 20 C resistance of the Horn = 4.40E-4 ohm

power dissipation = 44,000 watts

Thermal: a table and a plot of thermal stress vs coefficient of thermal expansion are shown in Table 1 and Fig.5

at 20 C

change of surface temperature of the Horn by one pulse of 200kA current

~ 20 C

Mechanical

maximum deflection after placing two supports =

critical buckling or hoop stress

a plot of critical stresses along the Horn is shown in Fig.4

resultant stress (without thermal stress)

= 6,000 N/cm^2 [PATRAN] = 5,700 N/cm^2 without gravity [ANSYS]

maximum deflection after placing the supports = 0.03 mm @neck

Vibration

natural frequency of vibration of the Horn

simplest mode = 92.5 Hz

2nd. mode = 92.5 Hz

3rd. mode = 140 Hz

4th. mode = 140 Hz

Conclusion

This study concludes that a Horn with a constant wall thickness of 2 mm is at an optimum thickness for low particle absorption loss and with sufficient strength to withstand the cyclic loadings. Two conditions which should be met for this design are (1) the Horn should be manufactured without detectable cracks and with a very smoothly finished surface, (2) the cooling system in the Horn must be able to keep the operating temperature in the range of 0~20 degree Celsius since the change in surface temperature of the Horn from one pulse of 200kA current is about 20 degree Celsius. These two conditions have been studied and suggest that the problems are workable. Mechanically, with this design the Horn will work for at least one year with confidence and reliability.

This report not only optimized the thickness of this particular Horn, but also presented a method for analyzing an electrical conductor subjected to a unique loading condition such as the Horn.

Discussion

Because of the uniqueness of the loading and boundary conditions, a combination of methods commonly used in engineering design was used. From this study I propose that to analyze this Horn, the factors contributing to a failure and all the forces acting on the Horn must first be carefully visualized and studied. The primary concern for the failure of this horn should be the cyclic magnetic pressure. Because of the conical shape of the Horn the magnetic pressure increases quadratically as the radius decreases.

The use of FEA software, PATRAN and ANSYS

The function for magnetic pressure due to 200kA, $P=636.62$ newton/r^2 was derived from $P=B^2/2m$ [A4.1], and this function was used in PATRAN to describe the varying pressure on the Horn. One important thing about this function is the radius. The radius, r, should be the distance from the axis of azimuthal symmetry to the *outer radius* of the Horn, because the magnetic pressure generated by the current is largest at the outer surface.[A4.2] To verify the validity of this function, ANSYS which has the capability of modeling magneto-static was used to find the resultant stress, Von Mises stress caused by the magnetic pressure. The input for ANSYS using magneto-static analysis was 200kA current and the software generated the magnetic field; on the other hand the input for PATRAN using static analysis was the function $P=636.62$newton/r^2 .[A4.2] The resultant stress calculated using the two different approaches was the same. This indicates that the function used to describe the magnetic pressure was appropriate. The resultant stress calculated by the two software should compare to the fatigue stress limit.

The big picture about the Horn

We know that the magnetic pressure tends to compress the Horn radially while stretching it axially, and the flanges at the ends which purposely constrain all degrees of freedom at the particle entrance and constrain all degrees of freedom but one in axial direction which is only partially constrained . The constraints would create reaction forces which tend to prevent the Horn from stretching. Because of the symmetry and the conical shape, the opposing reaction forces would decrease the stretching and compression; thus the reaction forces would compensate some stresses due to the magnetic pressure. This effect is seen by using PATRAN. Over constraining is a good idea only when the Horn is aligned perfectly symmetrically. This is possible when two supports are placed at the suggested locations. The maximum deflection due to gravity after placing the support is only 0.03 mm , which is negligible. Also the operating temperature must be kept at constant in the range, 0~20 C.

The magnetic field

We know that the direction of the applied current is axial from the particle entrance to the particle exit, and this creates a magnetic field outside the cone only. There is no magnetic field inside the cone.[A4.1]

The induced current

The magnetic field which is created by the 200kA applied current, decreases slowly after removal of the applied current. During the time after the removal of the applied current until the beginning of the next cycle, 1.995 seconds, the left-over magnetic field induces current which circulates radially instead of moving through the Horn axially. This induced current then generates a magnetic field inside the cone which creates an internal magnetic pressure pushing the Horn outward. The exact magnitude of the induced magnetic pressure has not been calculated but the pressure is expected to be much less than the applied magnetic pressure.[7] The effect of the induced pressure has been taken into account in the fatigue design.

The shape of the Horn

The shape of the Horn was designed by Anthony Malensek in order to optimize the focus of the particles.[10]

Vibration

The natural frequencies of the Horn calculated using PATRAN are 92.5 Hz and 140 Hz.

Supports

The supports should be located 122 cm from the ends. The location of the supports was determined by considering the deflection, critical hoop stress, or buckling stress, and the natural frequency of vibration.

The uncertainty in this design

Since the supports and the flanges have not yet been analyzed, their coefficients of thermal expansion are uncertain. There are uncertainties in the effects of vibration, such how the fluid used in the cooling system effects the loading or vibration.

Recommendation

The key to the success of this design relies heavily on the successful manufactured and machining of the Horn, and on a reliable cooling system. The Horn should be manufactured with no detectable cracks under an X-ray crack detector and with a very smoothly finished surface. The Horn is best if it can be manufactured in one continuous pieces along with its end flanges.[11] If the Horn is to be manufactured in separate piece, the separation should be at the supports.

The cooling system for the Horn must be reliable and able to keep constant operating temperature between 0~20 C. This eliminates the problems of thermal stress, thermal fatigue, and also maintains the constant mechanical strength of the material. When designing the cooling system, the ideal operating temperature for the Horn should be determined with considering the mechanical properties of aluminum at low temperatures, such as the strength, resistivity, as well as the type of fluid used in the system. Although the strength of the aluminum would increase at low temperature without loosing its ductility, thermal stress would be created at low temperatures as well as at high temperature.

The alignment of the flanges, Horn, and its supports is critical because the reaction forces created by the ends are expected to compensate for about 20 to 30 % of the applied stresses.[PATRAN] If the system is not aligned properly, the reaction forces will create either torsion, shear stress, or both. That would amplify the resultant stress, thus bad alignment is not favorable.

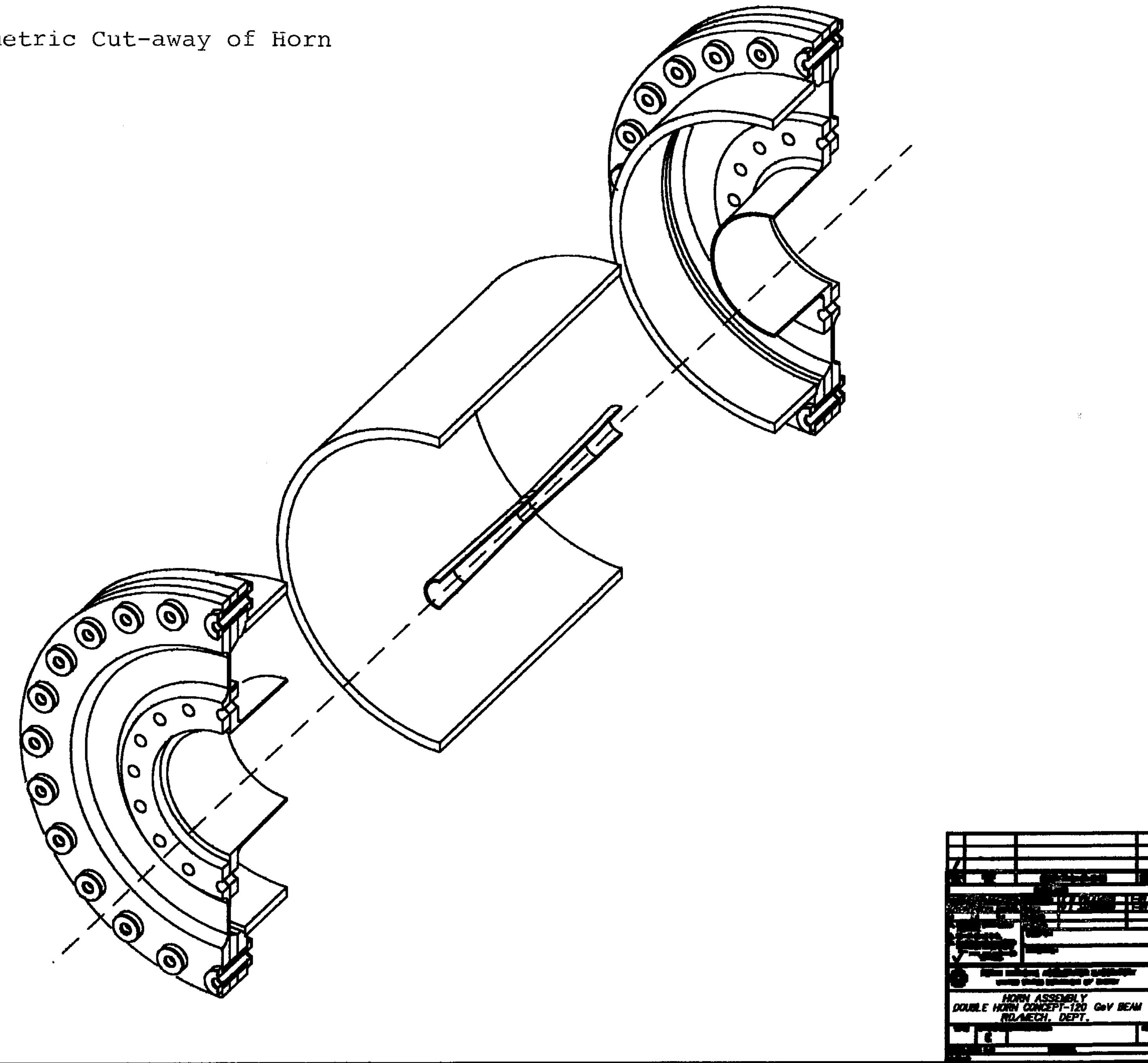
Acknowledgments

I would like to thank Bob Wonds, James Crab, and Ang Lee for their assistances on ANSYS, and Anthony Malensek, Ernie Villesges, Age Visser, W. Gajewski, and Luca Marinelli for their ideas and useful conversations, and especially Anne Jaeckel for helping me editing this paper.

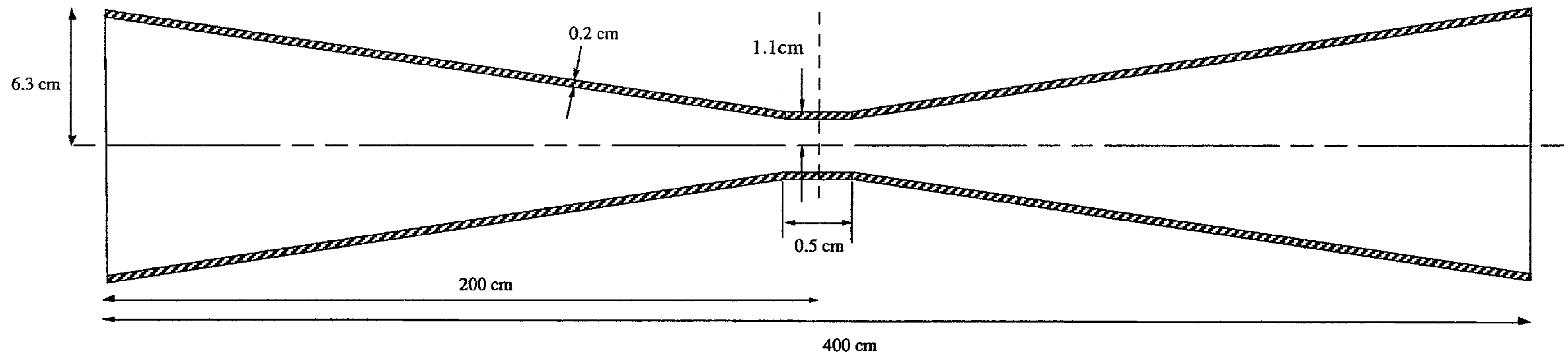
Table 1. Thermal Stress

	Thermal Stress as a function of coeff. of Thermal expansion at ends and supports											
Assuming HORN has initial suface temperature of 20 C												
	Thermal exp.	**Surface Temperature of the HORN, degree C, or Kelvin**										
	coef.of HORN	120	110	100	90	80	70	67	65	60	50	40
	, m/mC											
	2.36E-05	**Change of Temperature of HORN, degree C, or Kelvin**										
		dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre	dT, degre
		100	90	80	70	60	50	47	45	40	30	20
%of thermal	Expected,											
exp.coef.at	Therm.ex.coef	**Thermal Stress**										
ends&support	@ends,&suppt	**N/cm^2**										
100%	2.36E-05	0	0	0	0	0	0	0	0	0	0	0
95%	2.24E-05	1,480	1,332	1,184	1,036	888	740	695	666	592	444	296
90%	2.12E-05	2,959	2,663	2,368	2,072	1,776	1,480	1,391	1,332	1,184	888	592
85%	2.01E-05	4,439	3,995	3,551	3,107	2,663	2,220	2,086	1,998	1,776	1,332	888
80%	1.89E-05	5,919	5,327	4,735	4,143	3,551	2,959	2,782	2,663	2,368	1,776	1,184
75%	1.77E-05	7,398	6,659	5,919	5,179	4,439	3,699	3,477	3,329	2,959	2,220	1,480
70%	1.65E-05	**8,878**	**7,990**	7,103	6,215	5,327	4,439	4,173	3,995	3,551	2,663	1,776
65%	1.53E-05	***10,358***	*9,322*	**8,286**	7,251	6,215	5,179	4,868	4,661	4,143	3,107	2,072
60%	1.42E-05	11,838	***10,654***	*9,470*	**8,286**	7,103	5,919	5,564	5,327	4,735	3,551	2,368
55%	1.30E-05	13,317	11,986	***10,654***	*9,322*	7,990	6,659	6,259	5,993	5,327	3,995	2,663
50%	1.18E-05	14,797	13,317	11,838	***10,358***	**8,878**	7,398	6,955	6,659	5,919	4,439	2,959
45%	1.06E-05	16,277	14,649	13,021	11,394	*9,766*	**8,138**	7,650	7,324	6,511	4,883	3,255
40%	9.44E-06	17,756	15,981	14,205	12,429	***10,654***	8,878	**8,345**	7,990	7,103	5,327	3,551
35%	8.26E-06	19,236	17,312	15,389	13,465	11,542	*9,618*	9,041	**8,656**	7,694	5,771	3,847
30%	7.08E-06	20,716	18,644	16,573	14,501	12,429	10,358	*9,736*	*9,322*	**8,286**	6,215	4,143
25%	5.90E-06	22,195	19,976	17,756	15,537	13,317	***11,098***	10,432	9,988	8,878	6,659	4,439
20%	4.72E-06	23,675	21,308	18,940	16,573	14,205	11,838	***11,127***	***10,654***	*9,470*	7,103	4,735
15%	3.54E-06	**25,155**	22,639	20,124	17,608	15,093	12,577	11,823	11,320	10,062	7,546	5,031
10%	2.36E-06	**26,635**	23,971	21,308	18,644	15,981	13,317	12,518	11,986	***10,654***	7,990	5,327
5%	1.18E-06	**28,114**	25,303	22,491	19,680	16,869	14,057	13,214	12,651	11,246	**8,434**	5,623
0%	0.00E+00	**29,594**	**26,635**	23,675	20,716	17,756	14,797	13,909	13,317	11,838	8,878	5,919
	Buckling Stress(hoop) @ Supports = 8,500 N/cm^2						**NOTE:**	Yield Stress = 27,500 N/cm^2 @ 0~100 C				
	Fatique Stress Limit (Mechanical) @ 5.0E8 cycle = 9,600 N/cm^2							Yield Stress = 26,700 N/cm^2 @ 110 C				
	Buckling Stress (hoop) @ ends = 11,000 N/cm^2							Yield Stress = 25,500 N/cm^2 @ 120 C				
								Yield Stress = 24,100 N/cm^2 @ 130 C				
								Yield Stress = 22,700 N/cm^2 @ 140 C				

Fig.1: Isometric Cut-away of Horn



The sketch is not drawn in scale

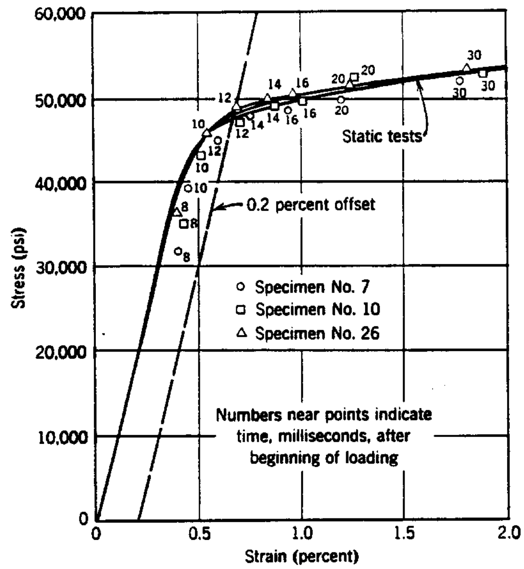


Outer radius at neck = 1.1 cm
Outer radius at end = 6.3 cm
Constant wall thickness = 0.2 cm
Tube section at neck with length = 0.5 cm

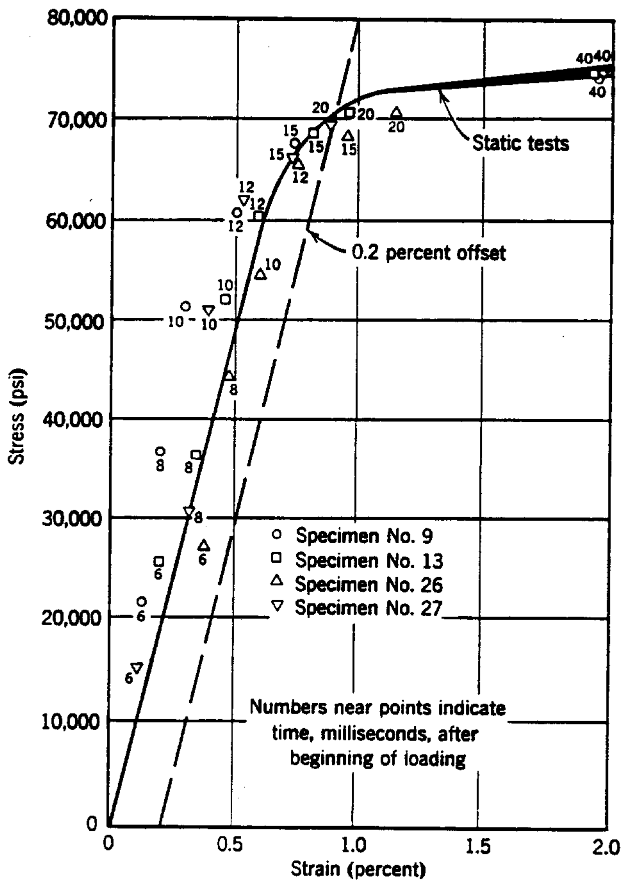
Fig.2: Sketch of the HORN with dimensions in cm

the sketch here is the center piece, inner conductor, shown in Fig.1

Fig.3:



Stress versus strain for three rapid load tests on 2024–T4 aluminum alloy. [After Clark and Wood (9), courtesy of The American Society for Testing and Materials.]



Stress versus strain for four rapid load tests on 7075–T6 aluminum alloy. [After Clark and Wood (9), courtesy of The American Society for Testing and Materials.]

Fig.4: **Compare critical stress using (finite tubes) with applied resultant stress (Von Mis), Excluding Thermal stress**

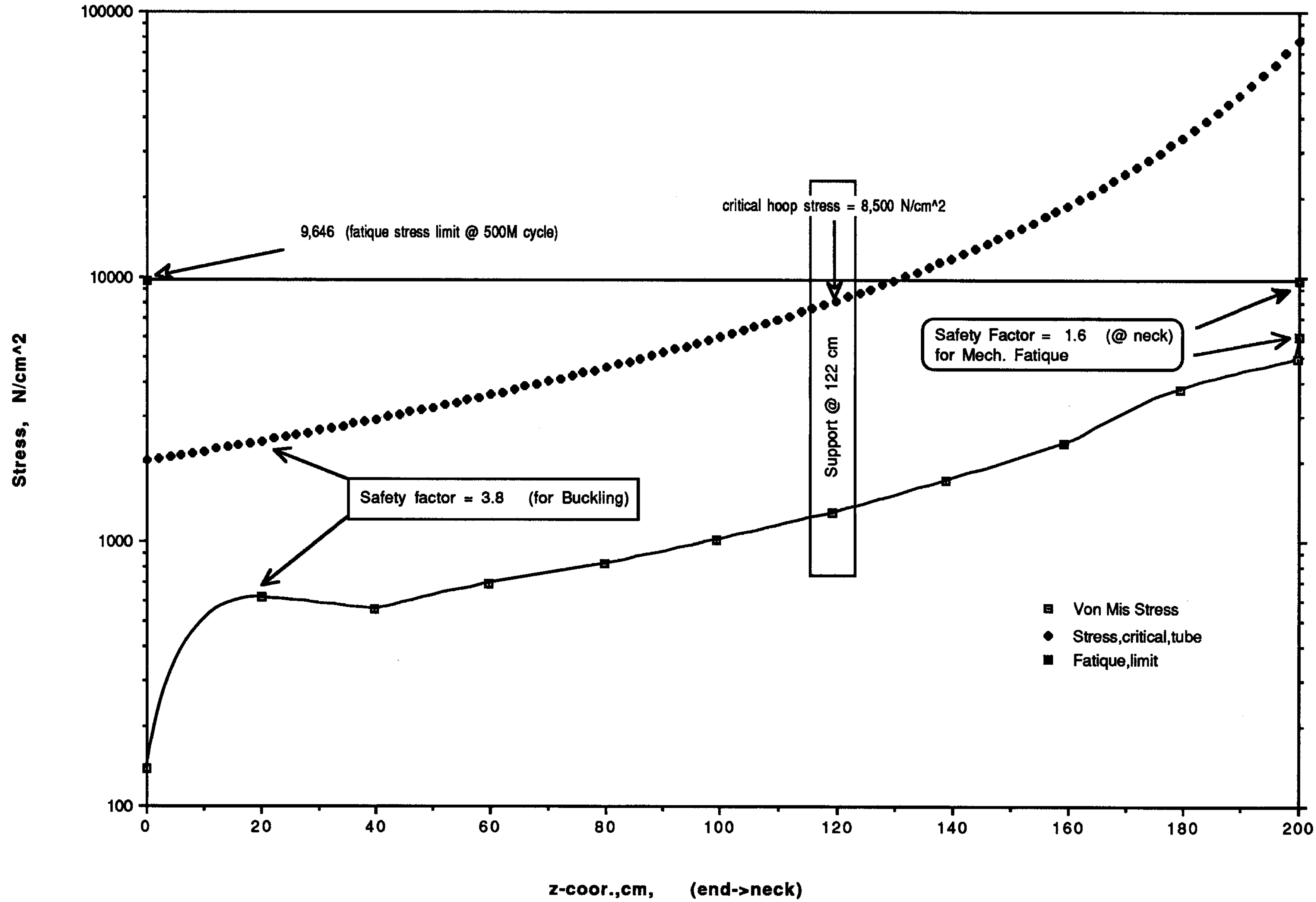


Fig.5: **Thermal Stress as a function of % of Thermal Expansion Coeff. at ends and supports @ 100 C**

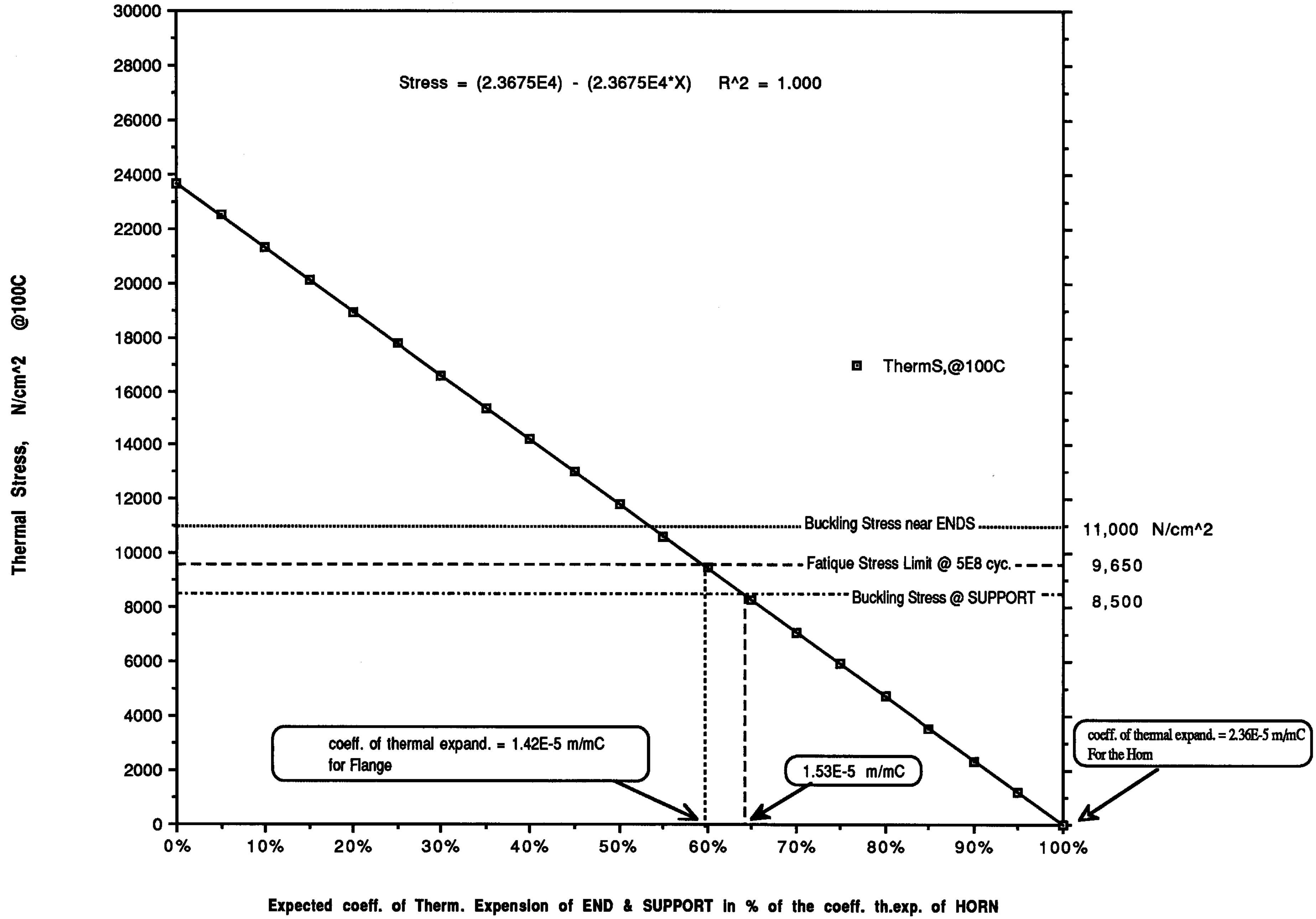


Fig.6: Fatigue Curve

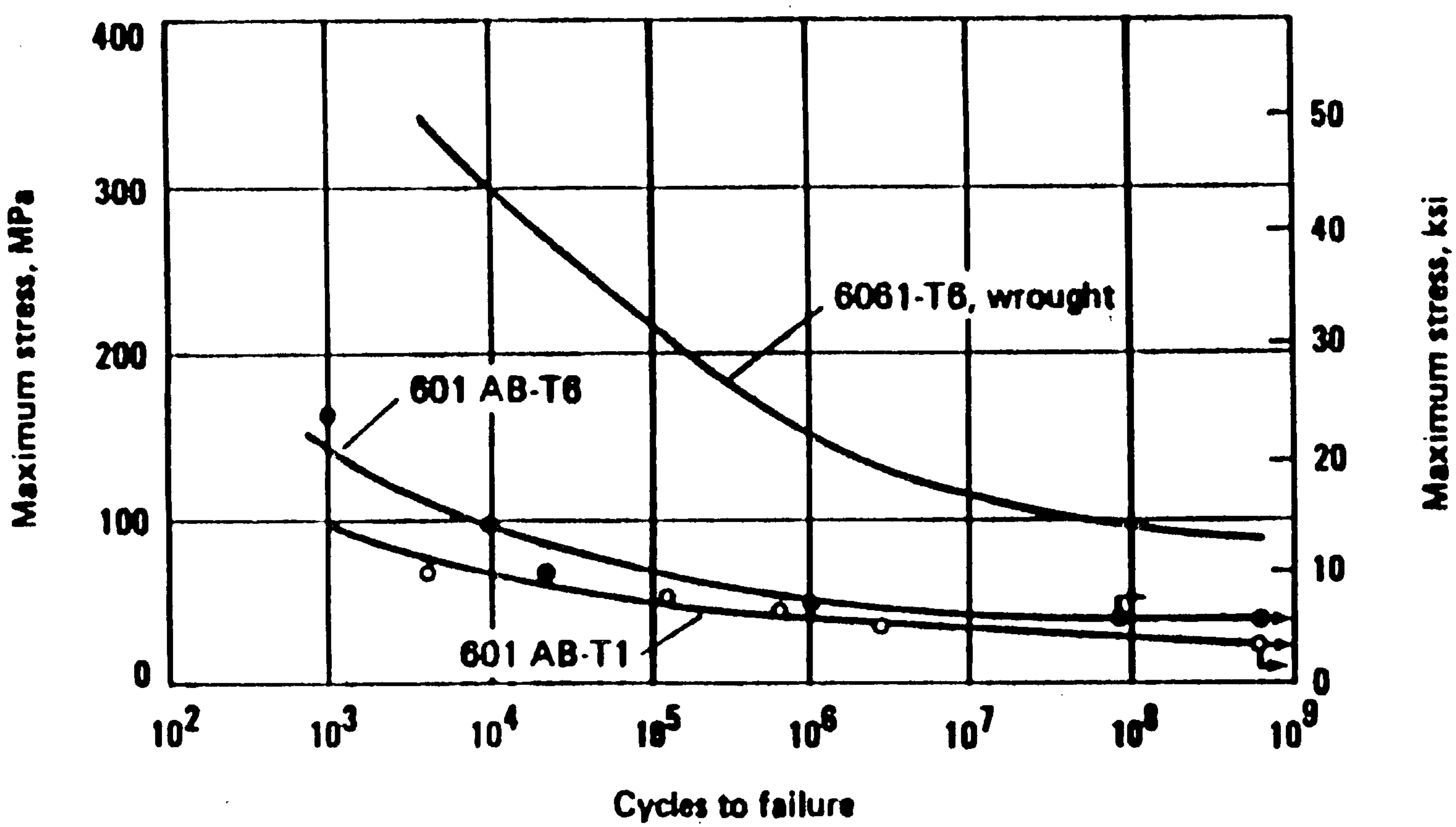


Fig.7. Von Mises Stress (ANSYS)

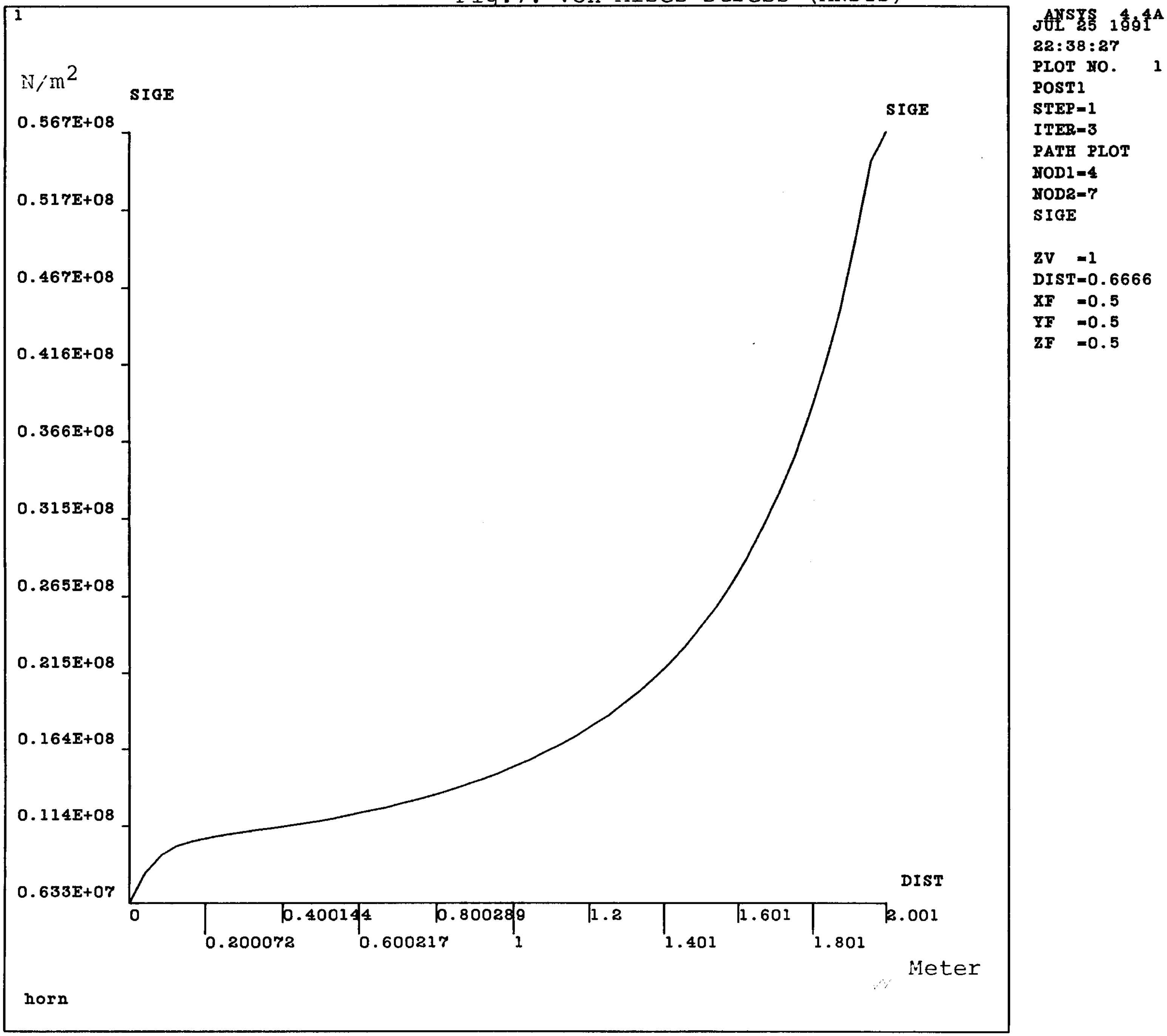


Fig.8: **Von Mises Stress distribution along the Horn in axial direction obtained from (PATRAN)**

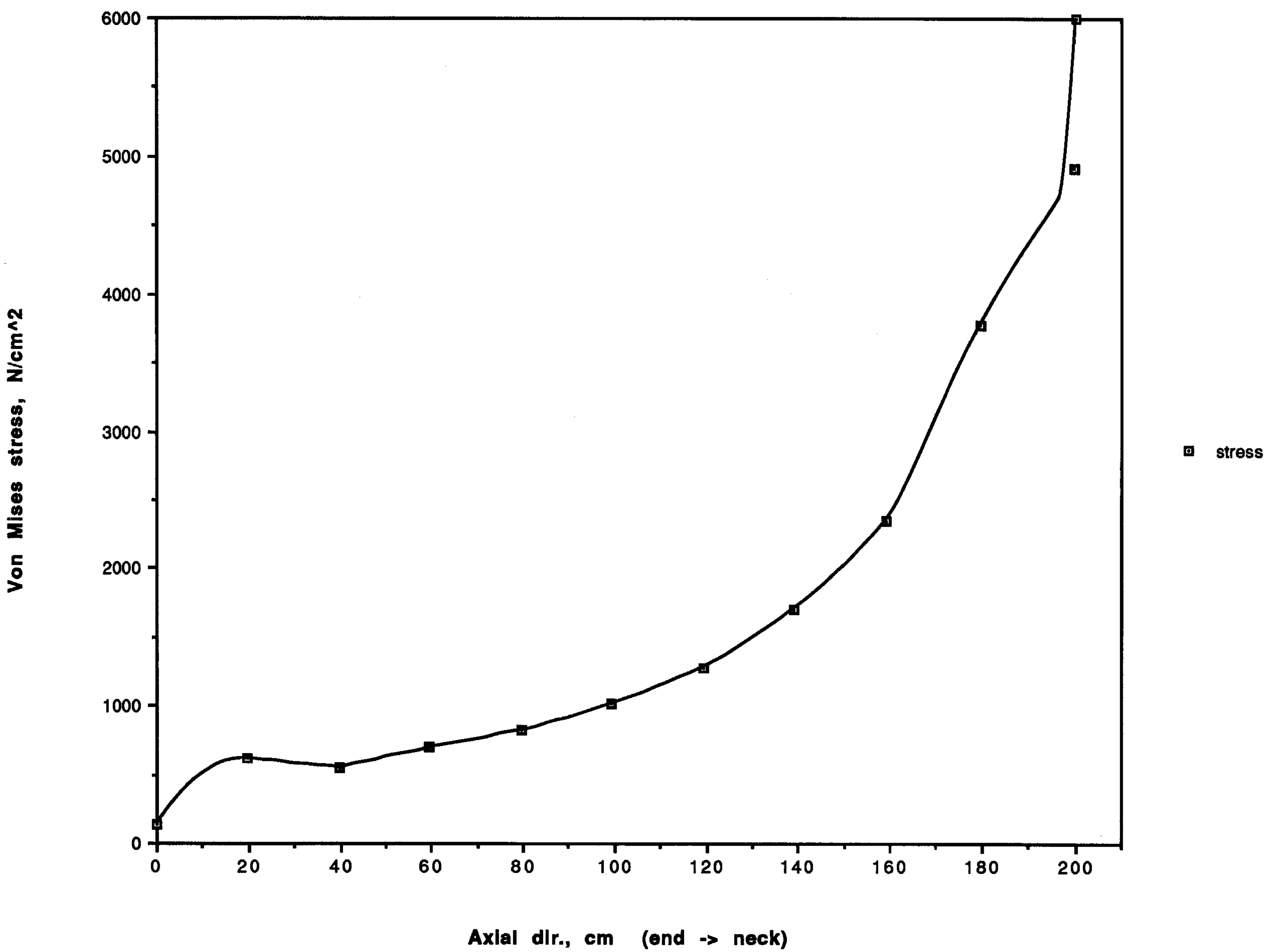
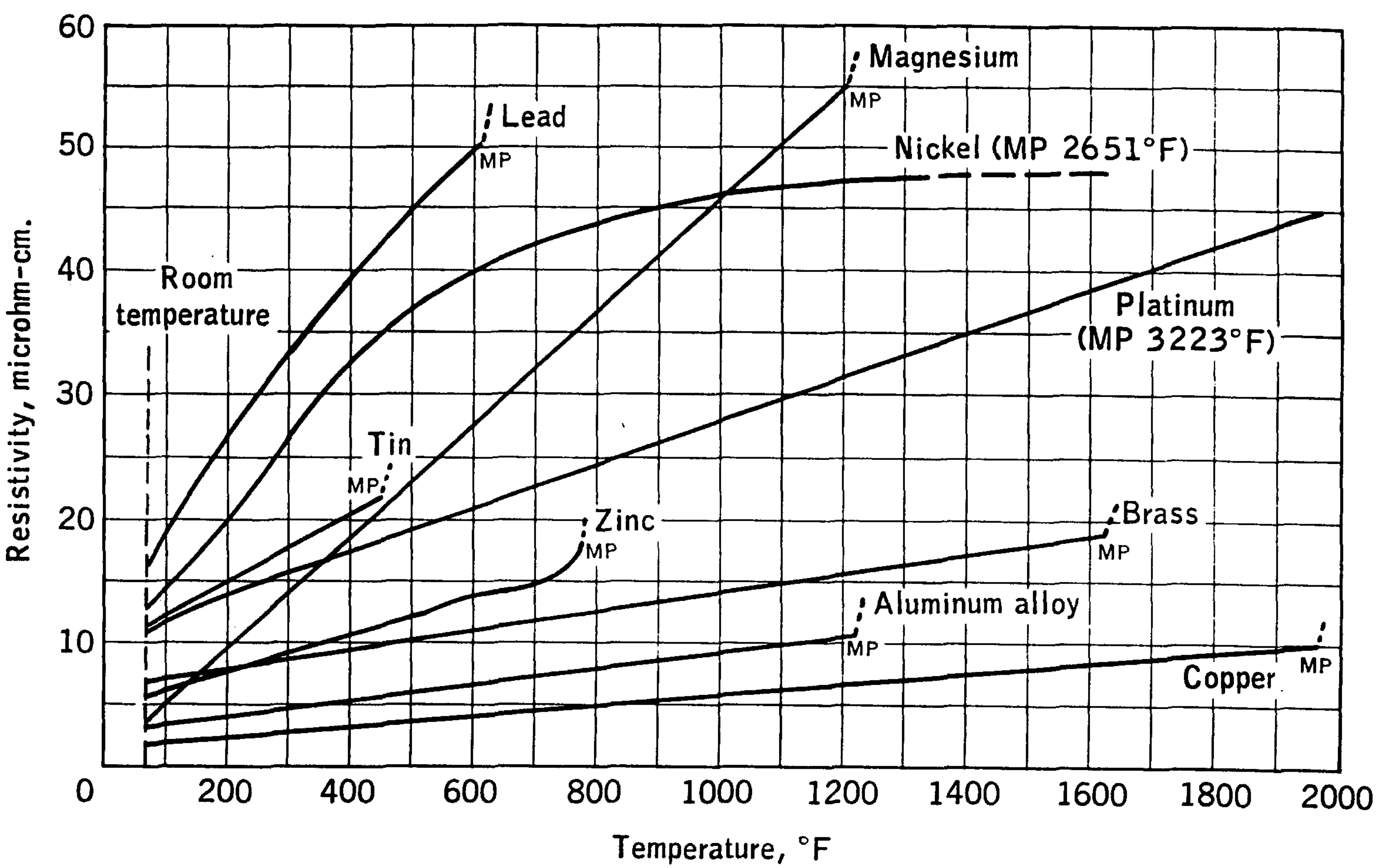


Fig.A3.1: Resistivity vs. temperature (21)



A1. Properties of the HORN [5,8,12,13,18]

material: Aluminum Alloy 6061-T6

Mechanical Properties:

	psi, lb/in^2	Pa, N/m^2	N/cm^2
Tensile Strength,min.	42,000	289M	28,900
Yield Strength,min.	35,000	241M	24,100
Modulus of Elasticity, E	10E+6	68.9G	6.89M
Modulus of Rigidity, G	3.76E+6	25.9G	25.9M
Fatique Stress Limit @ 5E8 cycle	14,000	96.46M	9,646

Poisson's ratio	0.334	isotropic

Physical Properties:

Density	0.098 lb/in^3	2.70 Mg/m^3
Specific gravity	2.7	
Specific Heat	896 J/kg/C	896 J/kg/Kelvin
Thermal Conductivity @25C	96.5 Btu/ft/hr/F	167 w/m/C
Thermal Expansion Coeff. @ 20~100 C	1.31E-5 in/in/F	2.36E-5 m/m/C or cm/cm/C
approx. melting range	1080~1200 F	582~649 C

Calculated Physical Properties:

Volume	1.81E-3 m^3
Surface Area	0.80 m^2
Mass	4.888 kg

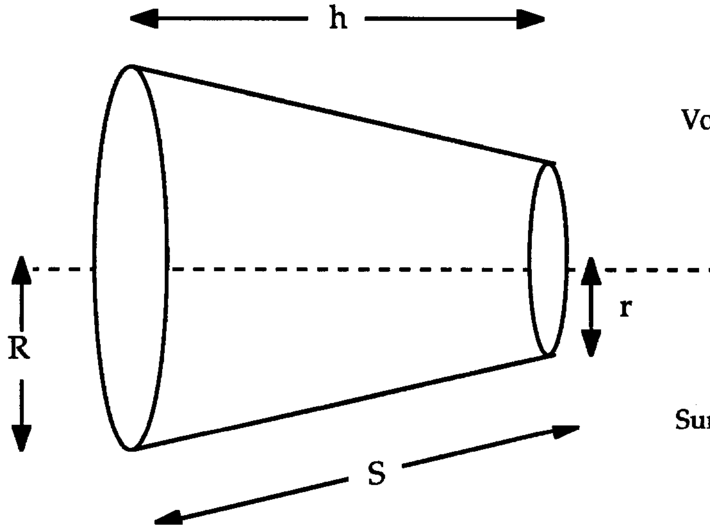
Electrical Properties:

Electrical Conductivity @20 C	43 % of Copper
Electrical Resistivity @20 C	4.0E-6 ohm*cm

Calculated Electrical/Thermal properties:

Resistance of HORN @ 20 C	4.40E-4 $\pm$ 0.3% ohm
Power Dissipation by 10KA rms @ 20 C	44k watts
Change of Temp. of HORN by 1 pulse of current	20 C

A2. Calculation of physical properties [14]

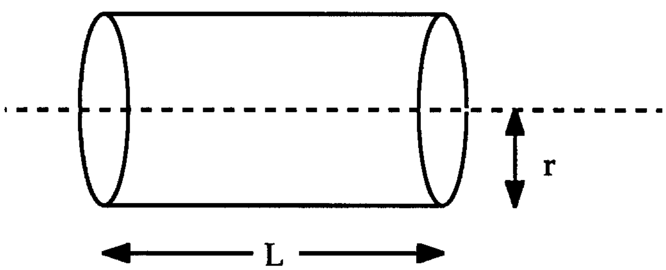


Right Circular Cone

$$\text{Volume} = \frac{pi*h}{3}\,(R^2 + r^2 + R*r)$$

$$S = [h^2 + (R-r)^2]^{1/2}$$

$$\text{Surface Area} = pi*S\,(R + r)$$



Cylinder

$$\text{Volume} = pi*r^2*L$$

$$\text{Surface Area} = 2*pi*r*L$$

Horn has thickness of 2mm everywhere

Let outer radius at ends = R_o = 6.3 cm outer radius at support = r_o = 1.1 cm

inner radius at ends = R_i = 6.1 cm inner radius at support = r_i = 0.9 cm

each cone has length, L_{cone} = 199.75 cm

Length of cylinder at neck, $L_{cylinder}$ = 0.5 cm

$$\text{Volume} = 2\{\ \frac{\pi L_{cone}}{3}\left[\left(R_o^2 + r_o^2 + R_o r_o\right) - \left(R_i^2 + r_i^2 + R_i r_i\right)\right] + \pi\left(r_o^2 - r_i^2\right)L_{cylinder}\ \} = 1.81\text{x}10^{-3}\ \text{m}^3$$

$$\text{Outside Surface Area} = \left[\pi(R_o + r_o)\sqrt{L_{cone}^2 + (R_o - r_o)^2}\right] + \left(2\pi r_o L_{cylinder}\right) = 0.80\ \text{m}^2$$

$$\text{Mass of HORN} = \text{volume} * \text{density} = \left(1.81\text{x}10^{-3}\ \text{m}^3\right)\left(2.7\text{x}10^6\ \text{g}/\text{m}^3\right) = 4.888\ \text{kg}$$

A3. Calculations for Electrical/Thermal properties [6]

A3.1 Calculating the resistance of the HORN

$$\text{Resistance} = \text{resistivity x} \frac{\text{length of a section}}{\text{cross sectional area of the section}} \qquad \textit{unit:}\left[\Omega = \Omega \bullet cm \times \frac{cm}{cm^2}\right]$$

Since HORN has conical shape, the cross sectional area would vary. The conical angle of the HORN is only ~ 1.5 degree, so the calculated resistance of the HORN would be accurate if the HORN is divided into small tube elements.

Using the convergence test in finite element method, it is found that the resistance calculated by dividing the HORN into 100~200 pieces of small tubes would be a good approximation with $\pm$ 0.8 % error. Actually the result converges when the HORN is divided into ~ 140 small tube elements with only $\pm$ 0.23 % error. This is shown is Fig.A3.2.

Shown in Table A3.1, a spread-sheet program Excel was used to calculate the resistance of the HORN. The procedures and equations are described below.

(1) Due to symmetry, half of the HORN with conical shape was divided into 70 tube elements.

(2) At the neck of the HORN, which is a tube section with length of 0.5cm, the resistance is calculated separately.

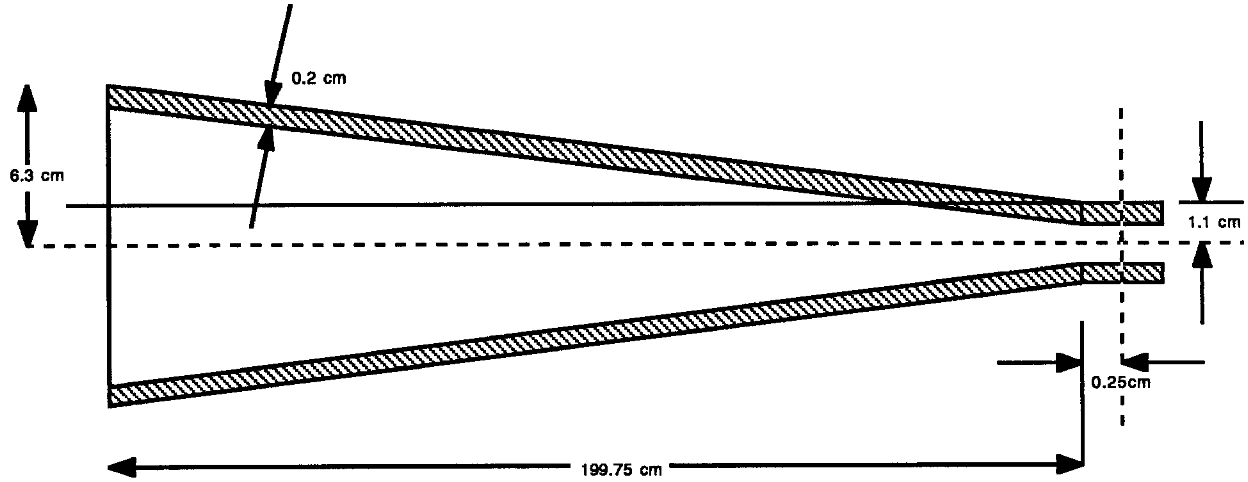
(3) Use Excel to calculate the resistance of the HORN, and the equations are given in the following page, as well as a sample calculation.

From Figure A3.1, the resistivity of aluminum alloy increases linearly as temperature of the material increases. Since resistance is linearly proportional to resistivity, resistance is also linearly proportional to temperture.

From chemistry and physics, we learned that the resistivity and resistance have the value of zero at 0 degree Kelvin or -273 Celsius. We can assume a linear slope for resistivity vs. temperature from 0 Kelvin to room temperature, about 293 K or 20 degree Celsius. Using Fig.A3.1, and convert the temperature from F to Kelvin, for alluminum alloy the resistivity at 350 R or 450 K is 5 microhm*cm and at 1130 F or 883 K, the resistivity is 10 microhm*cm. The slope from these two points is 1.15E-8 ohm*cm/K. If we take the points at 450K with 5 microhm*cm and the point at 0 K with 0 microhm*cm, the slope between these two points is 1.11E-8 ohm*cm/K. Then the difference between these two slope is 3.5 %. This value is the uncertainty or % error for assuming linear slope of resistivity vs. temperature from 0 K to 450 K.

Similarly, the resistance of the HORN, at 20 C is calculated from Excel to be 4.40E-4 ohm and at -273 C or 0 K, the resistance is 0 ohm, so the slope is (4.40E-4/293C or K) = 1.50E-6 ohm/C with $\pm$ 3.5 %. The values tabulated using this slope has uncertainty of $\pm$ 3.5 % as well.[Fig.A3.3,A3.4]

The sketch is not drawn in scale.



$$\text{Resistance} = \text{resistivity x} \frac{\text{length of a section}}{\text{cross sectional area of the section}} \qquad unit: \left[\Omega = \Omega \bullet cm \times \frac{cm}{cm^2} \right]$$

$$\text{resistivity @ } 20^\circ C = 4.0 \times 10^{-6}\ \Omega \bullet cm$$

$$\text{length of a section} = \frac{199.75}{70} cm \approx 2.85 cm$$

outer radius at end = 6.3 cm

outer radius at neck = 1.1 cm

length of conical section (≈ half of the HORN) = 199.75 cm

length of the tube section at neck = 0.5 cm

wall thickness = 0.2 cm everywhere

outer radius of the section

$$= (\text{outer } R_{neck}) + \left(\frac{\text{outer } R_{end} - \text{outer } R_{neck}}{\text{length of conical section}} \right) (\text{length of divided section})(N)$$

$N = 1,2,3,4,5,......70$ $\qquad$ $N = 1$ for the section nearest to neck

$$for\ N = 3, \qquad outer\ radius = (1.1cm) + \left(\frac{6.3cm - 1.1cm}{199.75cm}\right)\left(\frac{199.75cm}{70}\right)(3) \approx 1.32cm$$

$$inner\ radius = outer\ radius - thickness = 1.32cm - 0.2cm = 1.12cm$$

$$\text{cross sectional area} = \pi\left(outer\ radius^2 - inner\ radius^2\right)$$

$$= \pi\left(1.32cm^2 - 1.12cm^2\right) \approx 1.53cm^2$$

$$\text{resistance of the section} = \left(4.0 \times 10^{-6}\ \Omega \bullet cm\right) \times \left(\frac{2.85cm}{1.53cm^2}\right) \approx 7.45 \times 10^{-6}\Omega$$

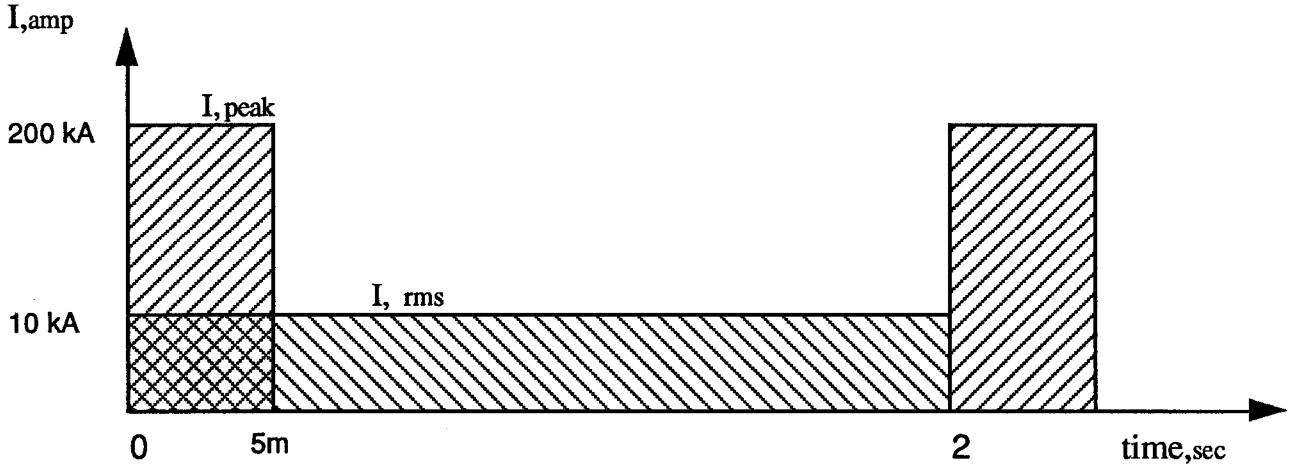
because of symmetry,

$$\text{resistance of HORN} = 2 \times \left(\sum_{N=1}^{70} \text{resistance of section}_{,N}\right) + \text{ resistance of tube section at neck}$$

$$\text{resistance of tube section at neck} = \left(4.0 \times 10^{-6}\ \Omega \bullet cm\right) \times \left(\frac{0.5cm}{\pi\left(1.1^2 - 0.9^2\right)cm^2}\right) \approx 1.59 \times 10^{-6}\Omega$$

A3.2 Calculation of Power dissipation [6]

Since the exact wave form has not been decided yet, using square wave to approximate the power dissipation is reasonablily accurate. [with reference to Age Visser of RD/EE]



given: peack current $= I_{peak} = 2\times10^5\,amp = 200kA$

$$I_{rms} = I_{peak}\sqrt{\frac{time\ of\ 1\ pulse}{period}} = \left(2\times10^5\,amp\right)\sqrt{\frac{5\times10^{-3}\,\mathrm{sec}}{2.0\,\mathrm{sec}}} = 10{,}000A = 10kA$$

Power dissipation per one cycle or one period:

$$Power = I_{rms}^2 \times \text{Resistance of HORN}$$

at 20 °C , HORN resistance $= 4.40\times10^{-4}\Omega$

$$Power = (10{,}000A)^2\left(4.40\times10^{-4}\Omega\right) = 44{,}000\ watts = 44kW$$

Power $\propto$ Resistance $\propto$ surface temperature of the HORN

Power dissipation would increase with an increase of surface temperature of the HORN. A plot of Power vs. surface temperature is shown in figure A3.5.

A3.3 Calculation for change of surface temperature of the HORN by 1 pulse of current [6]

From A3.2 the power dissipation per cycle = 44 kW.
The amout of energy generated in a cycle (or 2 seconds) is :

$$Energy = Power\ dissipation \times time$$

unit:

$$[Joules] = \left[\frac{J}{\sec}\right] \times [\sec] \qquad\qquad [watt] = \left[\frac{J}{\sec}\right]$$

The amount of energy needed to raise the temperature of the HORN by 1 degree Celsius is given in the physical properties of Aluminum 6061-T6, the *Specific Heat.*

At 20 C, Specific Heat = 896 J/kg/C or 896 J/kg/Kelvin

The mass of the HORN = 4.888 kg [A._]

The change of temperature by 1 pulse of 200kA peak current or 1 period(2 sec) of 10 kA rms current is :

$$\Delta T = \frac{Power \times time}{mass\ of\ HORN \times Specific\ Heat} \qquad unit: \quad [^{\circ}C] = \frac{\left[{}^{J}\!/_{s}\right][s]}{[kg]\left[{}^{J}\!/_{kg \bullet^{\circ} C}\right]}$$

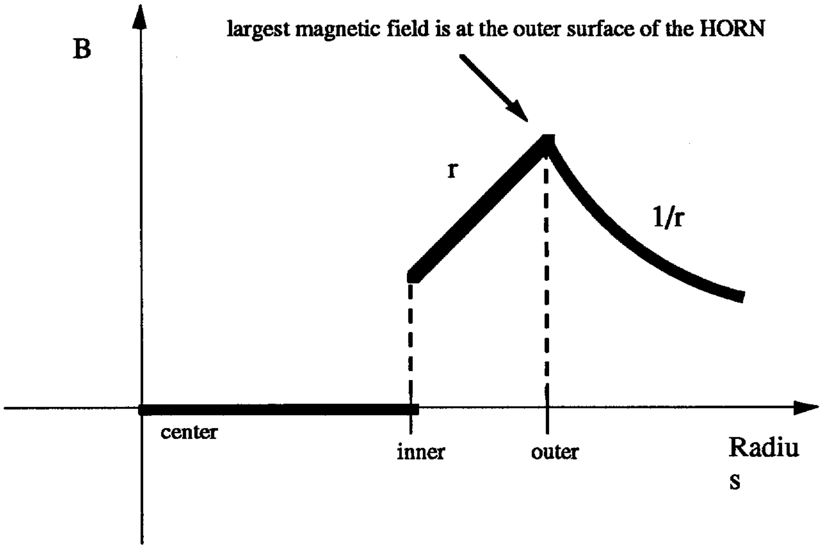
$$\Delta T = \frac{\left(44{,}000\,{}^{J}\!/_{\sec}\right)(2\sec)}{(4.888kg)\left(896\,{}^{J}\!/_{kg \bullet{}^{\circ}C}\right)} = 20.1^{\circ}C \approx 20\ ^{\circ}C$$

$$\Delta T \propto \text{Power} \propto \text{Resistance} \propto \text{surface temperature of the HORN}$$

Although Specific Heat does vary with temperature, the change of value would not be significant in the operational temperature of the HORN which is 0 ~ 100 C. Therefore the value of the specific heat is assumed to be constant. The uncertainty in the value of specific heat has been taken into account in the plot of dT vs. surface termperature where 0.5% error is added to the uncertainty in resistance, 3.5%, with net uncertainty of 4%.

A plot of dT vs. surface temperature is shown in figure A3.6.

A4.1 Magnetic field [13,15]



magnetic field of the HORN as a function of its radius

B = magnetic field

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\mu_0 / 4\pi = 10^{-7} \ \frac{N}{A}$$

Simplifying the equation

for

I = current is measured in kA

r = distance from axis of azimuthal symmetry to outer radius r, is measured in cm

B = magnetic field is obtained in kGauss

$$B = \frac{I}{5r}$$

given: $I = 200$ (kA)

$r = 1.1$ (cm) at the neck

obtained: $B = \dfrac{200}{5 \times 1.1} = 36.4$ (kGauss)

A4.2 Magnetic Pressure [16,17]

P = magnetic pressure
B = magnetic field
I = electric current
r = radius
μ_o = a constant
hydrostatic magnetic pressure

$$P = \frac{B^2}{2\mu_o} \quad , B = \frac{\mu_o I}{2\pi r}$$

$$\frac{\mu_o}{4\pi} = 10^{-7} \frac{N}{A}$$

yields,

$$P = \frac{636.62\ N}{r^2}$$

A5. Sample Calculation of Stresses [2]

A5.1 Buckling by external pressure for tube:

equations taken from [ref.2,eqn.9.246~9.249]

$P_{critical}$ = critical pressure

E = Elastic Modulus = $6.89\text{x}10^6\ \mathrm{N/cm^2}$

t = tube wall thickness = 0.2 cm

$$P_{critical} = \frac{Et^3}{4R^3(1-\upsilon^2)}$$

υ = poisson's ratio = 0.334

R = mid - wall tube radius

at neck, R = (1.1 - 0.1)cm = 1.0 cm

$$P_{neck,critical} = \frac{(6.89\text{x}10^6\ \mathrm{N/cm^2})(0.2\ \mathrm{cm})^3}{4(1.0cm)^3(1-0.334^2)} = 15{,}510\frac{N}{cm^2}$$

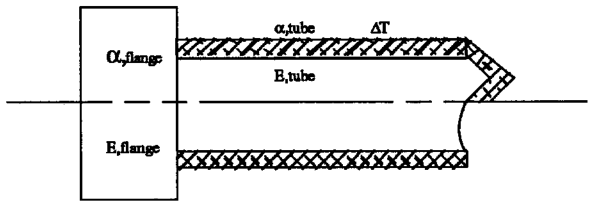
Critical Stress is defined by hoop stress

σ_{critical} = critical stress σ_{hoop} = hoop stress

$$\sigma_{\text{critical}} = \frac{P_{critical}R}{t} = \sigma_{hoop}$$

$$\sigma_{critical} = \frac{E}{1-\upsilon^2}\left(\frac{t}{2R}\right)^2 = \frac{(6.89\text{x}10^6\ \mathrm{N/cm^2})}{(1-0.334^2)}\left(\frac{0.2cm}{2(1.0cm)}\right)^2 = 77{,}551\frac{N}{cm^2}$$

A5.2 Thermal Stress [2]



Thin tube longitudinally connected to rigid section

equations taken from [ref.2,pg.918]

$\sigma_{thermal}$ = Thermal Stress

ΔT = change of temperature in °C or °*Kelvin*

α_{tube} = coefficient of thermal expansion = $2.3\text{x}10^{5}\ \dfrac{cm}{cm°C}$

α_{flange} = coefficient of thermal expansion of rigid flange or support

$$\sigma_{thermal} = 1.82E_{tube}(\Delta T)(\alpha_{flange} - \alpha_{tube})$$

Calculation for the Resistance of the HORN @ 20 C

Table A3.1

resistivity		HALF horn with		199.75	cm,is the length of	
at 20 C		#of div			the conical section	
4.00E-06		70				
ohm*cm		leng/div=,cm				
		2.85				
outer rad at						
neck= ,cm						
1.1						
outer rad at		outer radius	inner radius	x-sect area	resistance	
end= ,cm	N	cm,	cm,	cm^2	ohm	
6.3	1	1.17	0.97	1.35	8.46E-06	
thinckness=	2	1.25	1.05	1.44	7.91E-06	
0.2	3	1.32	1.12	1.54	7.43E-06	
cm,	4	1.40	1.20	1.63	7.00E-06	
	5	1.47	1.27	1.72	6.62E-06	
	6	1.55	1.35	1.82	6.28E-06	
1.5916E-06	7	1.62	1.42	1.91	5.98E-06	
is = resistance	8	1.69	1.49	2.00	5.70E-06	
of the mid.sec.	9	1.77	1.57	2.10	5.44E-06	The resistance
of the HORN	10	1.84	1.64	2.19	5.21E-06	of the HORN=
, ohm	11	1.92	1.72	2.28	5.00E-06	4.4027E-04
	12	1.99	1.79	2.38	4.80E-06	,ohm
	13	2.07	1.87	2.47	4.62E-06	
	14	2.14	1.94	2.56	4.45E-06	
	15	2.21	2.01	2.66	4.30E-06	
	16	2.29	2.09	2.75	4.15E-06	
	17	2.36	2.16	2.84	4.01E-06	
	18	2.44	2.24	2.94	3.89E-06	
	19	2.51	2.31	3.03	3.77E-06	
	20	2.59	2.39	3.12	3.65E-06	
	21	2.66	2.46	3.22	3.55E-06	
	22	2.73	2.53	3.31	3.45E-06	
	23	2.81	2.61	3.40	3.35E-06	
	24	2.88	2.68	3.50	3.26E-06	
	25	2.96	2.76	3.59	3.18E-06	
	26	3.03	2.83	3.68	3.10E-06	
	27	3.11	2.91	3.78	3.02E-06	
	28	3.18	2.98	3.87	2.95E-06	
	29	3.25	3.05	3.96	2.88E-06	
	30	3.33	3.13	4.06	2.81E-06	
	31	3.40	3.20	4.15	2.75E-06	
	32	3.48	3.28	4.24	2.69E-06	
	33	3.55	3.35	4.34	2.63E-06	
	34	3.63	3.43	4.43	2.58E-06	
	35	3.70	3.50	4.52	2.52E-06	
	36	3.77	3.57	4.62	2.47E-06	
	37	3.85	3.65	4.71	2.42E-06	
	38	3.92	3.72	4.80	2.38E-06	
	39	4.00	3.80	4.90	2.33E-06	
	40	4.07	3.87	4.99	2.29E-06	
	41	4.15	3.95	5.08	2.25E-06	
	42	4.22	4.02	5.18	2.20E-06	
	43	4.29	4.09	5.27	2.17E-06	
	44	4.37	4.17	5.36	2.13E-06	
	45	4.44	4.24	5.46	2.09E-06	
	46	4.52	4.32	5.55	2.06E-06	
	47	4.59	4.39	5.64	2.02E-06	
	48	4.67	4.47	5.74	1.99E-06	
	49	4.74	4.54	5.83	1.96E-06	
	50	4.81	4.61	5.92	1.93E-06	
1	51	4.89	4.69	6.02	1.90E-06	
	52	4.96	4.76	6.11	1.87E-06	
	53	5.04	4.84	6.20	1.84E-06	
	54	5.11	4.91	6.30	1.81E-06	
	55	5.19	4.99	6.39	1.79E-06	
	56	5.26	5.06	6.48	1.76E-06	
	57	5.33	5.13	6.58	1.74E-06	
	58	5.41	5.21	6.67	1.71E-06	
	59	5.48	5.28	6.76	1.69E-06	
	60	5.56	5.36	6.86	1.66E-06	
	61	5.63	5.43	6.95	1.64E-06	
	62	5.71	5.51	7.04	1.62E-06	
	63	5.78	5.58	7.14	1.60E-06	
	64	5.85	5.65	7.23	1.58E-06	
	65	5.93	5.73	7.32	1.56E-06	
	66	6.00	5.80	7.42	1.54E-06	
	67	6.08	5.88	7.51	1.52E-06	
	68	6.15	5.95	7.60	1.50E-06	
	69	6.23	6.03	7.70	1.48E-06	
	70	6.30	6.10	7.79	1.47E-06	

Fig.A3.2: **Convergence test using finite element method for calculation HORN resistance @ 20 C**

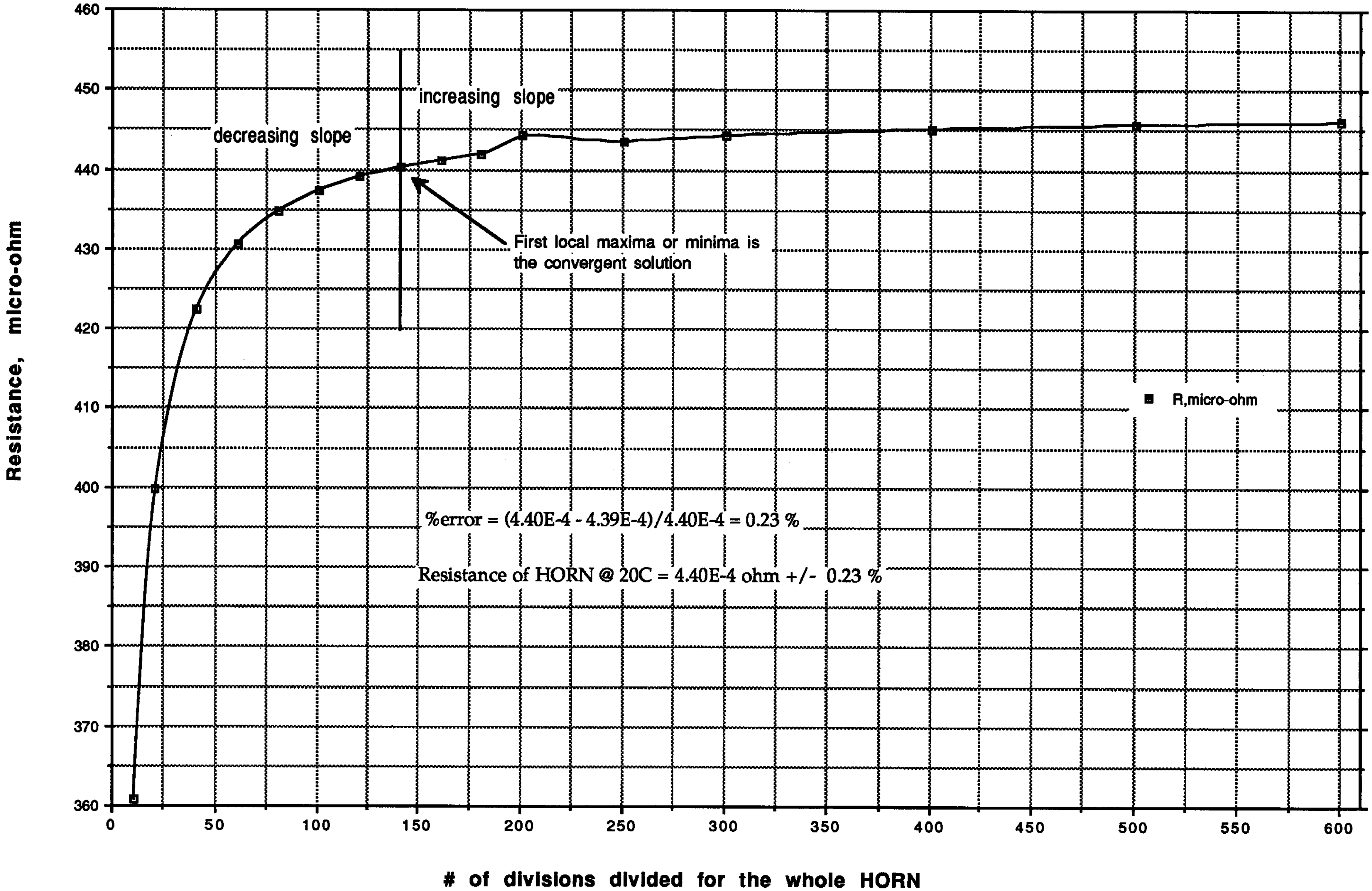
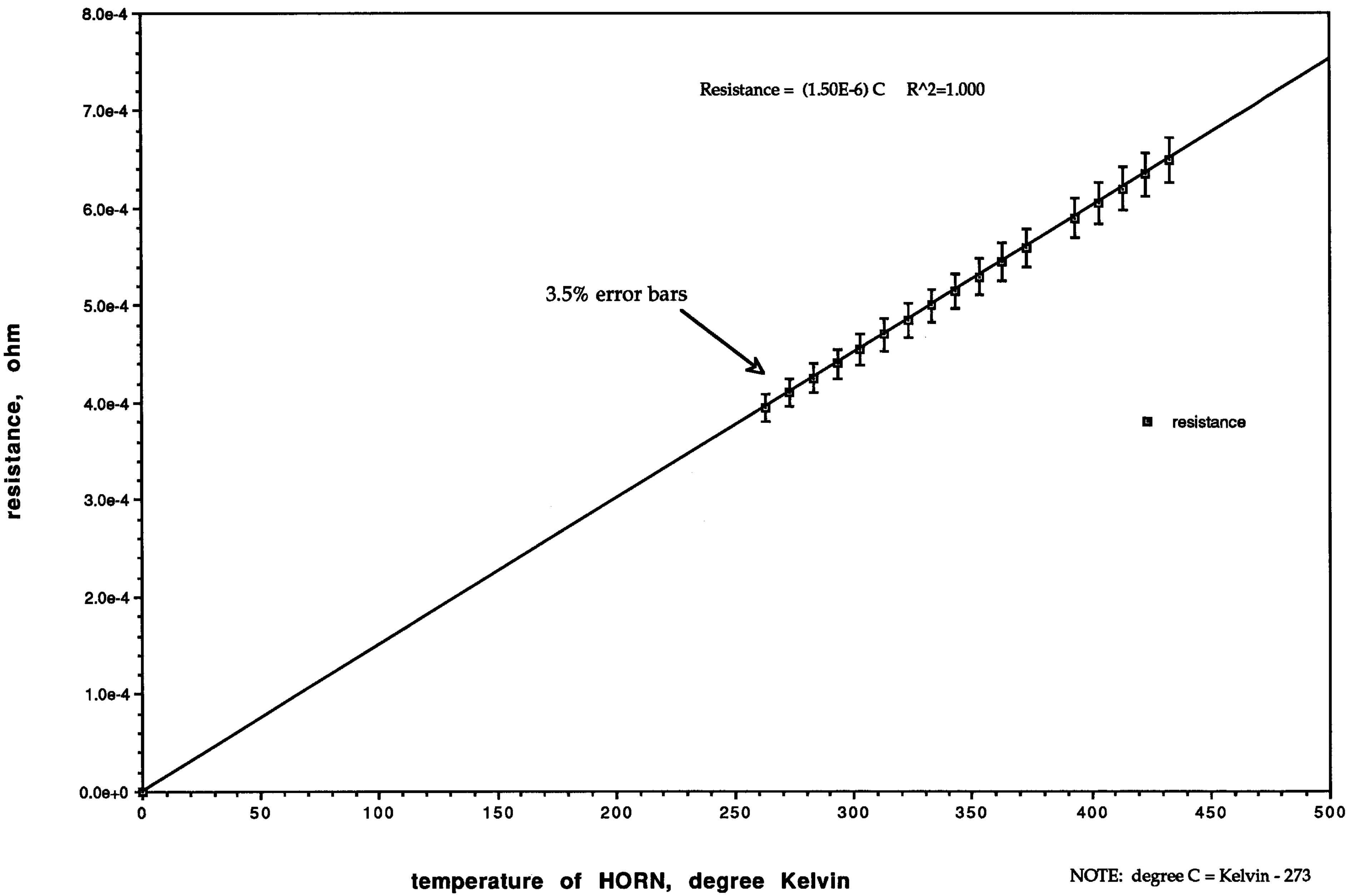
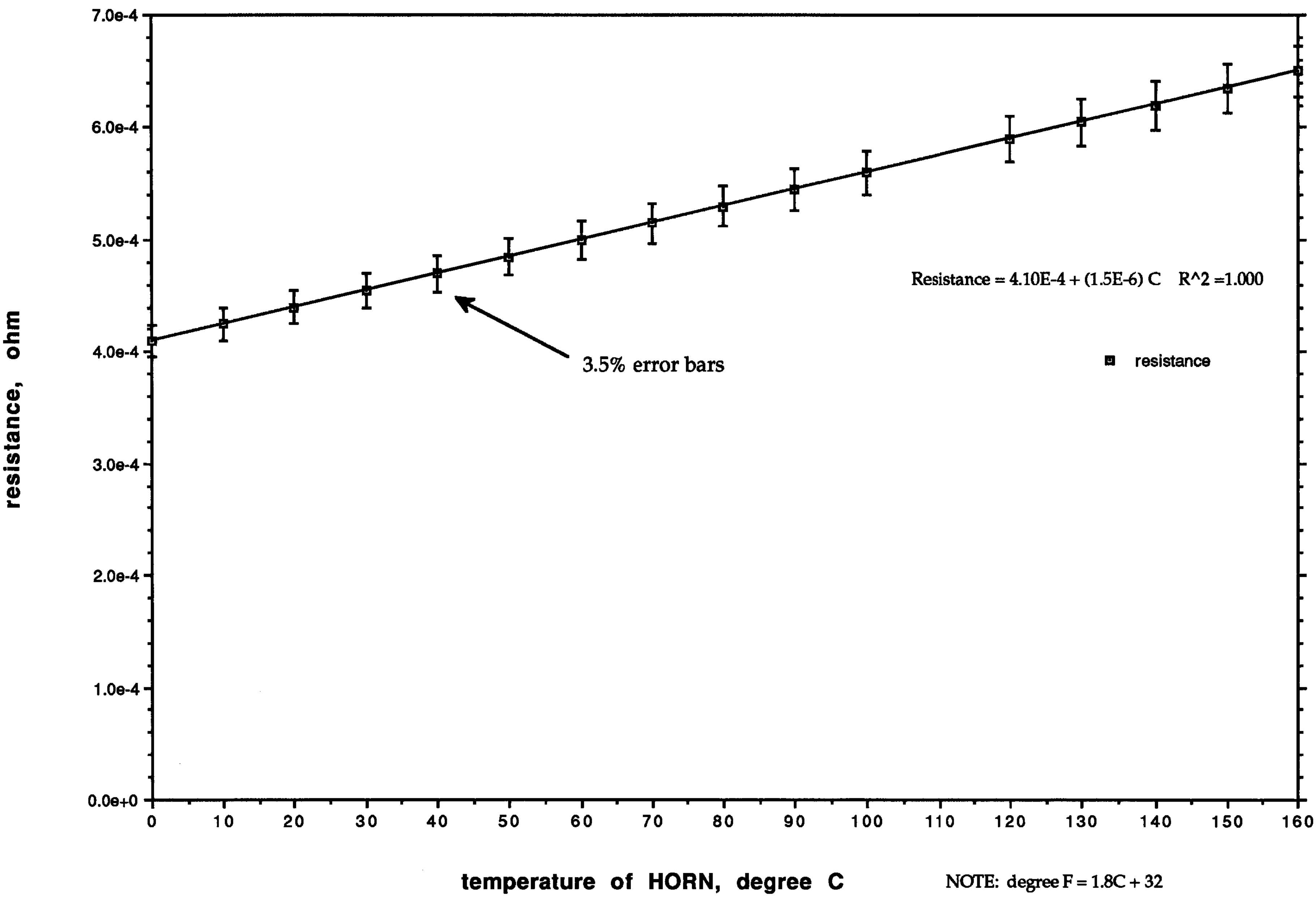


Fig.A3.3 **Resistance of the HORN as a function of Temperature in (Kelvin)**



NOTE: degree C = Kelvin - 273

Fig.A3.4: **Resistance of the HORN as a function of Temperature in (Celsius)**



NOTE: degree F = 1.8C + 32

Fig.A3.5: **Power dissipation per cycle by10KA of rms current as a fuction of surface temperature of the HORN**

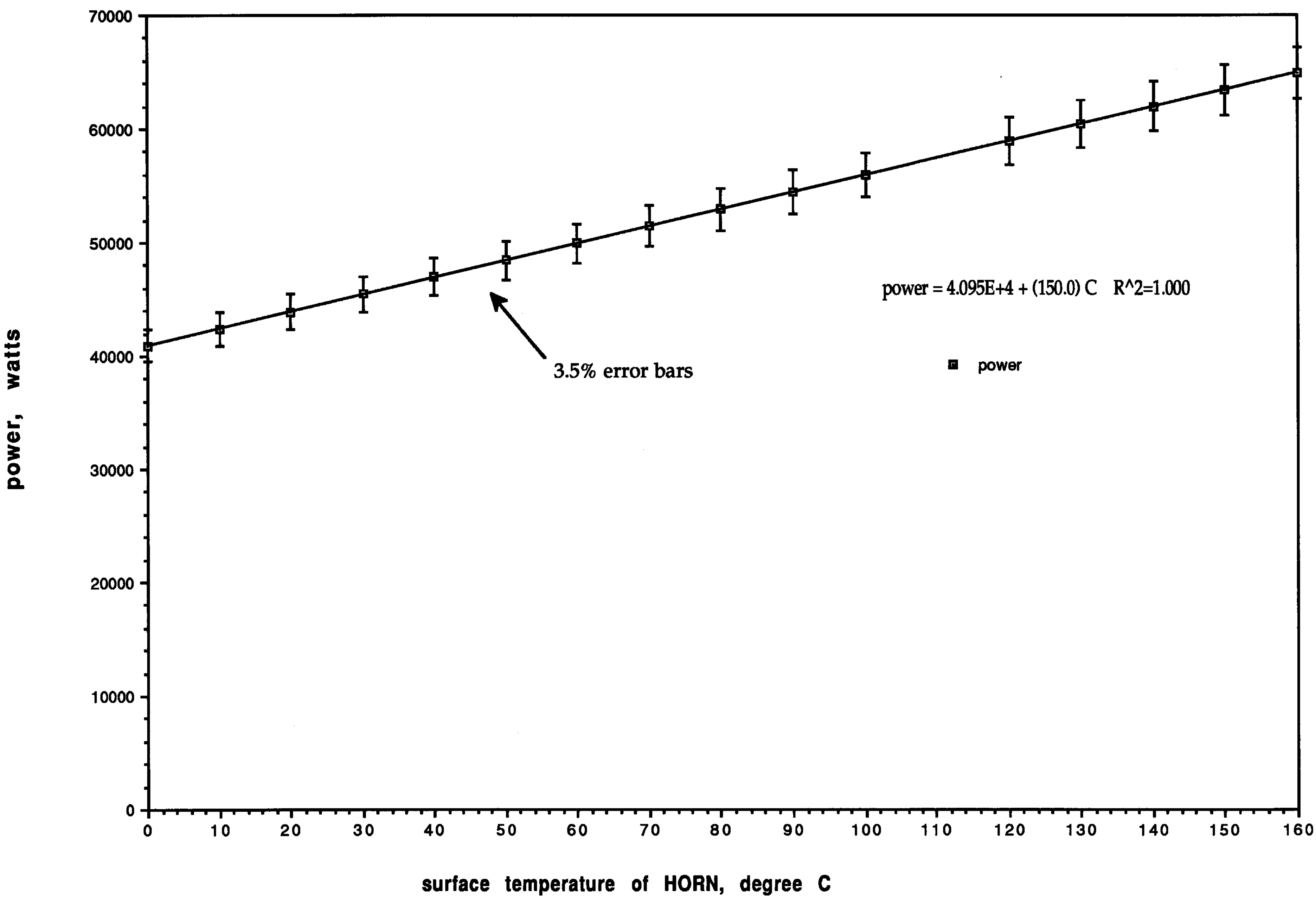


Fig.A3.6: **Change of surface temperature of the HORN by 1 pulse of 200kA peak current as a function of initial surface temperature**

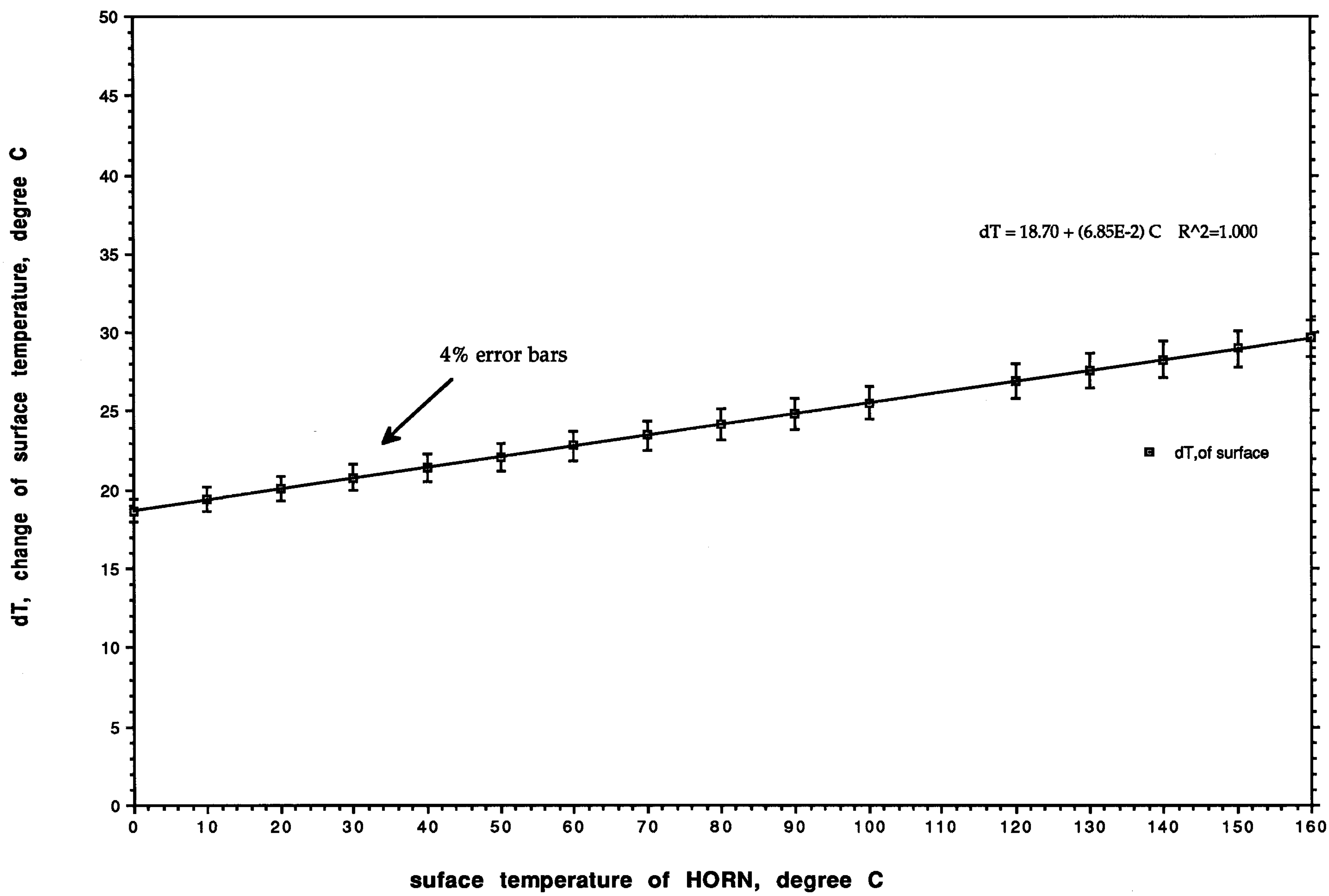
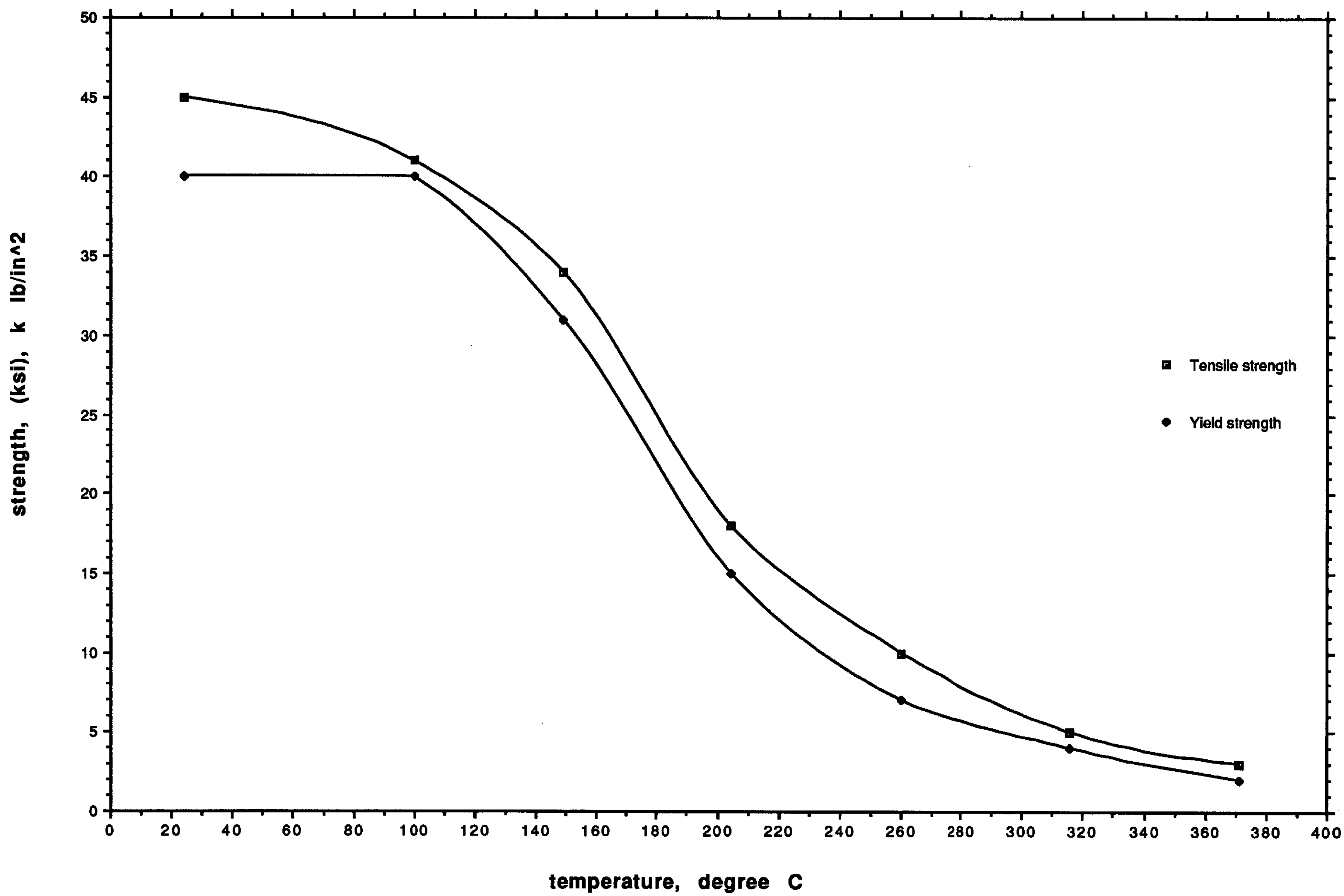


Fig.A5.1

Mechanical Strength of HORN, Al 6061 T-6, as a function of Temperature



Reference

1. S. Dye, W. Gajewski, J. Schultz, & R. Svoboda, Conceptual Design Report for Neutrino Physics after the Main Injector Upgrade.(1991)

2. J. Faupel & F. Fisher, Engineering Design, 2nd. ed.,Wiley & Sons, Inc. New York, N.Y.(1981)

3. Ansys 4.4A, Swanson Analysis System, Inc., Houston, PA, copyright 1991.

4. PATRAN4.2 plus, by PDA Engineering, Costa Mesa, CA. copyright 1989.

5. Lawrence Livermore Laboratory, Engineering Standard Reference, ESR NO. 215C.(1977)

6. Age Visser. Private communication.

7. Luca Marinelli. Private communication.

8. Annual Book of ASTM(American Society for Testing and Material) Standards, Nonferrous Metal Products, vol. 2.02,N.Y.(1988)

9. K. Horn, Aluminum, vol. 2,Design and Application, American Society for Metals, Metals Park, Ohio.(1967)

10. Anthony Malensek. Private communication.

11. Ernie Villesges. Private communication.

12. Shigley & Mischke, Mechanical Engineering Design, 5th ed., McGraw-Hill, Inc. N.Y.(1989)

13. W. Gajewski, University of California Irvine Tech. Note 2/91 (1991).

14. W. Eshbach, Handbook of Engineering Fundamentals, 2nd. ed., Wiley & Sons, Inc., N.Y.(1952)

15. W. Gajewski. Private communication.

16. E. Purcell, Electricity & Magnetism, vol.2,1965, McGraw-Hill, co., N.Y.

17. W. Panofsky & M. Phillips, Classical Electricity and Magnetism, 2nd. ed., Addison-Wesley, Inc., Massachusetts.(1962)

18. Metals Handbook, 9th ed., (1979)vol. 2, Properties and Selection: Non-Ferrous Alloys and Pure Metals.

19. Bob Wonds. Private communication.

20. Anne Jaeckel. Private communication.

21. P. Simpson, Inducting Heating, McGraw-Hill, Inc., N.Y. (1960) Fig.B-3.