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Repetitive Stern-Gerlach Effect

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ABSTRACT

I show that two spin rotators 180° apart may be desirable for the repetitive Stern-Gerlach effect. I also calculate the effect of depolarization resonance on the repetitive Stern-Gerlach effect. It is shown that to first order in resonance strength, we can avoid the imperfection resonance if the energy of the beam is at $G\gamma = n+1/2$. The time available for accumulating the Stern-Gerlach kick is then limited by the intrinsic resonance.

I . Stern-Gerlach Spin Splitter[1]

In the Stern-Gerlach experiment, an atomic beam is kicked when passing a inhomogeneous magnetic field. The kick in the non-relativistic limit is the familiar

$$\vec{F} = \nabla(\vec{\mu} \cdot \vec{B}) \tag{1}$$

where μ is the magnetic moment of the particle. For the proton $\mu = 8.8 \times 10^{-14}$ MeV T^{-1}. The beam is separated into two beams of opposite spin orientation after the kick is applied.

The quadruple magnets in a storage ring are inhomogeneous fields. We can write

$$B_x = by\text{, and}$$
$$B_y = -bx\,. \tag{2}$$

Then the angle of the kick is

$$x' = \frac{\mu_y bL}{p\beta c}\text{, and}$$
$$y' = \frac{\mu_x bL}{p\beta c}, \tag{3}$$

where μ_x and μ_y are projections of magnetic moment on the x and y axis respectively.

To coherently add up the kicks turn after turn, the spin tune and betatron tune have to satisfy the resonant condition

$$q_s = q_\beta\,. \tag{4}$$

We will use Q for tune and q for the fractional part of the tune.

Notice that b/p is proportional to the lattice constant K so that normally the kick is independent of the beam momentum. The accumulated kick for a 0.2 GeV/c beam with bL=20 T is 4.2 mrad/sec if the revolution frequency is 1MHz. The beam will be separated into two beams with opposite spin orientation. The separation is 4.2 mm/sec if the betatron function is 10 m for the above example.

II. Siberian Snake

The spin tune is $G\gamma$, where G is the anomalous magnetic moment of the particle. It is 1.79 for proton. Obviously the spin tune depends on the energy of the particle. For a particle beam with energy spread, it will be impossible to keep the resonant condition for all the particles. We can use a Siberian snake to eliminate this energy dependent spin tune spread.

Remember that a snake is a solenoid or a combination of magnets that rotates the spin 180° around the z axis. The spin rotation matrix is $\exp(-i\pi\sigma_z/2)$. The bending magnets rotate the spin around the y axis. Its spin rotation matrix is $\exp(-i\phi\sigma_y/2)$. The σ's are Pauli matrices. For a perfect machine, we can calculate the one turn rotation matrix of the spin at the location symmetric to the solenoid to be

$$M=\exp(-i\phi\sigma_y/2)\exp(-i\pi\sigma_z/2)\exp(-i\phi\sigma_y/2)=\exp(-i\pi\sigma_z/2). \tag{5}$$

So the contributions from the two bending sections cancel. The net spin rotation is 180° around the z axis independent of $\phi(=G\gamma\pi)$. The spin tune is 0.5. The condition for the repetitive Stern-Gerlach kick is $q_\beta=0.5$. This is undesirable.

We can add another solenoid that rotates the spin by φ at the location symmetric to the snake. The arrangement now is a bending-solenoid-bending-new solenoid. The matrix is

$$\begin{aligned} M&=\exp(-i\phi\sigma_y/2)\exp(-i\pi\sigma_z/2)\exp(-i\phi\sigma_y/2)\exp(-i\varphi\sigma_z/2)\\ &=-i\sigma_y\cos(\varphi/2)-I\sin(\varphi/2)\\ &=\exp(-i\omega\sigma_y/2), \end{aligned} \tag{6}$$

where $\cos(\pi Q_S)=\cos(\omega/2)=-\sin(\varphi/2)$. Now the net rotation is ω around z axis. The spin tune Q_S is $\omega/2\pi$. We can adjust the new solenoid's strength φ to match the q_s to the desired betatron tune.

III. Single spin depolarization resonance model

We treated the case of a perfect machine in the previous section. In a real machine, the misalignment and vertical betatron motion will cause the spin to precess around the x axis. The edge field of the dipole will also cause the spin to precess around the z axis. The spinor Ψ now satisfies the equation[2]

$$\frac{d}{d\theta}\Psi=-\frac{i}{2}\begin{bmatrix}\gamma G & -\varsigma\\ -\varsigma^* & -\gamma G\end{bmatrix}\Psi, \tag{7}$$

where θ is the angle around the ring and ζ characterize the precession around x and z axis.

In a single depolarization resonance model, $\zeta=\varepsilon\exp(iK\theta)$, where K=integer for the misalignments (so called imperfection resonance) and K=integer $\pm\, q_\beta$ for the vertical betatron motion (so called intrinsic resonance). The motion of the spinor Ψ between θ_i and θ_f is described by

$$\Psi(\theta_f)=t(\theta_f,\theta_i)\Psi(\theta_i), \tag{8}$$

where $t(\theta_f,\theta_i)$ is the 2×2 spin transportation matrix. The elements are

$$t_{11}(\theta_f,\theta_i) = a\exp(i[c - K(\theta_f - \theta_i)/2]),$$
$$t_{12}(\theta_f,\theta_i) = ib\exp(-i[d + K(\theta_f + \theta_i)/2]),$$
$$t_{11}(\theta_f,\theta_i) = t_{22}^*(\theta_f,\theta_i),$$
$$t_{12}(\theta_f,\theta_i) = -t_{21}^*(\theta_f,\theta_i), \qquad (9)$$

where

$$\lambda = (\delta^2 + |\varepsilon|^2)^{1/2},$$
$$\delta = K - \gamma G,$$
$$\varepsilon = |\varepsilon| e^{-id},$$
$$ae^{ic} = \cos[\lambda(\theta_f - \theta_i)/2] + i\tfrac{\delta}{\lambda}\sin[\lambda(\theta_f - \theta_i)/2],$$
$$b = (1-a^2)^{1/2} = \tfrac{|\varepsilon|}{\lambda}\sin[\lambda(\theta_f - \theta_i)/2]. \qquad (10)$$

The spin transportation matrix with two solenoids is

$$M(\theta_0+2\pi, \theta_0) = t(\theta_0+2\pi, \theta_0+\pi)\ \exp(-i\pi\sigma_z/2)\ t(\theta_0+\pi, \theta_0)\ \exp(-i\varphi\sigma_z/2). \qquad (11)$$

We get

$$M_{11} = -\sin(\varphi/2) + 2ab\cos(\varphi/2)\sin[K(\theta_0+\pi)+d]\exp[i(c-K\pi)]$$
$$-2ib^2\sin(\varphi/2)\sin[K(\theta_0+\pi)+d]\exp[i(K(\theta_0+\pi)+d],$$
$$M_{12} = -\cos(\varphi/2) - 2ab\sin(\varphi/2)\sin[K(\theta_0+\pi)+d]\exp[-i(c-K\pi)]$$
$$+2ib^2\cos(\varphi/2)\sin[K(\theta_0+\pi)+d]\exp[-i(K(\theta_0+\pi)+d],$$
$$M_{22} = M_{11}^*,$$
$$M_{21} = -M_{12}^*. \qquad (12)$$

The eigenvalues are

$$\mathrm{Re}(M_{11}) \pm \sqrt{\mathrm{Re}^2(M_{11}) - 1} \quad , \qquad (13)$$

and the spin tune is

$$\cos(\pi Q_s) = -\sin(\varphi/2)$$
$$+ 2ab\cos(\varphi/2)\sin[K(\theta_0+\pi)+d]\cos(c-K\pi)$$
$$+ 2b^2\sin(\varphi/2)\sin^2[K(\theta_0+\pi)+d] \quad . \qquad (14)$$

Note that b is proportional to the strength of depolarization resonance $|\varepsilon|$. To first order in $|\varepsilon|$, $a \approx 1$. The spin tune now contains terms proportional to the first and second order in b. Both of these terms are dependent on θ_0 which is uncontrollable. Thus the spin tune will have a spread to first order in b equal to

$$\Delta Q_s \approx \frac{4ab\cos(\varphi/2)\cos(c-K\pi)}{\pi\sin(\pi Q_s)} \tag{15}$$

caused by vertical betatron motion and misalignment.

If K is an integer (imperfection resonance), the spread ΔQ_s is zero if $G\gamma = n+1/2$. In this case to first order in b, $\cos(K\text{-}c\pi)=0$ and the spin tune is independent of b. Since G=1.79, the lowest energy possible is $G\gamma=2.5$. If K=integer $\pm\, q_\beta$ (intrinsic resonance) we cannot eliminate spin tune spread. In order to keep the spin and betatron motion in phase, the number of turns needed to separate the beam will have to be less than $\approx 1/b \propto \lambda/|\varepsilon|$. The separation time is limited by the closest K=integer $\pm\, q_\beta$ resonance to $G\gamma$.

IV. Fermilab pbar accumulator and IUCF as an example

The idea to use the spin splitter to produce polarized anti-proton has been proposed to LEAR[3]. Here I calculated the intrinsic resonance strength of the Fermilab anti-proton accumulator. The resonance strength was calculated with program DEPOL[4]. The beam non-normalized emmitance was 1mm-mrad. In figure 1 I plotted the resonance strength as a function of γ. It is clear that the resonance strength is too large for effective separation of the beam.

The Indiana University Cooler Ring (IUCF) is a unique facility that it has a working Siberian snake. There is also a second solenoid 180° from the Siberian snake for electron cooling purpose. The two solenoids configuration proposed in section II is satisfied by the IUCF ring.

The following numbers are taken from the IUCF Siberian snake proposal[5]. The energy of the beam is such that $G\gamma=2.5$.

The tunes are $Q_x=5.13$ and $Q_y=5.12$. This gives $q_s=q_\beta=0.13$, and the rotation of spin by the second solenoid (the cooling solenoid) -0.74π. At $G\gamma=2.5$ the solenoid strength is 0.669T-m. The normal running strength of the cooling solenoid is 0.5 T-m.

We also have

$G\gamma$	γ	β	P	$B\rho$
2.5	1.397	0.698	914 MeV/c	3.05 T-m

The strength of the four quad pairs around the Siberian snake are

focal length (m)	kick angle (rad)
1.5	2.82×10^{-16}
1.18	3.58×10^{-16}
-1.57	2.69×10^{-16}
1.33	3.17×10^{-16}

The total kick around the location of the Siberian snake is 24.52×10^{-16} rad. Given the revolution frequency is 2.52×10^{6} Hz, the accumulated kick per second is 6.18×10^{-9} rad. If we use beta function 10 m, the separation per second is 6.18×10^{-5} mm. This separation is an order of magnitude smaller than the 7×10^{-4} mm/sec rate proposed by the LEAR spin-splitter group because the quadruple focal length is approximately a factor of 10 longer.

References

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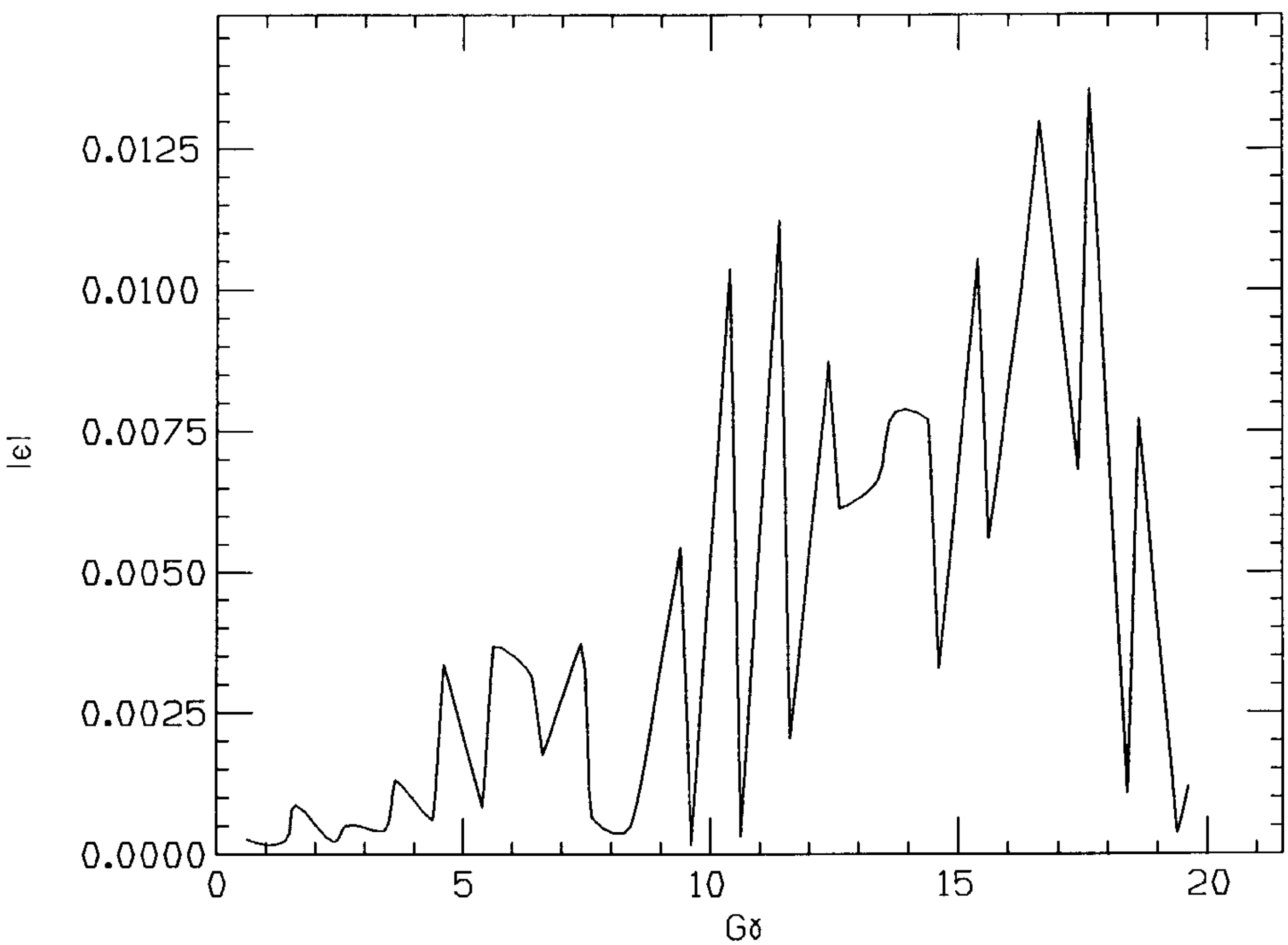

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