

Fermi National Accelerator Laboratory

TM-1645

A Nonlinear Oscillator

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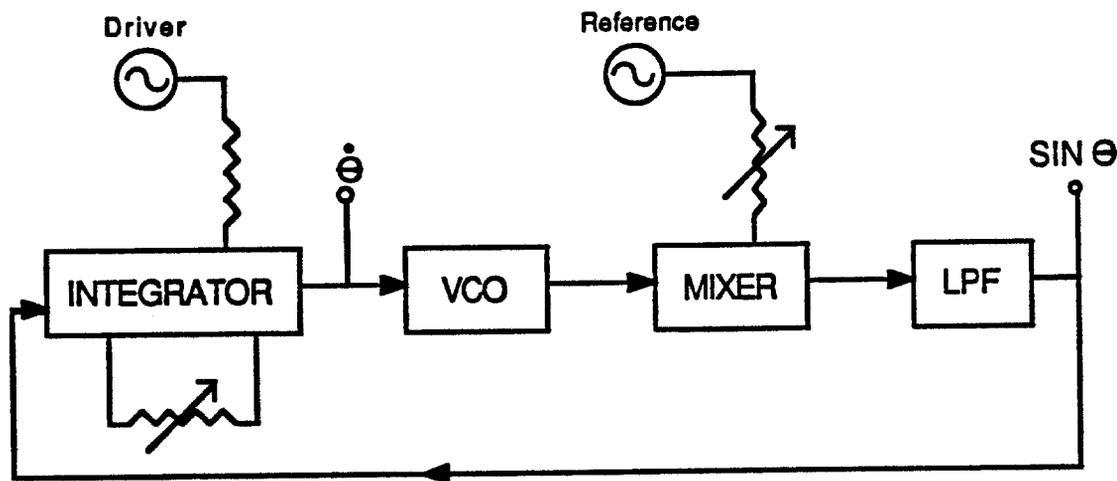
A Nonlinear Oscillator

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A nonlinear oscillator design was imported from Cornell, modified, and built for the purpose of simulating the chaotic states of a forced pendulum. Similar circuits have been investigated in the recent nonlinear explosion.

Circuit Function:

A block diagram shows the circuit which is a phase locked loop. The voltage controlled oscillator output is mixed with a reference signal. The result is filtered, integrated, and fed to the VCO as the error correction signal. This VCO locks over a range of approximately 23 to 27KHz. The natural operating frequency of the circuit is approximately 25KHz and is variable with the integrator setting. The low pass filter was breadboarded separately. Its response curve is included here. A VCO input versus frequency out curve for open loop is included.



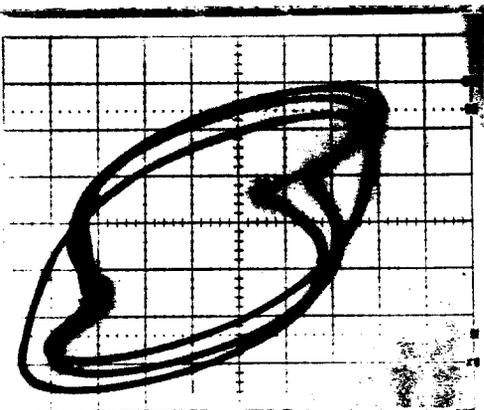
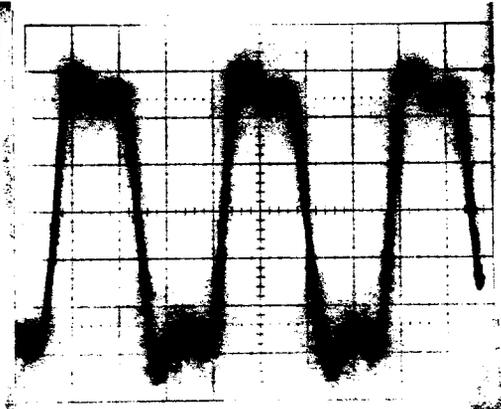
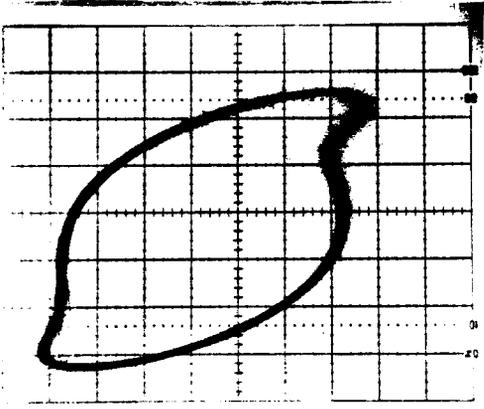
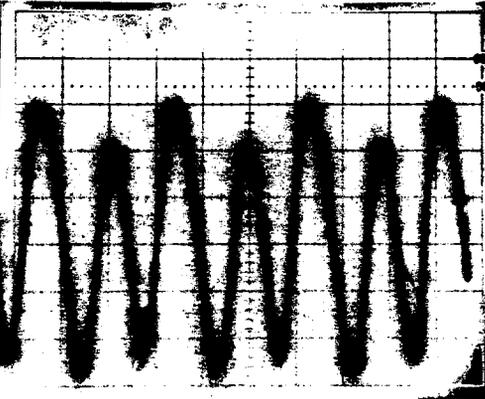
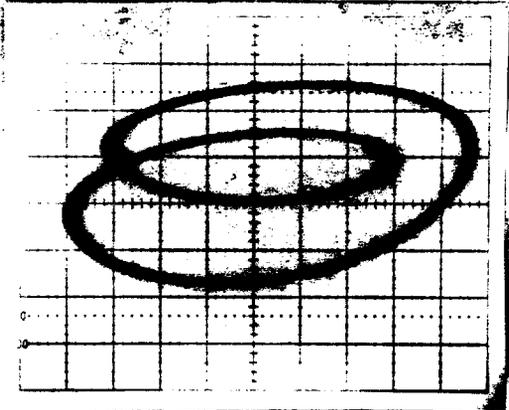
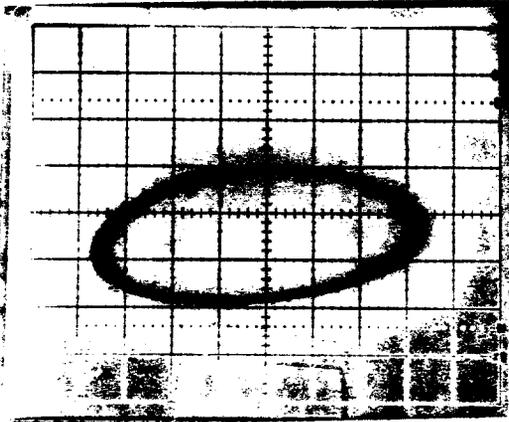
BLOCK DIAGRAM

The integrator IN and SINE inputs are fed with separate signals having variable frequency, amplitude, and DC offset settings.

A full range of nonlinear effects can be obtained. The SIN theta and theta dot signals were plotted as X versus Y on a TEK 454A to produce the bifurcation images included here. The accompanying time traces are also shown. More spectacular and fun things can be made to happen. For example, with a speaker at the integrator output we have generated bird calls like the whip-poor-will.

The oscillator was hooked to a TEK DSA 602 digitizing signal analyzer to plot theta dot versus SIN theta in XY fashion and simultaneously an FFT of theta dot. The fundamental frequency can be seen to go lower upon bifurcation. DSA 602 plots show a. a stable oscillation with second harmonic 30 dB down, b. bifurcation with the third frequency appearing and the spectrum shifting down, c. the oscillator tuned back to the stable condition, d. two bifurcations with four dominant frequencies and signs of more (chaos). In figure d the spectrum also shifts even lower.

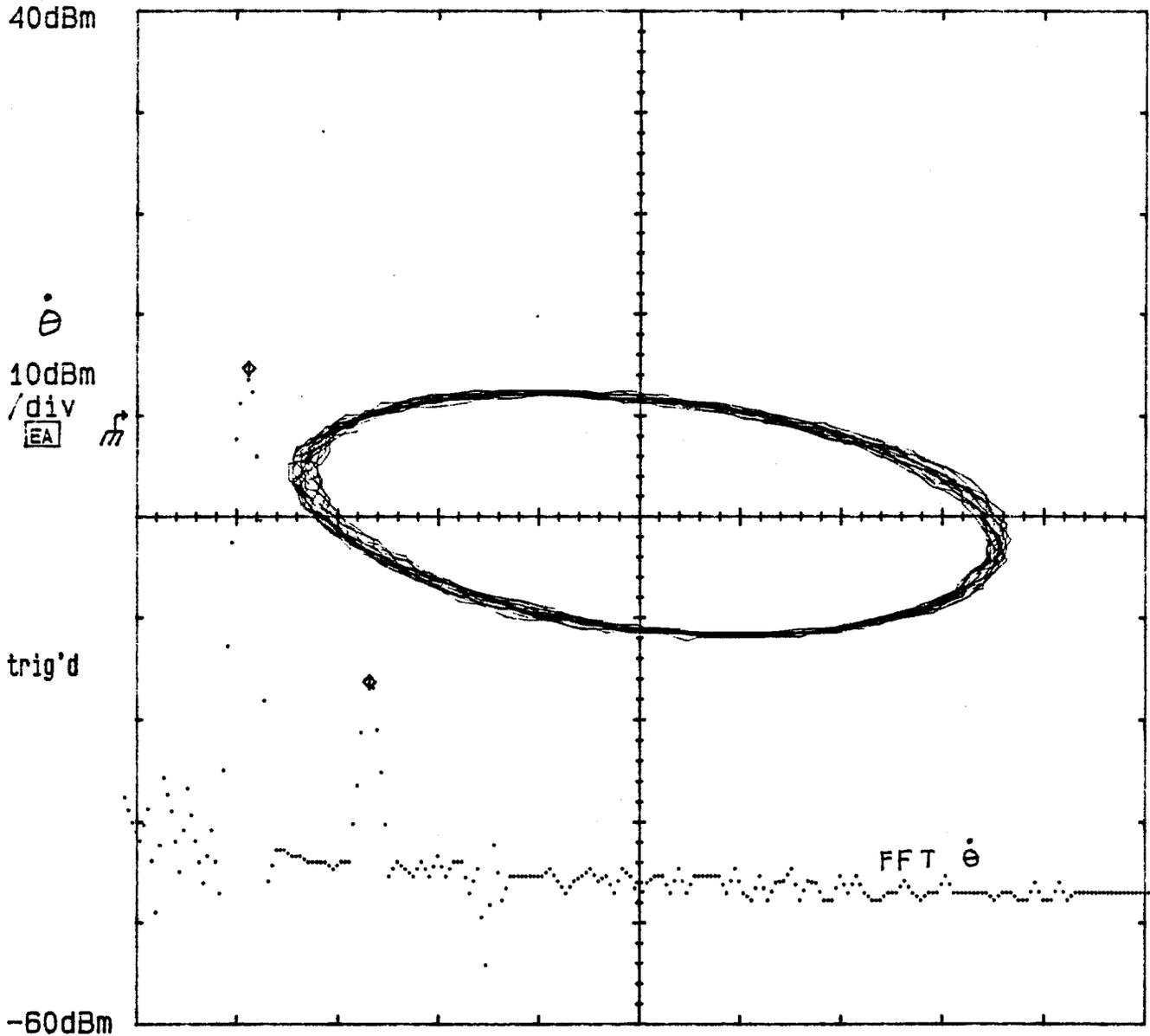
1. The circuit was obtained by private communication with Bob Siemann.
2. Chaotic States and Routes to Chaos in the Forced Pendulum. D. D'Humieres et al. Physical Review A (1982) 3483
3. Three-Frequency Quasiperiodicity, Phase Locking, and the Onset of Chaos. P. Linsay and A. Cumming. Physica D 40 (1989) 196-217



DSA 602 DIGITIZING SIGNAL ANALYZER
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Figure a.
 "Stable", with second harmonic.

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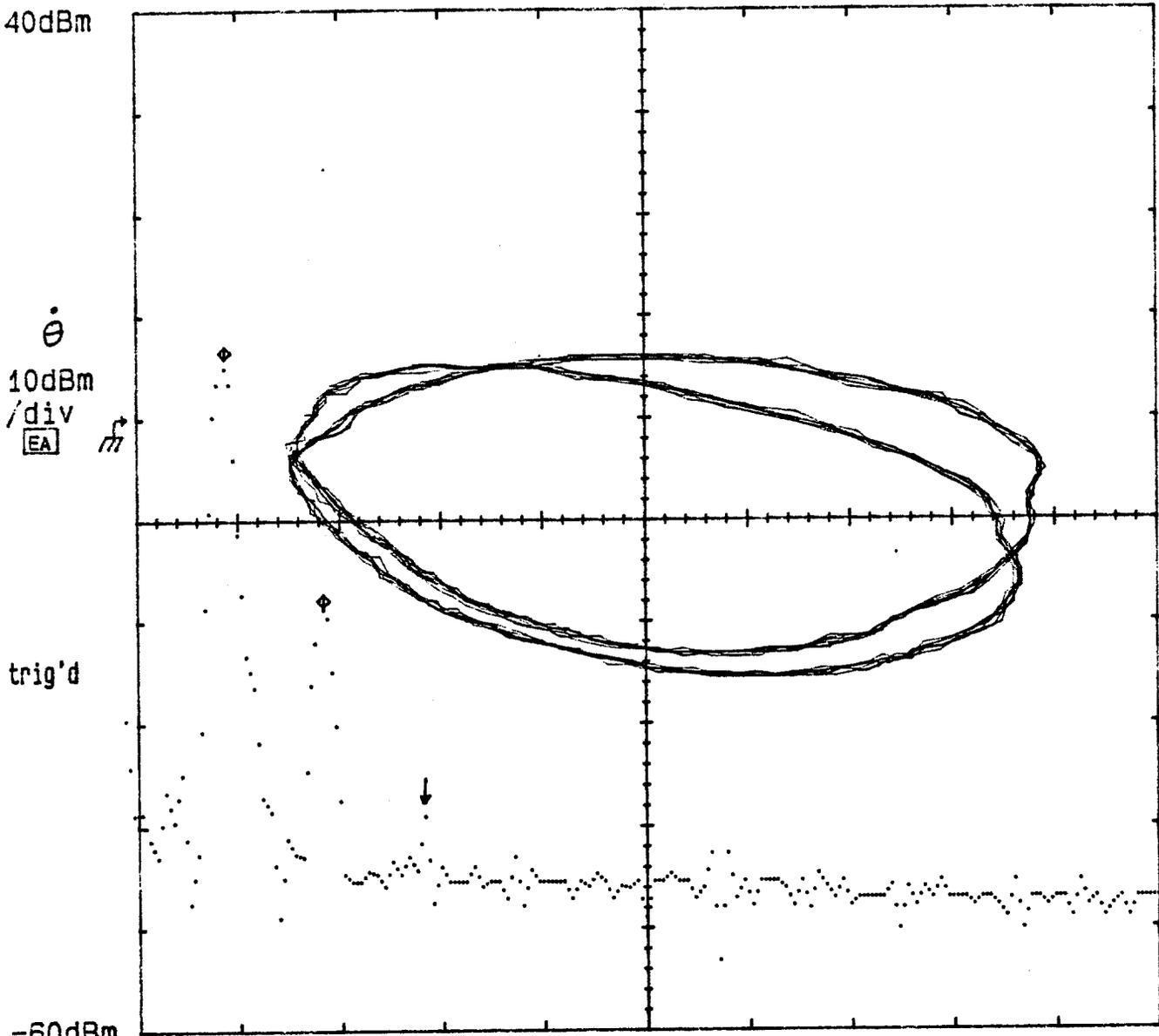


366.211Hz		3.0518kHz/div SIN θ		30.884kHz	
Cursor	v1= 4.672dBm	f1= 3.662kHz		Rem	
Type	v2= -26.26dBm	f2= 7.324kHz		Wfm 1	
Paired	Δv = -30.93dBm	Δf = 3.662kHz		FFTM	
Dots				Main	
			Cursor 1	Cursor 2	
			3.662kHz	7.324kHz	

DSA 602 DIGITIZING SIGNAL ANALYZER
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Figure b.
 Bifurcation with the third
 frequency appearing and
 spectrum shifted down.

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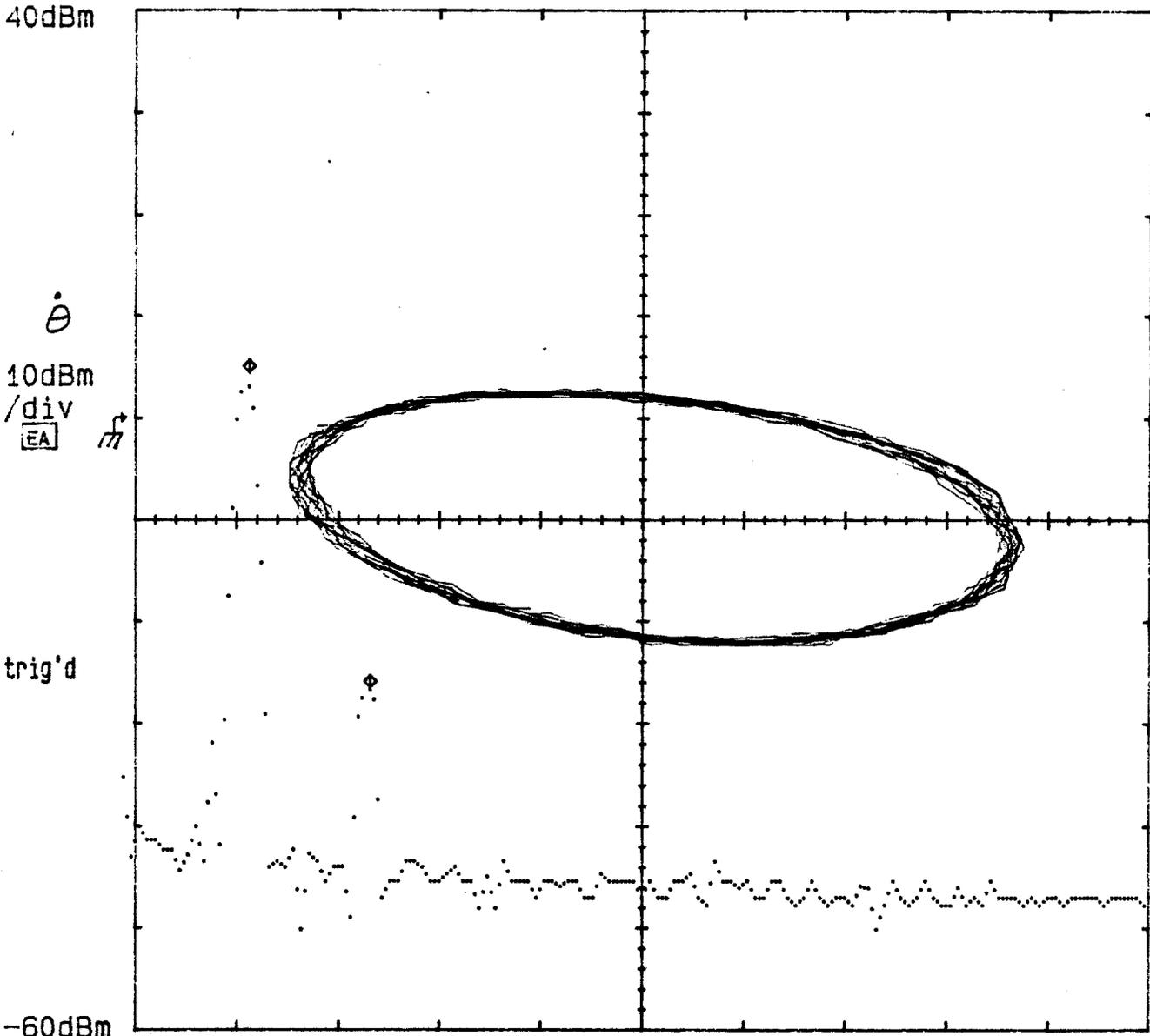
366.211Hz 3.0518kHz/div SIN θ 30.884kHz

Cursor Type Paired Dots	v1= 6.438dBm v2= -17.98dBm Δv = -24.41dBm	f1= 2.930kHz f2= 5.859kHz Δf = 2.930kHz	Rem Wfm 1 FFTm Main
	Cursor 1	Cursor 2	
	2.930kHz	5.859kHz	

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Figure c.
 Oscillator tuned back to
 condition a.

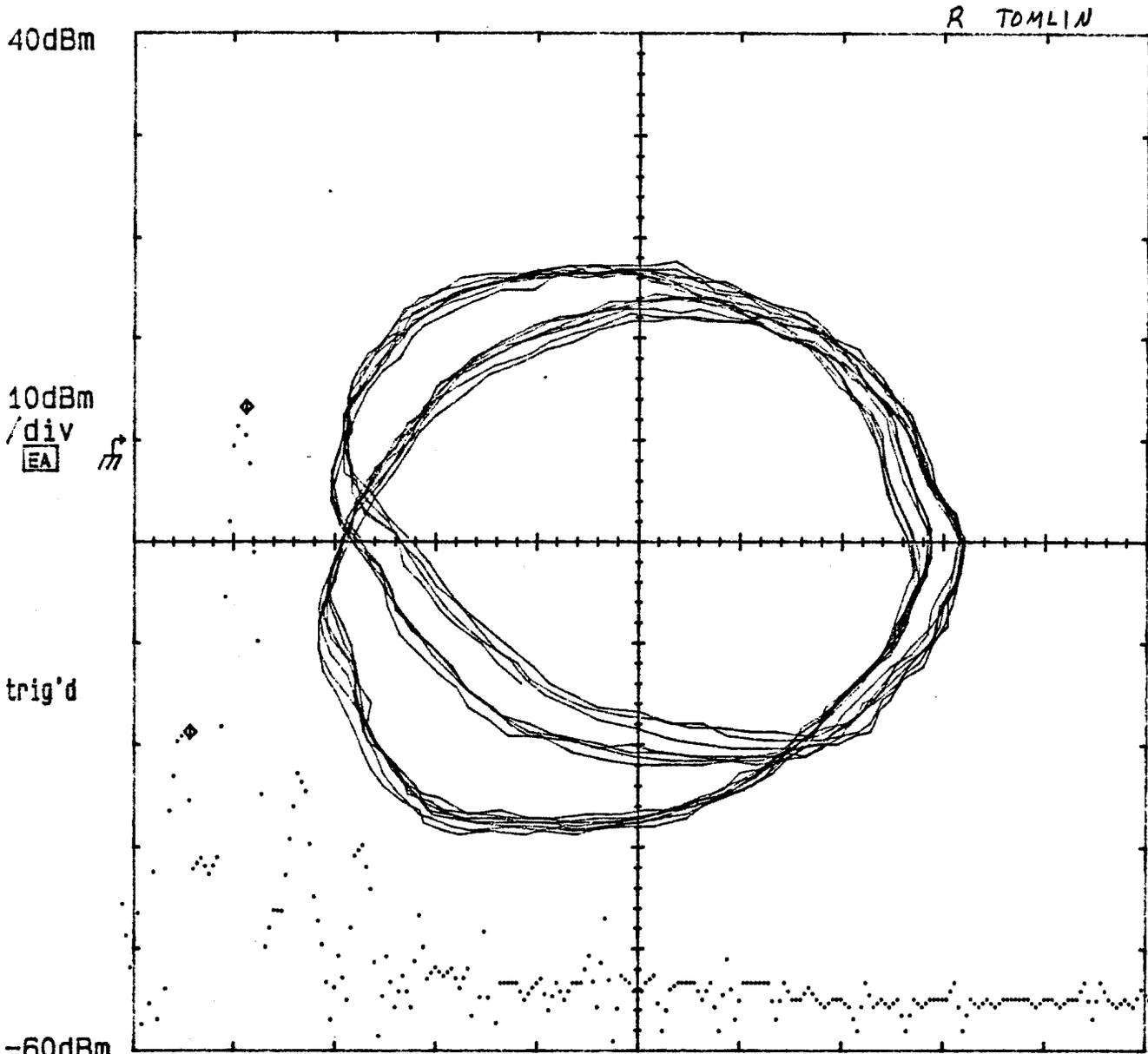
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366.211Hz		3.0518kHz/div SIN Θ		30.884kHz	
Cursor	v1= 5.125dBm	f1= 3.662kHz		Rem	
Type	v2= -25.81dBm	f2= 7.324kHz		Wfm 1	
Paired	Δv= -30.94dBm	Δf= 3.662kHz		FFTM	
Dots				Main	
		Cursor 1		Cursor 2	
		3.662kHz		7.324kHz	

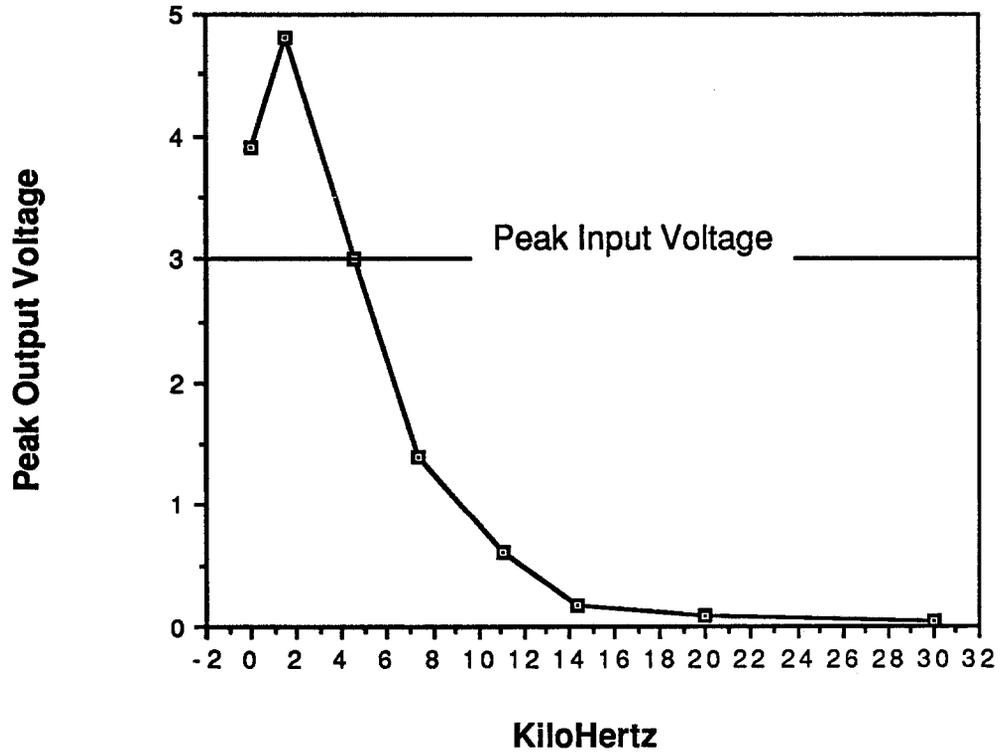
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Figure d.
 Two bifurcations. More
 frequencies appear.



366.211Hz		3.0518kHz/div		30.884kHz	
Cursor	v1= 3.219dBm	f1= 3.662kHz	Rem		
Type	v2= -28.77dBm	f2= 1.953kHz	Wfm 1		
Paired	Δv= -31.99dBm	Δf= -1.709kHz	FFTM		
Dots				Main	
f3 = 5.371	f7 = 14.65	Cursor 1		Cursor 2	
f4 = 7.324	f8 = 18.31	3.662kHz		1.953kHz	
f5 = 9.033					
f6 = 10.99					

Low Pass Filter Response



VCO Transfer Function

$$y = 9.3301 + 0.6989x + 0.3112x^2 - 0.018x^3 \quad R = 1.00$$

