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I. INTRODUCTION

The Tevatron dipoles and quadrupoles are at superconducting temperature while the low-beta insertion is at room temperature. In order to limit the heat flowing into the low temperature region, the low-beta insertion is joined to the ring through a special bellows shown in Fig. 1. The beam-pipe radius is 3.7 cm corresponding to a cutoff frequency of 3.10 GHz for the monopole mode and 2.38 GHz for the dipole mode. There are approximately 32 such bellows. Each bellows has a narrow gap of about 6 mm at the beam pipe and opens up into a $\sim$ 7 cm wide cavity at larger radius. The bellows ripples start at a radius of about 12 cm. We therefore expect the bellows to contribute sharp resonances below cutoff at frequency below 1 GHz. In this article, we are going to compute the longitudinal and transverse impedances and to check whether they can drive any collective instabilities of the beam bunches.

II. IMPEDANCES

The bellows is approximated by the geometry as shown in Fig. 2. We run URMEL[1] and obtain the lowest 5 resonances for the monopole and the dipole modes. They are listed in Tables I and II.

Frequency (GHz)	Shunt Impedance $Z_{\parallel sh}$ ($k\Omega$)	Figure of Merit Q	$Z_{\parallel sh}/Q$ (Ω)
0.885	3.009	296	10.17
1.932	1.379	477	2.891
3.065	90.59	2769	32.71
3.528	1.277	542	2.356
4.114	2.095	641	3.269

Table I: First 5 resonances of the longitudinal impedance

In the computation, the bellows wall is assumed to be of stainless steel having a resistivity of $\rho = 7.14\times10^{-7}$ Ω-m. As is usual in URMEL computation, the beam pipe is closed at both ends. As a result, we cannot trust any resonances with frequencies above 3.1 GHz (2.38 GHz), the monopole (dipole) cutoff frequency. The fields corresponding to

Frequency (GHz)	Shunt Impedance $Z_{\perp sh}$ ($k\Omega$/m)	Figure of Merit Q	$Z_{\perp sh}/Q$ (Ω/m)
1.313	117.9	477	247.2
2.107	27.64	509	54.31
3.153	0.092	5234	0.018
3.578	0.476	552	0.863
3.933	13.04	655	20.31

Table II: First 5 resonances of the transverse impedance

these resonances will leak out and propagate along the beam pipe. Thus, in reality they will be very much de-Qed. To demonstrate this, we run TBCI[2] in the time domain with beam pipe open at both ends. The longitudinal impedance and transverse impedance shown in Figs. 3 and 4 are obtained by Fourier transforms of the corresponding wake potentials.

For the longitudinal impedance, we see in Fig. 3 only the first two resonances of Table I. These resonances should have zero widths and infinite heights because infinite conductivity of the bellows walls has been assumed in the computation. However, due to the finite length $\ell = 3$ m of the wake potential, a δ-function resonance at ω_0 has the shape of $\sin[(\omega-\omega_r)\ell/2c]/[\omega-\omega_r]$ instead. In other words, the infinitely narrow width is widened to $\sim$ 200 Mhz and there are ripples of period $\sim$ 200 Mhz. The ripples on the broad resonances can be smoothed out by increasing the length of the wake or by truncating the wake at a length when it vanishes. However, the $\sin x/x$ deformation of the δ-function resonance cannot be avoided. Provided that the wake is of finite length, we always get the same envelope of $(\omega-\omega_r)^{-1}$ no matter how long the wake is computed. As a result, we should always interpret these impedance plots after averaging out the ripples. Only in this way will the real part of the longitudinal impedance be non-negative.

The third and strongest resonance at 3.06 GHz is absent. This is because it is very near to the cutoff frequency 3.1 GHz of the beam pipe and all the field corresponding to this mode leaks away as the lowest propagating TM mode of the pipe. At cutoff, the propagating mode has infinite phase velocity. Therefore, it does not interact with the particle beam and does not contribute to the longitudinal impedance. The spectrum at higher frequencies is completely different from the prediction of URMEL. We see a broad band at about 4 GHz with a peak of about 44 Ω.

The transverse impedance of Fig. 4 shows similar features. We see two sharp resonances below the dipole cutoff frequency of 2.38 GHz. In addition, there are broader resonances of about 1.3 kΩ/m near 4 GHz.

III. SINGLE BUNCH INSTABILITIES

Sharp resonances can drive microwave growths for both the monopole and dipole modes. The stability limits are given by[3]

$$\left|\frac{Z_{\|}}{Q}\right|_{sh} \leq \frac{4|\eta|(E/e)\delta^2}{I_{av}} , \tag{3.1}$$

$$\left|\frac{Z_{\perp}}{Q}\right|_{sh} \leq \frac{8\sqrt{2/\pi}|\eta|(E/e)\delta}{\bar{\beta}I_{av}} , \tag{3.2}$$

where e is proton charge and E is the particle energy. We take

frequency dispersion $\eta = 0.0028$,

particle per bunch $N = 10^{11}$,

revolution frequency $f_0 = 47.75$ khz ,

fractional rms energy spread $\delta = 3.18 \times 10^{-4}$,

average beta-function $\bar{\beta} = 57.6$ m .

The *average* current per bunch is therefore $I_{av} = eNf_0 = 0.765$ mA. At the injection energy of $E = 150$ GeV, we obtain the limits $|Z_{\|}/Q|_{sh} \leq 0.22$ MΩ and $|Z_{\perp}/Q|_{sh} \leq 19.3$ MΩ/m.

Microwave growths driven by broad-band impedances have stability limits

$$\left|\frac{Z_{\|}}{n}\right| \leq \frac{2\pi|\eta|(E/e)\delta^2}{I_p} , \tag{3.3}$$

$$|Z_{\perp}| \leq \frac{4\sqrt{2\pi}|\eta|(E/e)\delta n_c}{\bar{\beta}I_p} , \tag{3.4}$$

where n is the azimuthal harmonic number around the ring, n_c is the harmonic near cutoff and is usually taken as the ratio of the ring radius to the pipe radius, and I_p is the peak current of the bunch which is related to the average current I_{av} by

$$I_{av} = I_p\sqrt{2\pi}\sigma_t f_0 \ , \tag{3.5}$$

if the bunch is of gaussian shape with rms time length σ_t. For a bunch of rms length $\sigma_\ell = 30$ cm, $\sigma_t = 1$ ns and $I_p = 6.39$ A. At the injection energy of 150 GeV, the stability limits are $|Z_\parallel/n| \le 41.8\ \Omega$ and $|Z_\perp| \lesssim 98\ \mathrm{M\Omega/m}$.

Instabilities due to bunch-mode crossing are driven by impedances at frequencies comparable to the frequency range of the bunch. The limits for stability are

$$\frac{\mathcal{I}m\ \bar{Z}_\parallel}{n} \le \frac{8\sqrt{\pi}\ \eta|(E/e)}{I_{av}}\frac{\sigma_\ell}{R}\delta^2 \ , \tag{3.6}$$

$$\mathcal{I}m\ \bar{Z}_\perp \le \frac{4\sqrt{\pi}|\eta|(E/e)\delta}{\bar{\beta}I_{av}} \ , \tag{3.7}$$

In above, the *average* impedances with a bar on top are defined as

$$\frac{\mathcal{I}m\ \bar{Z}_\parallel}{n} = \mathcal{I}m \int_{-\infty}^{\infty} d\omega \frac{Z_\parallel(\omega)}{n} h(\omega) \ , \tag{3.8}$$

$$\mathcal{I}m\ \bar{Z}_\perp = \mathcal{I}m \int_{-\infty}^{\infty} d\omega Z_\perp(\omega) h(\omega) \ , \tag{3.9}$$

where $h(\omega)$ is the bunch power spectrum normalized to unity. For a gaussian bunch of rms length $\sigma_\ell = 30$cm, the power spectrum has an rms width of $\omega/2\pi = 1/\sqrt{8}\pi = 0.113$ GHz. When the frequency is below ~ 0.5 GHz, $\mathcal{I}m\ Z_\parallel/n$ and $\mathcal{I}m\ Z_\perp$ are roughly constant and can therefore taken out of the integral signs. The integrals then become trivial. Since the longitudinal impedance consists essentially of two sharp resonances and so does the transverse impedance, we can write approximately

$$\frac{\mathcal{I}m\ \bar{Z}_\parallel}{n} \approx \sum_{i=1}^{2} \frac{Z_\parallel}{Q}\frac{\omega_0}{\omega}\bigg|_{\text{at resonance}} \ , \tag{3.10}$$

$$\mathcal{I}m\ \bar{Z}_\perp \approx \sum_{i=1}^{2} \frac{Z_\perp}{Q}\bigg|_{\text{at resonance}} \ . \tag{3.11}$$

If we use the first two resonances in Tables I and II, we obtain for 32 bellows, $\mathcal{I}m\ \bar{Z}_\parallel/n = 0.0198\ \Omega$ and $\mathcal{I}m\ \bar{Z}_\perp = 9.65\ \mathrm{k\Omega/m}$.

From Eqs. (3.6) and (3.7), the limits of stability are $\mathcal{I}m\ \bar{Z}_\parallel/n = 236\ \Omega$ and $\mathcal{I}m\ \bar{Z}_\perp = 21.5\ \mathrm{M\Omega/m}$. Thus, if there are any mode-colliding instabilities, the contribution from these 32 bellows is negligible.

IV. MULTI-BUNCH GROWTHS

The two sharp resonances of the monopole and dipole modes can drive coupled-bunch growths. For M equal bunches, there are M independent modes. The growth rate for the μ-th monopole coupled-bunch mode is

$$\tau_\mu^{-1} = \frac{|\eta| M I_b f_0}{2\nu_s (E/e)} Z_{\|\mathrm{eff}} \ , \tag{4.1}$$

where ν_s is the synchrotron tune and the effective impedance is defined as

$$Z_{\|\mathrm{eff}} = \mathcal{R}e \sum_{k=-\infty}^{\infty} \nu_k Z_\|(\nu_{\|k}\omega_0) e^{-(\nu_{\|k}\omega_0\sigma_t)^2} \ , \tag{4.2}$$

for a gaussian bunch of rms time length σ_t, with

$$\nu_{\|k} = kM + \mu + \nu_s \ . \tag{4.3}$$

Because of the exponential factor in Eq. (4.2), most of the contribution comes from the first resonance at 0.885 GHz. We take the extreme condition that the resonances of the 32 bellows line up exactly at 0.885 GHz and lie exactly on top of one of the line defined by Eq. (4.3). This gives $Z_{\|\mathrm{eff}} = 6.65 \times 10^{-5}\ \Omega$ for all the 32 bellows. With $\nu_s \approx 0.025$ and a full ring of $M = 1113$ bunches, we obtain the worst growth rate $\tau_\mu^{-1} = 1.01 \times 10^{-11}\ \mathrm{sec}^{-1}$, which is negligibly small.

For the dipole mode, growth rate for the μ-th coupled-bunch mode is given by

$$\tau_\mu^{-1} = \frac{M I_b c}{4\pi\nu_\beta (E/e)} Z_{\perp\mathrm{eff}} \ , \tag{4.4}$$

where the effective transverse impedance is defined as

$$Z_{\perp\mathrm{eff}} = \mathcal{R}e \sum_{k=-\infty}^{\infty} Z_\perp(\nu_{\perp k}\omega_0) e^{-(\nu_{\perp k}\omega_0\sigma_t)^2} \ , \tag{4.5}$$

with

$$\nu_{\perp k} = kM - \mu + \nu_\beta \ . \tag{4.6}$$

Again we take the extreme case that the first dipole resonances of the 32 bellows line up exactly and fall exactly on top of a line defined by Eq. (4.6). Then $Z_{\perp\mathrm{eff}} = 1.04 \times 10^{-29}\ \mathrm{M}\Omega/\mathrm{m}$ and the worst growth rate is $\tau_\mu^{-1} = 7.26 \times 10^{-29}\ \mathrm{sec}^{-1}$.

The smallness of these growth rates is a result of the long bunches of the Tevatron. A $\sigma_t = 1$ ns bunch has a power spectrum of rms width 0.113 GHz only. But the lowest resonances of the bellows are 0.885 GHz and 1.313 GHz for respectively the monopole and dipole modes. The overlaps are therefore negligibly small.

V. SUMMARY AND DISCUSSION

We summarize the stability limits and the impedance contribution of the 32 bellows in Table III. We can therefore conclude that the introduction of the 32 bellows joining the warm sections to the cold sections will not contribute to any single-bunch and multi-bunch instabilities. As a result, no shielding will be needed.

	Stability Limits	Bellows Contribution (32 bellows)
Microwave		
Sharp Resonance $m = 0$	$\left\|\frac{Z_\parallel}{Q}\right\|_{sh} = 220$ kΩ	$\left\|\frac{Z_\perp}{Q}\right\|_{sh} = 325$ Ω at 0.885 GHz
$m = 1$	$\left\|\frac{Z_\perp}{Q}\right\|_{sh} = 19.3$ MΩ/m	$\left\|\frac{Z_\perp}{Q}\right\|_{sh} = 7.91$ kΩ/m at 1.31 GHz
Broad Band $m = 0$	$\left\|\frac{Z_\parallel}{n}\right\| = 41.8$ Ω	$\left\|\frac{Z_\parallel}{n}\right\| = 0.017$ Ω
$m = 1$	$\|Z_\perp\| = 98$ MΩ/m	$\|Z_\perp\| = 42$ kΩ/m
Mode Mixing		
$m = 0$	$\frac{\mathcal{I}m\, Z_\parallel}{n}\Big\|_{\omega=0} = 236$ Ω	$\frac{\mathcal{I}m\, Z_\parallel}{n}\Big\|_{\omega=0} = 0.020$ Ω
$m = 1$	$\mathcal{I}m\, Z_\perp\|_{\omega=0} = 21.5$ MΩ/m	$\mathcal{I}m\, Z_\perp\|_{\omega=0} = 9.7$ kΩ/m
Coupled Bunch		
$m = 0$	Worst growth rate driven at 0.885 GHz $= 1.0 \times 10^{-11}$ sec^{-1}	
$m = 1$	Worst growth rate driven at 1.313 GHz $= 7.3 \times 10^{-29}$ sec^{-1}	

Table III: Stability limits and bellows contributions

REFERENCES

1. C. Palm, U. Van Rienen, and T. Weiland, DESY M-85-11, 1985.

2. T. Weiland, DESY 82-015, 1982; Nucl. Instrum. Meth. **212**, 13 (1983).

3. For the stability limits, see for example, K.Y. Ng Fermilab Report TM-1385 (1986).

4. For the coupled-bunch growth rates, see for example, A.W. Chao, in *Summer School on High Energy Particle Accelerators, SLAC, Stanford, Aug. 1982, Ed. M. Month.*

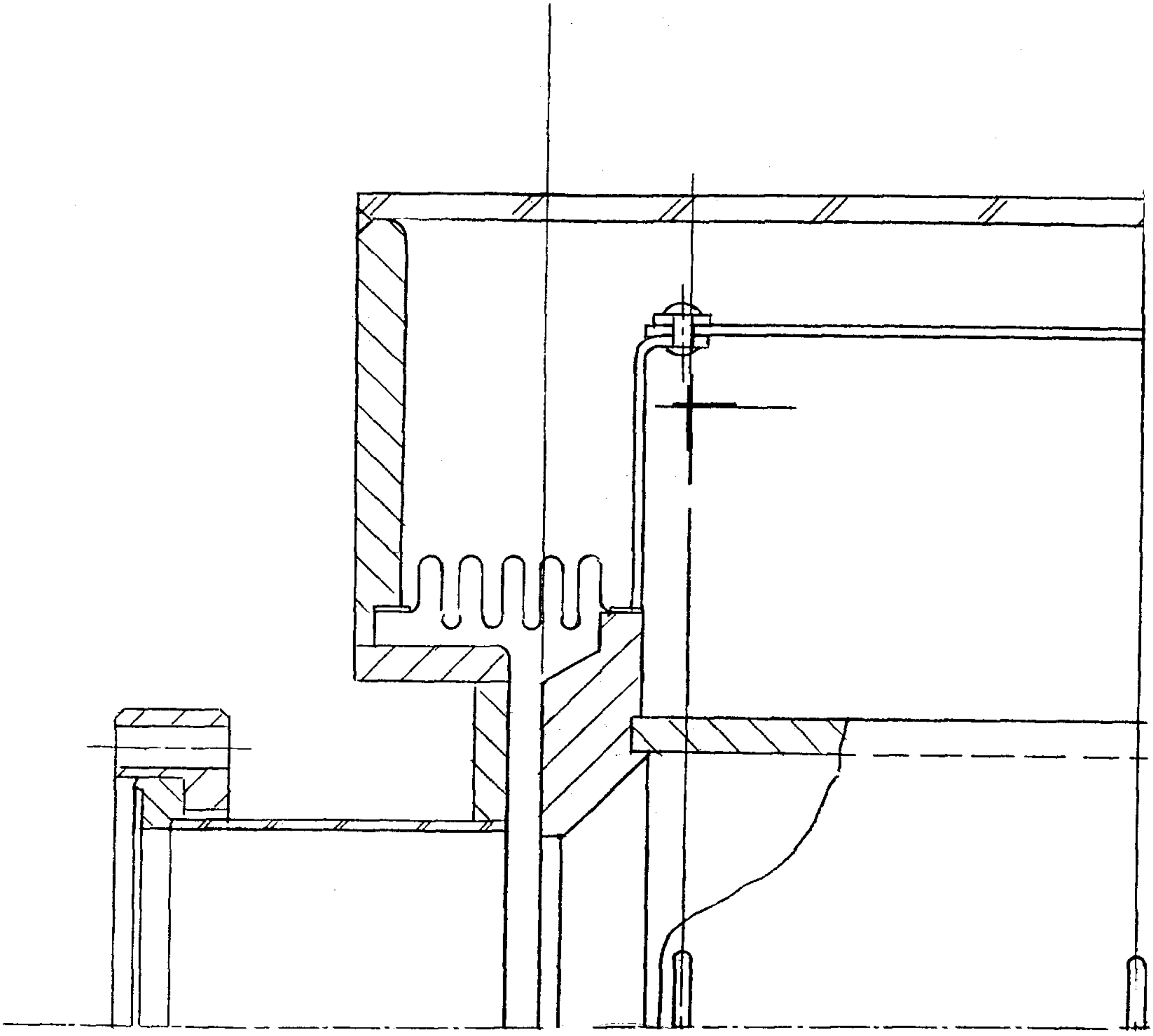

Figure 1. The bellows at low beta

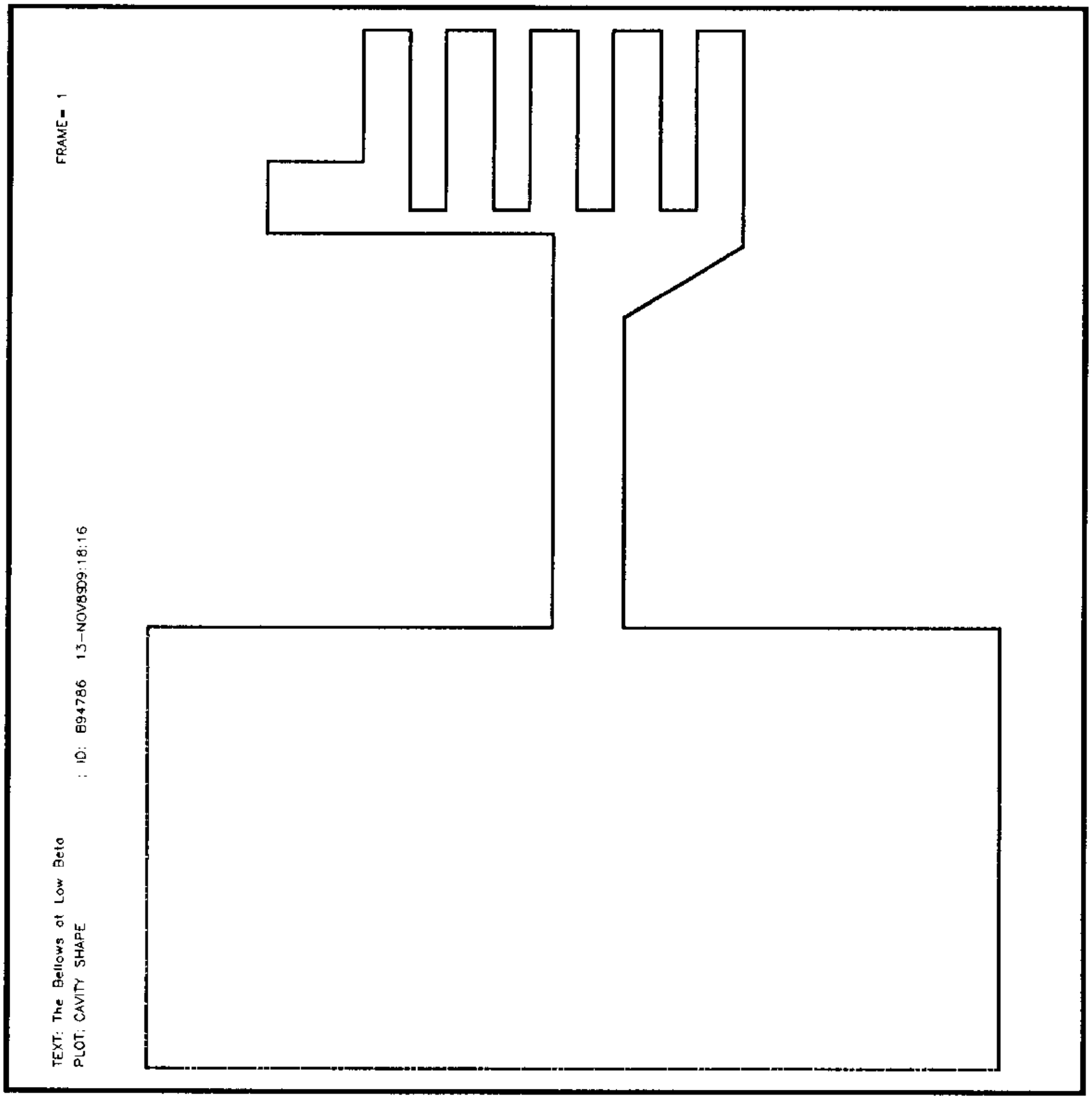

Figure 2. The geometry of the bellows used in URMEL computations

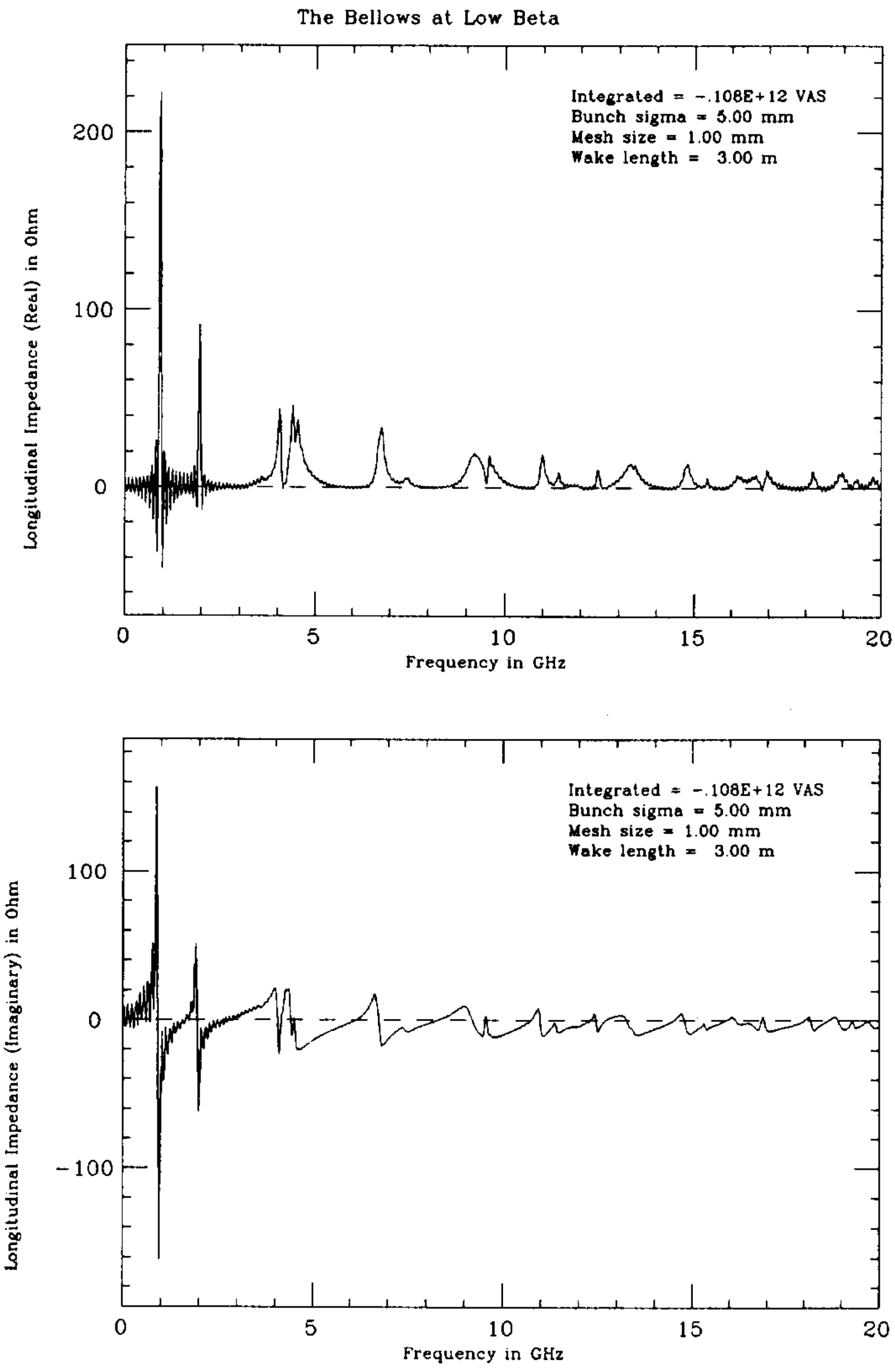

Figure 3. Longitudinal impedance of one bellows

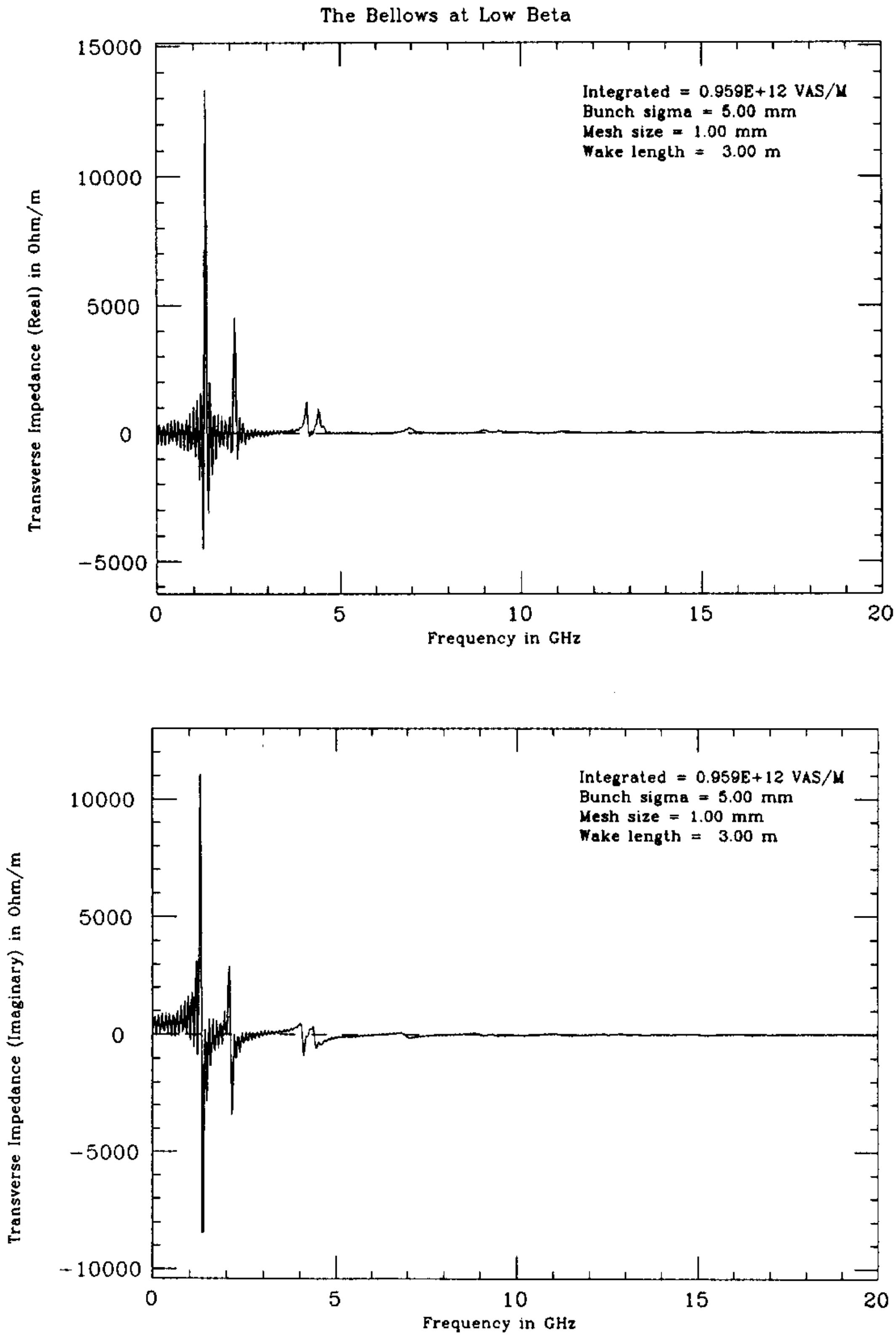

Figure 4. Transverse impedance of one bellows