Fermilab

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AUTOMATIC BEAM CENTERING AT THE SSC INTERACTION REGIONS

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In the SSC interaction regions, the two colliding beams, each only a few microns in size, will have to be centered and maintained in good alignment over many hours, in order to provide the maximum possible luminosity and to minimize off-center beam-beam focussing effects.

It is unlikely that sufficiently good alignment can be achieved without some kind of active feedback system, based on the beam-beam interaction rate. This memo describes such a system.

In the proposed scheme, one of the beams is moved continuously and in a circular fashion about its mean transverse position. The radius of this motion is approximately 0.01 of the rms beam size at the interaction point. The motion is achieved with two sets of crossed high frequency dipole magnets, one on each side of the interaction region, suitably phased.

As a consequence of this motion, the beam-beam interaction rate is modulated in synchronism with the beam motion when the beams are not centered on one another. The amplitude and phase of this modulation yields information on the magnitude and direction of the misalignment between the beams, allowing continuous display and automatic correction of any misalignment.

Luminosity versus Misalignment

Let us take two round, Gaussian shaped beams, separated in the horizontal (x) plane by a distance $\mathcal{D} = 2\,\delta$.

The overlap integral of the two beams, proportional to the luminosity L, can be written as

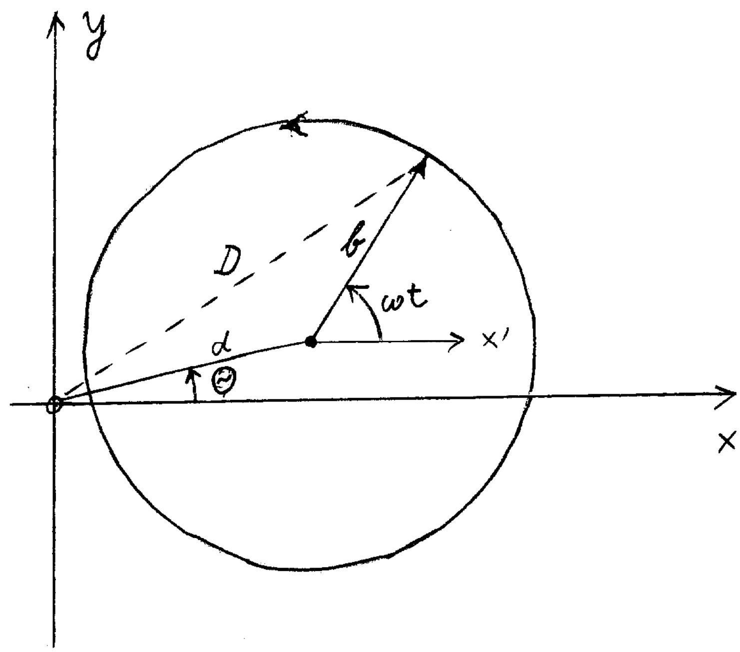
$$L \sim \iint_{-\infty}^{+\infty} e^{-\frac{(x+\delta)^2+y^2}{2\sigma^2}}\, e^{-\frac{(x-\delta)^2+y^2}{2\sigma^2}}\, dx\, dy =$$

$$= e^{-\frac{y^2}{\sigma^2}} \int_{x=-\infty}^{+\infty} e^{-\frac{(x+\delta)^2}{2\sigma^2}}\, e^{-\frac{(y-\delta)^2}{2\sigma^2}} =$$

$$= e^{-\frac{x^2+y^2}{\sigma^2}}\, e^{-\frac{\delta^2}{\sigma^2}} = L_0\, e^{-\frac{\delta^2}{\sigma^2}} = L_0\, e^{-\frac{(D/2)^2}{\sigma^2}}$$

where L_o is the luminosity for perfectly centered beams.

Geometry



In this diagram, the center of beam 1 is at the origin. Beam 2 has a misalignment described by the vector d, assumed to be slowly varying. In addition, beam 2 is made to follow the large circle of radius b with an angular frequency ω. The luminosity depends on the time-varying distance D, which we will now calculate:

$$
\begin{aligned}
D^2 &= (d\cos\Theta + b\cos(\omega t))^2 + (d\sin\Theta + b\sin(\omega t))^2 = \\
&= (d^2 + b^2) + 2\,d\,b\sin\Theta\sin(\omega t) + 2\,d\,b\cos\Theta\cos(\omega t) = \\
&= (d^2 + b^2) + 2\,d\,b\cos(\Theta + \omega t).
\end{aligned}
$$

As shown above, the luminosity depends on D as $L = L_0 \exp(-\frac{D^2}{4\sigma^2})$.
For a small distance $D \ll \sigma$, as is appropriate here, we write

$$L \cong L_o \left(1 - \frac{D^2}{4\sigma^2}\right) = L_o \left[\left(1 - \frac{d^2 + b^2}{4\sigma^2}\right) - \frac{2\,d\,b\cos(\Theta + \omega t)}{4\sigma^2} \right]$$

If we write it as a constant term plus an oscillating term, $L = L_o\,(1 - A\cos(\Theta + \omega t))$, we find that

$$A = \frac{2\,d\,b}{4\sigma^2\left(1 - \frac{d^2 + b^2}{4\sigma^2}\right)} = \frac{2\,d\,b}{4\sigma^2 - d^2 - b^2}$$

Finding Offset and Phase from the Counting Rates

The luminosity counter runs for a time interval T, containing many cycles of the beam rotation. A total of N_t counts is accumulated into k bins of the phase angle $\varphi \equiv \omega t$ (modulo 2π). We Fourier-analyze the data by convoluting with $\sin\varphi$ and $\cos\varphi$:

$$S = \frac{1}{N_t} \sum_{i=1}^{k} N_i \sin_i\varphi \qquad \text{and similarly for } \cos\varphi \ .$$

The error on S is $(\Delta S)^2 = N_t^{-2} \sum_i (\Delta N_i \sin_i\varphi)^2 = N_t^{-2} \sum_i N_i \sin_i^2\varphi$.

For the case of a small modulation we can calculate **S** and its error:
Take equal counting rates in all bins, $N_i \cong \frac{N_t}{k}$ to get

$$S = \frac{1}{N_t} \sum_{i=1}^{k} \frac{N_t}{k} (1 + A\sin\varphi)\sin\varphi = \frac{1}{2\pi} A\, 2\pi \langle \sin_i^2 \rangle = A/2.$$

$$(\Delta S)^2 = N_t^{-2} \sum_{i=1}^{k} \frac{N_t}{k} \sin_i^2\varphi = (2\,N_t)^{-1}; \qquad \Delta S = (2\,N_t)^{-1/2} .$$

Sensitivity

We find the minimum detectable offset by setting the signal S equal to its error and solving for d:

Set $\Delta S = S$.

$$A = 2S = \frac{2\,d\,b}{4\sigma^2 - d^2 - b^2} = 2\,\Delta S = 2(2N_t)^{-\frac{1}{2}} .$$

For small misalignment and small offset, $d \ll \sigma$ and $b \ll \sigma$, we simplify to

$$\frac{d\,b}{2\sigma^2} = 2\,(2N_t)^{-\frac{1}{2}}; \qquad d = \frac{4\,\sigma^2}{\sqrt{2N}\,b} .$$

Numerical example

Take luminosity $L = 10^{37}$ events/m^2 s, of which half is detected in the counters. Take the total cross section as $\sigma_{tot} = 1.3 \times 10^{-29}\ m^2$ and assume we operate the beam rotation at $b = 0.05\,\sigma$, corresponding to an average 0.[illegible]5% reduction in luminosity. The design beam size at a $\beta^* = 1$ m is $\sigma = \sqrt{\frac{\varepsilon\beta^*}{\gamma}} = 6.8 \times 10^{-6}$ m.

In a 1 sec measurement, yielding $\frac{1}{2} \cdot L \cdot \sigma_{tot} \cdot T = 6.5 \times 10^7$ counts we determine d to

$$\frac{4\,(6.8\ 10^{-6})^2}{\sqrt{2 \times 6.5 \times 10^7} \times 0.05 \times (6.8 \times 10^{-6})} = 4.8 \times 10^{-8}\ m = 0.007\,\sigma .$$

Under the same conditions, but a luminosity of only 10^{35} events/m^2 s we can still determine the misalignment to 7% of σ in a one second measurement. This error in d actually represents the radius of an error circle in the x-y-plane.