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FIELD QUALITY OF DOUBLER DIPOLES AND ITS POSSIBLE IMPLICATIONS*

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I. Introduction

A report[1] on the same subject was issued more than a year ago when there were only nine doubler dipoles built and measured. Since then, there have been many important changes in the overall design of the superconducting ring[2] and significant improvements in the field quality of dipoles. A statement made in the previous report - "One should therefore never imagine that the ultimate qulity of the doubler would be represented by these nine dipoles." - is indeed a correct one thanks to many people in the (then) Doubler Magnet Division and in the Accelerator Division. The most important design change has been to reduce the dipole length by one foot in order to create a space for more flexible and powerful correction systems. Before this reduction of the dipole length was implemented, fifty-five magnets had gone through a complete set of measurements. At present, there are twenty-two dipoles of the proper size of which eight are already in the tunnel, four have been accepted and four more are likely to be accepted soon. Although these numbers are small compared to 774, the total needed to complete the ring, it seems safe to assume that one is now in a position to speculate on the possible performance of the doubler.**

The emphasis in this report is on the resonance widths when the ring is used as a fixed-target accelerator since the evaluation of resonance widths is more or less straightforward (but not completely so).

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* "A View from Mezzanine"

**Of course this is true only if we strictly observe the present acceptance criteria for all magnets in the future.

The experience in the main ring as well as in the CERN SPS has clearly taught us that, during the injection flatbottom, the effective resonance width is a function of time. Near a resonance, the beam seems to feel an ever increasing resonance width as it is kept circulating. One casually invokes the effects of synchrotron oscillation, power supply ripples and even residual gas scattering to "explain" the phenomenon but there is as yet no satisfactory theory. It is therefore difficult if not downright impossible (for me) to predict the expected performance of the doubler as a storage ring. One can probably argue that, for pp or $\bar{p}p$ collider, the beam-beam interaction dominates the beam lifetime and the nonlinearity of the external field is but a small perturbation. This certainly is the case for the CERN ISR which is the only available source of information for proton storage rings. Speculations are (timidly) presented here regarding this aspect of the doubler with a full realization that it would be quite disappointing to those who refuse to consider anything but a clear-cut "yes" or "no" answer. They would have to go to someone who is wise enough and courageous enough to make such a statement.

I am again indebted to Dan Gross, Bob Peters, Alvin Tollestrup and Masayoshi Wake for clarifying many points on the data used here. The effort to improve the field qulity and the effort to utilize the data in the best possible manner are continuing. It is hoped that the next report will bring us a less ambiguous picture of the doubler both as an accelerator and as a storage ring.

II. Summary of Data

A. Integrated Bend Field $\int B\, d\ell$

Three methods are currently used: 1) stretched wire, 2) NMR and 3) combination of Hall probe and NMR. The NMR measurement is useful in finding any unusual "bump" in the field which almost always indicates a structural defect. The field flatness in the body of the magnet is typically within ±0.025%. The combination of NMR-Hall probe is neccesary to study the field fall-off near the edges and to find the precise value of effective field length. This method is still in the process of getting refined. With proper cares, it should also be an accurate measurement of the integrated bend field. Data given below are all based on the measurement by a stretched wire.

2,000A	61.065	- 0.06% (G-m/A) + 0.05%	(12 magnets)
4,000A	61.035	- 0.07% + 0.06%	(12 magnets)
4,100A	-0.066% change from 2,000A		(4 magnets)
4,200A	-0.073% change from 2,000A		(3 magnets)

B. Maximum Quench Current

At present, three types of quench current are measured for each magnet: 1) current goes up linearly at a certain value of dI/dt until the magnet quenches, 2) current goes up to a certain value with dI/dt = 200A/s and then held constant for ∿15 sec., 3) current is ramped ∿ ten cycles with dI/dt = 200A/s and with a flattop of ∿ 20 sec simulating the anticipated doubler ramp. The maximum quench current is influenced by the available refrigeration on measurement stands and by the quality of warm bores used for the measurement. For example, stand No. 6 tends to give lower values for the maximum quench current. The acceptance criterion, which is still somewhat flexible, is 4,100A (corresponding to 924 GeV/c) or higher for all three types of quenches. The maximum value of the first type is often more than 4,300A. In order to reach the goal of 1,000 GeV/c, it may become necessary later to replace a number of dipoles, how many to be replaced depending on the available refrigeration in the tunnel and on the number of "sub-standard" magnets installed. The present guidance is to minimize this number without imposing an unrealistic acceptance condition.

C. AC Loss

The design goal is 500 J/cycle from 0 to 4,000A with 200A/s. The average of 13 magnets is 521 J/cycle. Data on one dipole, TA0208, show that this value will be reduced by ∿ 50 J/cycle if the lowest current of the cycle is ∿400A instead of zero.

D. Vertical Plane Angle

This has been one of the most troublesome problems in the measurement. The vertical plane angle is typically a few mrad and shows no appreciable change when the excitation current is varied. Although field ramping does not seem to affect the angle, there is a large change (up to ∿3 mrad) caused by warm-up, cool-down cycles. An intense study is now in progress at MTF by C. Hojvat to understnad this change and to find ways of reducing it. Until this problem is completely solved, one must assume that dipoles installed in the tunnel would have an uncertainty of a few mrad. Obviously, one cannot build the entire ring in this manner; the resulting vertical distortion of the closed orbit will be too large to be compensated for by the correction system.[3] For example, if thirty dipoles were installed in series with a uniform distribution of vertical angles within ±3 mrad, the expected maximum distortion from these dipoles alone would be 5 mm at $\beta_y = 100$ m when one desired the probability of not exceeding this value to be 85%. It is essential that the problem associated with the vertical plane be solved before we install more than ∿30 magnets.

E. <u>Multipole Field Components</u>

Components of nonlinear field are represented by two parameters, b_n for the normal field and a_n for the skew field,

$$B_y^{(n)} \text{ (at +1", median plane)} = 10^{-4} B_o b_n$$

$$B_x^{(n)} \text{ (at +1", median plane)} = 10^{-4} B_o a_n$$

where B_o is the dipole field and n = 1 for quadrupoles, n = 2 for sextupoles, etc. In Table I, which is a summary of sixteen dipoles, the average, the rms, and the range of all samples are given at 500A, 1,000A and 4,000A. The last column gives the number of magnets for which the corresponding parameter goes beyond ±2×(rms). Numbers in the table should be compared with the criteria given in Table 3-I of the design report:[2]

$I \geqq 2{,}000$A	b_1, a_1	±2.5
	b_2	±6
	a_2, b_3, a_3, a_4	±2
	b_4	1.1±2

A few magnets with some parameters slightly outside the acceptance range have been accepted. It should be mentioned here that it is not meaningful to have a rigid set of criteria for all magnets to satisfy. The decision to accept or not to accept a particular magnet is influenced by many factors and the multipole components are just a part of these. What is important is to judge each magnet independent of those already accepted and a few exceptions should never be an excuse to alter the criteria.

Table I. Multipole Components

500A	average	rms	range			average	rms	range	
b_1	-.73	1.45	(-2.8,3.3)	(1)	a_1	-.27	1.90	(-3.8,2.8)	(0)
b_2	-8.59	2.86	(-12.6,-3.3)	(0)	a_2	-.17	1.19	(-2.5,2.2)	(0)
b_3	-.63	0.88	(-3.1,.65)	(1)	a_3	-.67	2.39	(-5.6,5.7)	(2)
b_4	1.61	1.61	(-1.3,4.7)	(0)	a_4	-.25	0.83	(-2.6,.57)	(1)
b_6	5.87	0.72	(4.8, 7.3)	(0)	a_6	-.28	0.37	(-1.0,.58)	(2)
b_8	-17.3	0.50	(-18.2,-16.2)	(1)	a_8	.15	0.59	(-.97,1.1)	(0)
b_{10}	5.84	0.47	(5.1, 6.7)	(0)	a_{10}	-.46	0.37	(-.97,.18)	(0)

1,000A	average	rms	range			average	rms	range	
b_1	-.63	1.41	(-2.2,3.3)	(1)	a_1	-.18	1.87	(-3.5,2.9)	(0)
b_2	-2.85	2.84	(-6.7,2.7)	(0)	a_2	-.12	1.15	(-2.3,2.1)	(0)
b_3	-.47	0.86	(-2.9,.85)	(1)	a_3	-.65	2.41	(-5.5,5.8)	(2)
b_4	1.61	1.65	(-1.2,4.7)	(0)	a_4	-.12	0.68	(-1.9,.72)	(1)
b_6	6.39	0.69	(5.0, 7.8)	(1)	a_6	-.15	0.30	(-.8, .4)	(1)
b_8	-17.3	0.34	(-18.1,-16.8)	(1)	a_8	.24	0.45	(-.6, .9)	(0)
b_{10}	5.69	0.37	(5.2, 6.3)	(0)	a_{10}	-.31	0.31	(-.8, .3)	(0)

(continued)

Table I (continued)

4,000A	average	rms	range			average	rms	range	
b_1	-.92	1.47	(-2.5,3.2)	(1)	a_1	.12	1.88	(-3.1,2.9)	(0)
b_2	-.90	2.86	(-4.5,4.8)	(0)	a_2	-.11	1.11	(-2.0,2.0)	(0)
b_3	-.34	0.82	(-2.7,1.0)	(1)	a_3	-.59	2.44	(-5.5,5.9)	(2)
b_4	1.38	1.73	(-1.5,4.8)	(0)	a_4	.00	0.56	(-1.3,1.0)	(1)
b_6	7.25	0.74	(5.7, 8.7)	(1)	a_6	-.05	0.27	(-.5, .4)	(0)
b_8	-17.5	0.30	(-18.3,-17.0)	(1)	a_8	.31	0.39	(-.4, .9)	(0)
b_{10}	5.64	0.32	(5.1, 6.3)	(0)	a_{10}	-.18	0.33	(-.8, .3)	(0)

Note 1. As the number of measured magnets increases, the average values will change but the rms and the range should stay more or less as they are now.

Note 2. A large difference in the normal sextupole component b_2 at various currents is due to the hysteresis. It is affected by the quality of superconducting filaments. Before 200-series, the average difference from 500A to 1,000A was 4.3 and from 4,000A to 1,000A was 0.74.

III. Dependence of Tunes on the Momentum Deviation ($\Delta p/p$) and on the Betatron Oscillation Amplitude

The tune of the betatron oscillation is in general a complicated function of the momentum deviation ($\Delta p/p$) and the oscillation amplitudes in both transverse directions. Because of the skew field components, the closed orbit is not confined in the median plane. It may not be possible to eliminate the orbit distortion completely by steering dipoles, especially at high energies, and this further complicates the dependence. In order to include all possible effects, it is necessary to run many cases numerically and to sort out the relative importance of these effects. This is certainly very time consuming.

It is not difficult to find analytical expressions for the tune shifts $\Delta\nu_x$ and $\Delta\nu_y$ if one assumes that the closed orbit is confined in the median plane. This assumption eliminates the effects of all skew components $\{a_n\}$ and also the dependence on the finite amplitude of the vertical betatron oscillation. The distortion in the horizontal closed orbit does not introduce an essential difficulty but the resulting expression becomes messy. Since one is interested in the dependence on the betatron oscillation amplitude (radial only), the amplitude $(\hat{\beta}W)^{1/2}$ should be a well-defined quantity of the particle. This is the case when the operating point (ν_x, ν_y) is away from all resonances. Near nonlinear resonances, W ("emittance" of each particle) itself is a function of the azimuthal angle θ. When the particle goes out of the stable area, W may grow rapidly, a typical example being resonance extractions. Expressions given below, which must have been derived by many people,[4] cannot be used under these conditions. For the doubler with 774 dipoles, the contribution to the tune shift from each dipole is

$$\Delta\nu_x = (\beta_x/774) \sum_n b_n \sum_k^n (\delta_p \eta_x)^{n-k} (\hat{x}_\beta)^{k-1} \binom{n}{k} C_{k+1} , \tag{1}$$

$$\Delta\nu_y = -(\beta_y/2\times774) \sum_n b_n \sum_{k=1}^{n} (\delta_p \eta_x)^{n-k} (\hat{x}_\beta)^{k-1} k \binom{n}{k} C_{k-1} \tag{2}$$

where β_x, β_y, η_x = betatron functions and momentum dispersion function at the magnet, $\delta_p = (\Delta p/p)$, $\hat{x}_\beta = \sqrt{\beta_x W_o} \equiv$ the average* betatron amplitude at the magnet, $b_n (n=1, 2, \ldots)$ = normal field components of the magnet,

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} ,$$

$$C_m = 0 \qquad \text{for odd } m$$

$$= \frac{m!}{2^m [(m/2)!]^2} \qquad \text{for even } m.$$

The tune shifts are the summation of these contributions from all dipoles. Eqs. (1) and (2) clearly show the importance of higher multipole fields in the magnets where β_x and η_x are large. With low-beta insertions, it

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* When the radial excursion at θ is written as $x(\theta) = \sqrt{\beta_x(\theta) W} \cos\phi$, W_o is the average of W over θ and ϕ.

is quite possible that contributions from a few special quadrupoles are comparable to or even greater than the contributions from all other magnets. Possible dependence of tunes on δ_p and on W_o for each multipole is shown in Table II.

Table II. Possible dependence of tunes on δ_p and W_o.

A. "natural" multipoles - odd powers of δ_p

n = 2	sextupole	δ_p
= 4	decapole	δ_p^3 , $\delta_p W_o$
= 6	14-pole	δ_p^5 , $\delta_p^3 W_o$, $\delta_p W_o^2$

B. "error" multipoles - even powers of δ_p

n = 1	quadrupole	independent of δ_p or W_o
= 3	octupole	δ_p^2 , W_o
= 5	12-pole	δ_p^4 , $\delta_p^2 W_o$, W_o^2

Effects of quadrupole and sextupole components will be corrected by trim quadrupoles and chromaticity sextupoles, respectively. There will also be some octupole corrections but the final design of the system will depend on the average octupole field of all magnets. Unless drastic changes in the construction of dipoles or changes in the acceptance procedures are made in the future, the expected average sextupole components and the corresponding chromaticities are $(\xi \equiv \Delta\nu/\delta_p)$

660A (150 GeV/c injection)

$(b_2)_{av} = -6 \sim -4$, $\xi_x = -160 \sim -107$, $\xi_y = 146 \sim 97$

1,000A $(b_2)_{av} = -3 \sim -1$, $\xi_x = -80 \sim -27$, $\xi_y = 73 \sim 24$

4,000A $(b_2)_{av} = -1 \sim +1$, $\xi_x = -27 \sim +27$, $\xi_y = 24 \sim -24$

Note: The natural chromaticity arising from the quadrupole aberration is - 22 in both directions.

It is perhaps instructive to find the tune shifts from the sum of Eq. (1) or Eq. (2) over all dipoles assuming many random sets of $\{b_n\}$. At the same time, since what is essential is the average value of b_n rather than its fluctuation from magnet to magnet, one probably gains almost all necessary information by listing the contribution of constant b_n for each n and for combinations of (δ_p, W_o). Figs. 1 - 3 are computed with $b_n = +1$. The momentum spread and the transverse emittance of the injected beam at 150 GeV/c are expected to be ±0.03% or less and 0.15π mm-mr, respectively. The value $\delta_p = 0.2\%$ is an example of possible momentum stacking and $W_o = 2.5$ mm-mr corresponds to the amplitude of 16 mm at $\beta_x = 100$ m.

IV. Widths of Various Resonances*

In the past twenty-five years or so, there have been numerous papers and reports on the subject of nonlinear resonances. The most recent one is a two-part report by G. Guignard who included the effects of longitudinal field component.[5] In a way, the calculation of resonance widths is a well-established technique and one simply substitues an appropriate set of parameters in the existing formulas. The formalism for obtaining such formulas is not always the same[6] but one standard recipe goes as follows. Equations of motion with linear and nonlinear magnetic field are easy to write down and the corresponding Hamiltonian can be found without any difficulty. Coordinates are x, dx/ds, y, dy/ds and s which is the path length variable. One then transforms these variables to the so-called action-angle variables which are essentially the amplitudes and phases of the betatron oscillation. The independent variable is usually $\theta \equiv s/R$, $R \equiv$ average machine radius. For two-dimentional problems (no horizontal-vertical coupling), it is customary and convenient to use the normalized phase $\phi \equiv$ phase advance/2π instead of θ as the independent variable but this is not essential. Since the machine parameters and the field are all periodic in θ (or ϕ) with the period 2π, one can Fourier-analyze the nonlinear part of the Hamiltonian. The formalism up to this point is "exact" to the extent that the original equations

* I have tried to avoid writing down equations. As a consequence, this section may be too wordy for some people.

are "exact". The transformed Hamiltonian still depends explicitly on the independent variable so that it is not a constant of the motion. The most important approximation in almost all treatments of nonlinear resonances is to retain in the Hamiltonian only those terms that are independent of the angle variables and θ ("phase-independent terms") and those that can drive the resonance under consideration ("driving terms"). It is usually argued that all other terms are rapidly oscillating so that their contributions are insignificant when averaged over many betatron oscillations. Clearly this should be a good approximation if the tune is close to one resonance and far away from all others. It is not valid at all when the tune is equally close to two or more resonances. The approximation enables one to make a simple canonical transformation (essentially a transformation to a rotating coordinate system) such that the resulting Hamiltonian is independent of θ, that is, a constant of the motion. For two-dimensional cases, this Hamiltonian describes the familiar flow diagram in phase space with stable and unstable regions. Given the beam emittance, one can find the resonance width by setting the area of the inner stable region equal to the beam emittance. For horizontal-vertical coupled resonances, one can find, in addition to the Hamiltonian, another constant of the motion by simply inspecting the equations of motion for two action variables.

When the resonance is of the form

$$n\,\nu_x + m\,\nu_y = p, \quad (n,\ m,\ p = \text{positive integers*}) \tag{3}$$

two constants of the motion can be written in the form

$$C_1 \equiv u^2 + Au^n v^m w + f(u^2,\ v^2) \tag{4}$$

$$C_2 \equiv v^2 + Au^n v^m w + f(u^2,\ v^2) \tag{5}$$

where $f(u^2, v^2)$ is the contribution from phase-independent terms in the Hamiltonian, u and v are proportional to two oscillation amplitudes, A (>0) is linearly proportional to the strength of nonlinear field that

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* It is well-known that, for difference resonances $n\cdot m < 0$, two amplitudes are always limited and the motion never becomes truely unstable. As a consequence, there is no unique way of defining the resonance width. See ref. 5 for one definition.

is driving the resonance, and

$$w = + \text{ or } - \cos(na_x + ma_y + \text{constant}) \tag{6}$$

with two phase variables a_x and a_y. Values of C_1 and C_2 are completely specified by the initial conditions for amplitudes (u, v) and phases (a_x, a_y). In the four-dimensional phase space, the motion is governed by the requirement that

$$u, v \geq 0 \quad \text{and} \quad |w| \leq 1. \tag{7}$$

Note that $u^2 - v^2 = C_1 - C_2$ = constant of motion. With this relation, one can eliminate v and study the behavior of w as a function of u. Specific examples are given elsewhere.[6,7] It is worth mentioning here that two phase variables appear in C_1 and C_2 in the combination $na_x + ma_y$ so that separate information on each is not available. However, this is not a very serious limitation. One usually assumes that the particle distribution in a_x and a_y is uniform and the stability is a question of amplitudes only. The problem of evaluating the resonance width is here reduced to solving Eqs. (4) and (5) with the condition (7). Algebraic solutions are possible only for low-order resonances and without contributions from the phase-independent terms.

The procedures Guignard uses to derive the general expression for resonance width are quite different from the ones given here.[5,8] His formula can be obtained from the conditions that the derivatives of two amplitudes and of $na_x + ma_y$ with respect to θ vanish at $|w| = 1$. For two-dimensional problems, this gives the fixed points in phase space and the area of the circle (in the properly normalized phase space) defined by these points can be set equal to the beam emittance. A particle outside the circle is unstable regardless of its phase. A particle inside the circle may be stable or unstable depending on its amplitude and phase. If one asked that a particle outside its emittance circle should be stable regardless of its phase ("non-adiabatic" model), it would lead to a larger resonance width than Guignard's. One can also argue that the onset of resonance is adiabatic and the beam emittance should be set equal to the area of the stable region which is generally non-circular.

In this model ("adiabatic" model), the beam which originally occupied a circular area in the normalized phase space adjusts its shape adiabatically to the shape of the non-circular stable region. The resonance width from this will be larger than Guignard's but smaller than the non-adiabatic width. T. Collins and D. Edwards[9] used this model and gave expressions (with numerical coefficients) for all resonances up to the fifth-order, $n + m \leq 5$. For coupling resonances, the conditions used by Guignard do not really define the true fixed points since a_x and a_y are not separately stationary. They define a curve on the (u, v) space and the width is obtained by the condition that, for a given set of beam emittance (E_x, E_y), the corresponding amplitudes (u_o, v_o) sit on the curve. A peculiar and hard-to-understand feature of the formula obtained by Guignard in this manner is that, for resonances with either $n = 1$ or $m = 1$, the inner part of a beam may have a larger resonance width compared to the outer part of the same beam. In using his formula, it is essential that one understands the nature of the width one is calculating.

It is unfortunate (for me) that Collins and Edwards[9] do not explain how they obtained numerical coefficients in their expressions for various resonance widths. Integral expressions for the driving term (A_k and B_k on p. 19, ref. 9) suggest that they used a formalism similar to the one used by Sturrock.[6] I have tried to reproduce their numerical values by solving Eqs. (4) and (5) but attained only a partial success. Two-dimensional cases, $n = 0$ or $m = 0$, are easy to handle and numerical values for them are all in agreement with what I have found. As for coupled resonances, I can reproduce their results (less than 10% difference) for sextupole ($n+m=3$) and normal octupole ($n+m=4$, n, m=even) resonances only. Resonances arising from skew octupoles ($n+m=4$, n, m=odd) and from decapoles have so far resisted my effort to solve Eqs. (4) and (5) algebraically. This is a rather embarrasing confession to make in a report.

It is probably fair to say that, during one-turn injection and acceleration, resonances of the order four and higher play a relatively minor role as far as the transmission efficiency of the beam is concerned.

In the remainder of this section, numerical results on the resonance width are given only for resonances of the order four and lower. Strengths of multipole components used for the calculation are either the average values or twice the rms values given in Table I. It is assumed that the upper limit of the width in the real doubler is less than the value calculated in this manner. A partial justification for this is given in the appendix where the strategy for magnet "shuffling" is explained. Non-adiabatic model is used throughout and the beam is assumed to occupy the emittance $E_x \leq E_o$ and $E_y \leq E_o$, this in contrast with the model of Collins and Edwards,[9] adiabatic and $E_x + E_y \leq E_o$ or $E_x + 4E_y \leq E_o$. The present estimate of the width is therefore "conservative". The width is always linearly proportional to the field strength. At present, there is no reason to suspect that the second-order effects[10] will not be negligible. On the other hand, one always worry about the resonances induced by the distortion of the closed orbit. Some of us remember the disastrous effect of correction sextupoles during the initial phase of main ring operation. When lumped sextupoles were turned on for the chromaticity correction, the beam almost disappeared. A large orbit distortion with rich 19th and 20th harmonic contents coupled with the 60th harmonic component of the strong sextupoles must have produced strong 40th and 41st quadrupole harmonic components to drive resonances $2\nu = 40$ and 41. The tune at the time was between 20 and 20.5. Estimates of the effect of a closed-orbit distortion are included here for cases that are expected to be important. For many reasons, a good closed orbit is probably more important than anything else in the initial operation of the doubler. After all, doubler magnets were never designed to have an "unnecessary" aperture for the beam.

Higher-order resonances are undoubtedly important in the storage mode and, to a lesser extent, during the extraction flattop and the multi-turn (momentum stacking) injection. Since one must deal with multi-resonance phenomena for such cases, the conventional treatment of resonances which contains the fundamental defect of one-resonance-at-a-time concept cannot adequately describe the situation. A speculative discussion on the expected performance of the doubler as a storage ring will be given in the next section.

The standard doubler lattice with two high-beta insertions at AØ and DØ has been used to calculate machine parameters, with $\nu_x = 19.4257$ and $\nu_y = 19.3877$. The full resonance width (f.r.w.) is twice the distance from the operating point to the resonance line,

$$n\nu_x + m\nu_y = p + \varepsilon, \quad \text{f.r.w.} \equiv 2|\varepsilon|/\sqrt{n^2 + m^2}$$

A. Quadrupole resonances: $2\times(b_1)_{rms}=2.9$, $2\times(a_1)_{rms}=3.8$

$|(a_1)_{av}|=0.3$

$2\nu_x = 39$: f.r.w.=0.0252 $\quad$ $2\nu_y = 39$: f.r.w.=0.0255

$\nu_x - \nu_y = 0$: c_o =0.0317

The maximum amount of coupling in amplitude is given by $|c_o|/\sqrt{(\varepsilon/2)^2 + c_o^2}$ where c_o is the coupling parameter[11] and $\nu_x - \nu_y = \varepsilon$. The total strength of the correction skew quadrupoles placed at all quadrupole locations should be $|B'\ell|$=250 kG at 1 TeV/c.

$\nu_x + \nu_y = 39$: $|c_o|$ =0.0147

The maximum amount of coupling in amplitude is $|c_o|/\sqrt{(\varepsilon/2)^2 + c_o^2}$ and $\varepsilon \equiv \nu_x+\nu_y-39$. If this is to be less than 10%, one must have $|\varepsilon|>0.3$. This resonance is probably important only during the slow extraction. If the magnet shuffling explained in the appendix is successful, we may not need any harmonic corrections for this.

B. Sextupole resonances : f.r.w. $\propto$ (beam emittance)$^{1/2}$

$2\times(b_2)_{rms}=5.7$, $2\times(a_2)_{rms}=2.3$

Since the lattice periodicity is two, there will be 58th harmonic component arising from $(b_2)_{av}$ and $(a_2)_{av}$. However, this effect is generally small.

$3\nu_x = 58$

injection:	$(b_2)_{av}=-5.9$,	emittance=$.15\pi$ mm-mr,	f.r.w.=6.1×10^{-4}
1,000 GeV/c	$= \pm1$	$=.03\pi$	$=4.6\times10^{-5}$

From $2\times(b_2)_{rms}$, f.r.w. = 0.00643 (emittance 0.15π mm-mr)
= 0.00287 (" 0.03π mm-mr)

$\underline{\nu_x + 2\nu_y = 58}$

Contributions from $(b_2)_{av}$ are small.
From $2\times(b_2)_{rms}$, f.r.w. = 0.0111 (emittance 0.15π mm-mr)
= 0.00497(" 0.03π mm-mr)

$\underline{3\nu_y = 58}$

Contributions from $(a_2)_{av}$ are small.
From $2\times(a_2)_{rms}$, f.r.w. = 0.00264 (emittance 0.15π mm-mr)
= 0.00118 " 0.03π mm-mr)

$\underline{2\nu_x + \nu_y = 58}$

From $2\times(a_2)_{rms}$, f.r.w. = 0.00446 (emittance 0.15π mm-mr)
= 0.00199 (" 0.03π mm-mr)

C. <u>Octupole resonances</u> : f.r.w. $\propto$ (beam emittance)

In evaluating the widths here, effects of the phase-independent terms are ignored. In a real machine, it is not very clear how much meaning one should give to resonance widths calculated in this manner.

$2\times(b_3)_{rms} = 1.7$, $2\times(a_3)_{rms} = 4.9$, $(b_3)_{av} = \pm 0.6$

$\underline{4\nu_x = 78}$, $\underline{4\nu_y = 78}$

From $(b_3)_{av}$, f.r.w. $\simeq 4\times10^{-5}$; from $2\times(b_3)_{rms}$, f.r.w.$=1.6\times10^{-4}$, both with emittance = 0.15π mm-mr.

$\underline{2\nu_x + 2\nu_y = 78}$

From $(b_3)_{av}$, f.r.w.$\simeq 1.4\times10^{-4}$; from $2\times(b_3)_{rms}$, f.r.w.$=4.3\times10^{-4}$, both with emittance = 0.15π mm-mr.

$3\nu_x + \nu_y = 78$, $\nu_x + 3\nu_y = 78$

From $2\times(a_3)_{rms}$, f.r.w.$\simeq$ 0.001 with emittance = 0.15π mm-mr. These resonances may lead to a large oscillation amplitude without becoming unstable.[7] I am not at all sure how one handles these resonances in the adiabatic model.

Effects of Closed Orbit Distortion

It is assumed here that the dominant component of the closed orbit x_c or y_c is its 19th harmonic,

$$x_c \simeq \sqrt{\beta_x W_x}\cos(19\phi_x + \text{const.}),$$

$$y_c \simeq \sqrt{\beta_y W_y}\cos(19\phi_y + \text{const.}).$$

A. Normal sextupole effects. f.r.w. $\propto W_x^{1/2}$ or $W_y^{1/2}$

$(b_2)_{av} = -5.9$ at injection, $2\times(b_2)_{rms} = 5.7$

$2\nu_x = 39$: From $(b_2)_{av}$, f.r.w.$\simeq 3.20\sqrt{W_x}$
From $2\times(b_2)_{rms}$, f.r.w.$\simeq 23.4\sqrt{W_x}$

For example, with $W_x = 1\times10^{-6}$m (max $|x_c|$=1 cm at β_x=100m), f.r.w. are 0.0032 and 0.0234, respectively.

$2\nu_y = 39$: From $(b_2)_{av}$, f.r.w. $\simeq 1.18\sqrt{W_x}$
From $2\times(b_2)_{rms}$, f.r.w. $\simeq 18.8\sqrt{W_x}$

These are comparable to what one expects from $2\times(b_1)_{rms}$.

$\nu_x - \nu_y = 0$: No contributions from $(b_2)_{av}$.

From $2\times(b_2)_{rms}$, $|c_o| \simeq 9.39\sqrt{W_y}$. For example, with $W_y = 1\times10^{-6}$m (max. $|y_c|$=1 cm at β_y=100m), $|c_o|$= 0.0094.

$\nu_x + \nu_y = 39.$: Contributions from $(b_2)_{av}$ are very small.

From $2\times(b_2)_{rms}$, $|c_o| \simeq 9.39\sqrt{W_y}$ which is comparable to what one expects from $2\times(a_1)_{rms}$.

B. Normal decapole effects.

Equivalent quadrupole and sextupole components are

$$\Delta b_1 = 4(x_c^3 - 3\,x_c y_c^2)\times b_4$$
$$\Delta a_1 = 4(-y_c^3 + 3x_c^2 y_c)\times b_4$$
$$\Delta b_2 = 6(x_c^2 - y_c^2)\times b_4.$$

With $(b_4)_{av} = 1.6$, $2\times(b_4)_{rms} = 3.4$ and $W_x = W_y = 1\times10^{-6}$m, all effects are small.

$\underline{2\nu_x = 39}$, $\underline{2\nu_y = 39}$: f.r.w. < 0.004

$\underline{\nu_x - \nu_y = 0}$, $\underline{\nu_x + \nu_y = 39}$: $c_o < 0.0016$

Contributions from Δb_2 are all negligible.

If one looks at these numbers, one may get two entirely opposite impressions depending on his (or her) experiences and background.

1. These widths are all so small that there is nothing to worry about.

2. These widths are much larger than one expected (or even calculated).

Unfortunately, both impressions are "right" and the dilemma once again points out the importance of knowing what one is evaluating and the limitations involved in the evaluation. Experiences in the main ring taught us that the straightforward estimate of widths is always optimistic. The model used in this report is in a way a reaction to the experiences and it tries to maximize the value. One reason for using this model is to find the width outside of which the beam is not only stable but also develops very little distortion. This is especially important for resonances of the type $\nu_x \pm \nu_y$, $\nu_x + 3\nu_y$ and $3\nu_x + \nu_y$. At the same time, all widths are indeed small. Aside from the skew quadrupole corrections, there seems to be no necessity of having harmonic correction systems if 1) the total tune spread coming from chromaticity, power supply ripples and oscillation amplitudes is confined to $\sim \pm 0.01$ and 2) the closed orbit distortion is less than $\sim$ 5mm at β= 100 m in both directions.

For the resonance widths based on the Collins-Edwards model or the Guignard prescription, my arithmetic gives the following factors that should be used to reduce the values given in this section.

	Collins-Edwards	Guignard
$3\nu_x$, $3\nu_y$	0.778	0.500
$4\nu_x$, $4\nu_y$	0.738	0.414
$\nu_x + 2\nu_y$, $2\nu_x + \nu_y$	0.645	0.518
$2\nu_x + 2\nu_y$	0.417	0.414
$\nu_x + 3\nu_y$, $3\nu_x + \nu_y$	?	0.433

V. Doubler as a Storage Ring

The desire of experimental physicists to use the doubler as a storage device for 1 TeV colliding experiments is universally accepted. In contrast with this, the apprehension and trepidation of some (but by no means all) accelerator builders are not necessarily appreciated. In predicting the expected performance of the doubler as a storage device, if one tried to be totally honest, he might reveal not only his ignorance but the ignorance of his colleagues as well.* On the other hand, if one made an unqualified statement, he might be accused of being less-than-honest or, worse still, of not knowing what he was talking about.**

* "What a fool Honesty is! ....." - Autolycus -

** This is not really fair. There are at present a number of serious and promising efforts going on to understand the long-time beam behavior in nonlinear fields. Sam Kheifets at SLAC developed a theory for electron storage rings and, in a somewhat similar manner, Fred Mills and Sandro Ruggiero are working on a comprehensive theory for electron and proton storage rings.

It is perhaps possible that a person who is neither honest nor intrepid may be able to strike a balance of some sort and this possibility is the only justification for adding this section in the report.

The starting point of discussions on colliding devices could be a flat statement that, in a "reasonable" ring, the beam lifetime or the luminosity lifetime is predominantly determined by the beam-beam interaction. This certainly is the case for the ISR in which the nonlinear field of magnets is much weaker than the field coming from the beam-beam interaction. Since not much can be done about the beam-beam interaction if one is to retain a certain luminosity, all accelerator builders can do is to build a storage ring with magnets which do not add nonlinear fields to the extent of changing the beam lifetime substantially. This immediately raises the crucial question: how much nonlinear field one can have in the ring for the beam to survive with the useful emittance for, say, twenty-four hours. Obviously, this cannot be answered in a quantitative manner unless the mechanism of beam loss and emittance increase is understood and this, unfortunately, is a problem of stability in nonlinear dynamics which is yet to be solved.

In 1961, Amman and Ritson[12] introduced a parameter called the linear tune shift to express the strength of beam-beam interaction and, ever since, arguments have been centered on the maximum tolerable value of this parameter. For proton storage ring, the number 0.005 came to be regarded as "the value" with names like Courant and Keil associated with it. Later numerimcal simulations with realistic models of the field began to cast a doubt on the meaning of this value and numbers as large as 0.04 were mentioned. At present, most people will still use 0.005 in designing a storage ring mostly because it is felt to be a conservative value. Another reason is that, for $\bar{p}p$ collidings, beams are most likely to be bunched and the collision to be head-on, thereby increasing the number of degrees of freedom to three. In the ISR, since beams are unbunched and the crossing angle is fairly large, there is essentially only one degree of freedom. Computer simulations as well as the current theoretical prediction give a lower limit of the tune shift when more dimensions in the phase space are involved. Recently, an entirely new approach has been suggested by S. Kheifets[13] and by Fred Mills and Sandro Ruggiero.[14] According to them, the gradual increase in the beam emittance is really caused not by the beam-beam interaction alone but by the combination of that with random noise felt by the beam between successive nonlinear kicks at the interaction points. Kheifets does not explicitly mention the physical origin of this noise but represents the effects with one phenomenological parameter. Mills and Ruggiero attempt to derive a diffusion coefficient from various physical effects such as Coulomb scattering by the residual gas, intrabeam scattering, power supply ripples and the synchrotron motion. The limiting tune shift in this model depends strongly on the condition of each machine and the experimental results from the ISR may not be applicable to the doubler or any other proton storage rings. Indeed, the role of the tune shift will be substantially reduced if this model turns out to be true.

Since the ISR is the only existing proton storage ring, one is forced to depend on its experiences for making any prediction on the effect of nonlinear field. One aspect of the problem in the work by Mills and Ruggiero is that the shape and the azimuthal distribution of the nonlinear field may play an important role. The field of the beam-beam interaction, which obeys the Poisson equation, is of course confined mostly in the beam itself if one beam is on top of the other. Azimuthally, it is point-like and therefore contains many harmonic components. In contrast, the external nonlinear field of magnets extends over the entire aperture of the machine and it is distributed almost continuously in the ring. Depending on whether one considers this difference to be important or not, one must look at results from various ISR beam experiments differently. For example, for estimating the effect of external field in the doubler, one may be tempted to use the results of experiments by Keil[15] who used an external nonlinear lens in the ISR. The field of this lens is the same as the field of magnets in its dependence on the coordinates, obeying the Laplace equation instead of the Poisson equation. However, the field does not cover the entire machine aperture. Because of the geometrical symmetry, only even multipoles (quadrupole, octupole, 12-pole, etc.) can exist and it covers a very short azimuthal region. The experiment by Zotter[16] is for the beam-beam interaction proper. The effective strength of the interaction is enhanced by changing the machine parameter at the interaction point. Results from this experiment may or may not be applicable for our purpose. Eventually we may have to repeat Keil's experiment in the main ring with distributed nonlinear lenses.

The importance of the difference between the beam-beam interaction and the magnet nonlinearity may have been demonstrated by the following experimental results. One surprising thing found in Keil's experiment is that the beam decay rate in the presence of the nonlinear lens is quite sensitive to the tunes. The nonlinear lens produced a linear tune shift of 0.054 and approximately the same amount of tune spread (FWHM) within the beam. When the tunes of the ISR were changed by ± 0.0025, which is much less than the shift or the spread, the decay rate changed by an order of magnitude or more. From this, Keil concluded that resonances of orders up to ten or so must be involved in the beam decay. A contrasting result has been obtained in the CERN SPS recently.[17] A beam with the momentum 240 GeV/c and the intensity of 1×10^{12} was stored with the lifetime of around ten hours. A scraper was used to limit the aperture and the corresponding acceptance was 0.12π mm-mr. They found that the beam lifetime is rather insensitive to tunes, chromaticities or rf on-off. The effect of the fifth and seventh order resonances was not observed at all and, without the aperture-limiting scraper, the beam lifetime was always forty to fifty hours, this even with the chromaticity of ~ -26. Nonlinear field in the SPS is of course rather weak; the tune-vs-$(\Delta p/p)$ curve is practically straight up to $(\Delta p/p) = \pm 0.6\%$. Nevertheless, the transmission of the beam in the regular injection flatbottom depends strongly on the tune. Perhaps the small acceptance created by the scraper made the difference. Perhaps it was something entirely different. Even admitting the difference in two machines and in the beam conditions, one is further confused on the issue of what is really responsible for the emittance growth and the beam loss.

I hope I have succeeded by now in making it abundantly clear to everyone that nothing is clear in this subject. The lengthy introduction is really meant to forestall expected critical comments on the naive approach I am going to take below and I hope that "caveat emptor" is redundant here.

We start with the experiment by Keil and Leroy.[15] Immediately we face the unfortunate circumstance that their lens produces the even multipoles (quadrupole, octupole, etc.) only while for the doubler dipoles these are the error multipoles and rather small. (See Table I, p. 5 and p. 6.) We should be more worried about the effects of odd multipoles (sextupoles, decapoles, etc.) which are the natural multipoles. Let us just look at the octupole resonance widths and compare them. It is not clear from their report what the beam emittance was for this experiment. Judging from the information in the report, I believe it was ∿ 0.5π mm-mr.* We take the strength of nonlinear lens that will give the linear tune shift of 0.005(!). Actually, their experiment covered the range of 0.01 to 0.09.

Keil-Leroy: liner tune shift = 0.005, emittance = 0.5π mm-mr

$4\nu_x$, $4\nu_y$	f.r.w. = 0.94×10^{-3}
$2\nu_x + 2\nu_y$	f.r.w. = 4.0×10^{-3}

Doubler (see p. 15)

$4\nu_x$, $4\nu_y$	f.r.w. = 2×10^{-4}
$2\nu_x + 2\nu_y$	f.r.w. = 5×10^{-4}

Note that the emittance used for the doubler is 0.15πmm-mr. Presumably this is an overestimate at 500 - 1,000 GeV/c. Even with this overestimate of the emittance (remember that the resonance width of order four is proportional to emittance), there is a difference of factor five to ten. No comparison has been made for higher order resonances but there is no obvious reason to suspect that the widths in the doubler would become larger than the ones from the Keil-Leroy lens.

Keil and Leroy concluded from their results that resonances of orders up to ten may be involved in the beam decay. In the doubler, it is expected that the tune will be in the range 19.37 - 19.42. The operating point is then safely away from the second- and third-order resonances.** One can compute the distance from an operating point within this range to all resonances up to the tenth order. From this point of view, the best

* I am asking Keil to give me more data on his experiments.

** Of course we may still need quadrupole and skew quadrupole harmonic corrections in the doubler.

operating points are

sum resonances only: $\nu_x = \nu_y = 19.414$

the minimum distance to resonance = 0.014

sum and difference resonances: $\nu_x = 19.420$, $\nu_y = 19.412$

the minimum distance to resonance = 0.0057

Naively speaking, if we controlled the tune spread and the tune fluctuation to within ~ 0.003, we should be safe. I hope this is not a very difficult requirement to meet. I am of course ignoring here any effect coming from the beam-beam interaction.

Questions on the odd multipoles are not easy to be answered simply because these multipoles do not exist in the ISR beam-beam interaction unelss two beams are off-centered. Here I will try to squeeze out some information from a report written by Guignard which is a short review of the ISR studies on the beam-beam interactions.[18] For one example,

momentum = 26 GeV/c, current = 30A, linear tune shift = 0.0014,
vertical beam size at the interaction point $\sigma_y^* = 1$ mm,

he calculates the resonance width (Δe) for orders from four to eight. His (Δe) is related to f.r.w. by the relation

$$\Delta e = (n^2 + m^2)^{1/2} \times (\text{f.r.w.}) \quad \text{for } n\nu_x + m\nu_y = p.$$

For resonances of odd orders, he assumes that a particle of beam 1 is away from the center of beam 2 by a distance y and $y \ll \sigma_y^*$ of beam 2. Although he calculates the beam size blow-up $\Delta\sigma_y/\sigma_y$ and the loss rate $dN_p/N_p dt$ with $\sigma_y^* = 1$ mm and $y = 0.2$ mm, they are based on a certain model[19] and results depend on machine conditions such as the rate of tune change and the tune diffusion constant associated with an intrabeam momentum diffusion. Therefore, we will just look at the width (Δe) and compare with what we expect from the doubler. In doing so, we will use (Δe) instead of f.r.w. and use the formula by Guignard. This is important since what one is really interested in is the relative values of resonance width rather than the absolute values. For the beam-beam interaction, Guignard gives, for $N\nu_y = p$,

N =	4	5	6	7	8
(Δe) =	1.9×10^{-3}	3.6×10^{-4}	4.0×10^{-4}	8.2×10^{-5}	6.1×10^{-5}

Note that he does not give (Δe) for $N = 3$, the skew sextupole resonance. It is not at all clear from his report if this indicates that any amount of sextupole resonance is intolerable and should be eliminated. For that matter, I am not certain if these resonance widths are tolerable in the doubler with conditions obviously different from the ISR. With this caveat, I will simply calculate (Δe) for $N = 3$ substituting Guignard's parameters into his formula,

$$N = 3 \qquad (\Delta e) = 0.84\times10^{-3}$$

In computing the width (Δe) for the doubler with Guignard's formula, the emittance is assumed to be 0.05π mm-mr which is a more realistic value than 0.15π mm-mr for the storage mode of doubler. Widths are calculated for all resonances up to the fifth order using b_n and a_n ($n = 3, 4, 5$). Again average values and $2\times$(rms) values are used.

$$3\nu_x = 5.6\times10^{-3} \qquad \nu_x + 2\nu_y = 7.4\times10^{-3}$$
$$3\nu_y = 2.2\times10^{-3} \qquad 2\nu_x + \nu_y = 3.0\times10^{-3}$$

$$4\nu_{x,y} = 1.1\times10^{-4}\ , \qquad 2\nu_x + 2\nu_y = 2.1\times10^{-4}$$

$$\nu_x + 3\nu_y\ ,\ \ 3\nu_x + \nu_y = 4.6\times10^{-4}$$

$$5\nu_x = 9\times10^{-6} \qquad 3\nu_x + 2\nu_y = 2.5\times10^{-5} \qquad \nu_x + 4\nu_y = 2.1\times10^{-5}$$

$$5\nu_y = 2.9\times10^{-6} \qquad 3\nu_y + 2\nu_x = 8\times10^{-6} \qquad \nu_y + 4\nu_x = 6.5\times10^{-6}$$

From these values, one may be able to say that, as far as resonances of orders four and up are concerned, the nonlinear fields of doubler dipoles are reasonably safe. At the same time, the widths for the third-order resonances are disturbing; they are three to nine times larger compared to the ISR value. My feeling is that we do need sextupole (normal and skew) harmonic corrections for the storage mode. One can compute the "equivalent linear tune shift" for the doubler by using the conversion factor

$$(\Delta\nu)_{\text{linear}}/(\Delta e)_{N=3} = 0.0014/0.84\times10^{-3} = 1.7$$

	$(\Delta\nu)_{\text{linear}}$		$(\Delta\nu)_{\text{linear}}$
$3\nu_x$	0.009	$\nu_x + 2\nu_y$	0.012
$3\nu_y$	0.004	$2\nu_x + \nu_y$	0.005

Since there will be substantial contributions from the beam-beam interaction, these numbers are not at all comfortable.

What conclusions can one draw from all these numbers? Perhaps it is best to leave that task for each reader to complete. There will be ten (or even more) different versions coming from ten readers. I am afraid "E Pluribus Unum" is hardly applicable to the doubler.

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17. SPS Improvement Report No. 163, August 24, 1979.
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19. For example, he uses the model proposed by M. Month, Proceedings of the 9th International Conference on High Energy Accelerators, SLAC, 1974, p. 402.

Appendix Magnet "Shuffling"

For those who are by now convinced that the theory is incapable of predicting anything on nonlinear resonances, the following story may be of some interest. In 1973, there were very strong sextupole resonances $3\nu_x = 61$ and $\nu_x + 2\nu_y = 61$ during the injection in the main ring. The strengths of these resonances were so large that the operation was quite unstable. We had a set of correction sextupoles for suppressing them but the necessary setting of these correction sextupoles indicated that the amplitude and phase of the resonance driving force changed from one week to the next for no obvious reasons. We finally traced this mystery to air-core sextupoles which were then used for the chromaticity correction. Initially, they were arranged identically in all six sectors so that there was no 61st harmonic component. From time to time, a sextupole would be removed to make a space for other devices. One should then remove the sextupole at the same station in the directly opposite sector in order to preserve the even symmetry. This precaution was not always followed and the 61st harmonic grew steadily. Realizing this, we computed the harmonic component and rearranged sextupoles such that the harmful harmonic component is very small. The result of this change was simply unbelievable.*

Since multipole components of the field are known in all dipoles, it is theoretically possible to arrange the magnets in the ring such that the driving force of a resonance is practically non-existent. In reality, of course, one is very much limited by various restrictions.

1. Usually, there are only $\sim$ five magnets available for the installation and they have to be placed within three to four stations (twelve to sixteen locations).

2. It is often impossible to place a magnet such that driving forces of all resonances are simultaneously reduced.

3. The cancellation should be made locally as much as possible. For example, it is clearly better to cancel the contribution from the station A22 with the contribution from A26 compared to the one from A42. This means that the accumulation of effects should be avoided at each step.

* I was rather embarrassed by this incident but I understand Ted Wilson, who was with the main ring group at that time, received a high praise from John Adams for this excercise.

In view of a certain amount of uncertainties in the phase advance, it is probably meaningless to cancel a large driving force generated in A sector with an equally large force in B sector.

The goal of the magnet mixing is therefore rather modest. It simply tries to avoid disastrous accumulation of bad effects which are statistically possible. For example, it is unreasonable to reject a magnet simply because one multipole component is slightly outside the specified bounds. The practical approach is to place such a magnet at a place where the contribution reduces rather than increases the driving force. Again it must be emphasized that this is not always possible. There is another factor which goes into the overall judgement. Occasionally, a magnet with all multipoles satisfying the criteria may have a rather steep fall-off of the field along the median plane. This is possible when several low-order multipoles have the same sign. The tentative criterion for the overall shape is

$$|\Delta B_y(\pm 2\ \mathrm{cm})/B_o| < 4\times 10^{-4} .$$

If a magnet is slightly outside this range, it is placed at a location where both β_x (radial amplitude function) and η_x (dispersion function) are small to minimize harmfull effects. As for the variation in $\int B\cdot d\ell$ which is responsible for the radial closed orbit distortion, it is expected that the correction system is capable of taking care of the problem. Besides, other contributions (for example, quadrupole misalignment) will be far more important for the distortion. Consequently, no shuffling is planned on this basis.

So far, only fifteen dipoles have been either installed in the tunnel or assigned the locations between A19 and A24 (twenty possible locations). The number is still too small to be statistically meaningful but the success (or failure) of the shuffling can be judged from the following results. The figure of merit for each resonance, all at 4,000A, is the value which is calculated from the ring of fifteen magnets in their places divided by the value of the same parameter expected from the rms value of the relevant multipole component. Somewhat arbitrarily, one might say that numbers smaller than $\sim$0.5 indicate a success and larger than $\sim$1.5 a failure.