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THE POTENTIAL OF EXISTING SYNCHROTRONS

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To survey the potential of existing proton synchrotrons as kaon factories we need to know the following information:

1. The range of kaon momentum which is of interest,
2. The production spectra of kaons,
3. The acceptance of the kaon beam,
4. The energy and intensity of existing synchrotrons.

We shall discuss these items one by one.

1. Kaon momentum of interest

For most of the applications considered, one requires kaons of ~1 GeV energy. In this study we will, therefore, concentrate on kaons of momenta 1 and 2 GeV/c.

2. Kaon production spectrum

Most of the existing proton synchrotrons which can conceivably be used to produce 1-GeV kaons have energies in the range of a few GeV to tens of GeV. Synchrotrons in the hundred-GeV class will be fully occupied by elementary-particle physics experiments for the foreseeable future. The production spectra of 1-2 GeV/c kaons by protons from a few to tens of GeV are best represented by the empirical formula given by Sanford and Wang[1]. For an interacting proton of momentum P_o GeV/c, the production spectrum of kaons with momentum P GeV/c and at an angle of θ rad is

$$\frac{d^2N}{d\Omega dP} = F(P)G(P,\theta)$$

where

$$\begin{cases} F(P) = aP^b\left(1-\frac{P}{P_o}\right) e^{-cP^d/P_o^f} & = \text{forward production} \\ G(P,\theta) = e^{-g\theta(P-hP_o\cos^k\theta)} & = \text{angular dependence.} \end{cases}$$

The eight parameters are adjusted to fit the experimental data for each secondary particle type. Specifically for kaons they are

	a	b	c	d	f	g	h	k
K^+(or K^o)	.05897	.6916	3.744	4.520	4.190	4.928	.1922	50.28
K^-(or $\overline{K^o}$)	.02210	1.323	9.671	1.712	1.643	4.673	.1686	77.27

The proton beam is focused to a small spot on the production target, hence the kaons can roughly be considered as being produced from a point source. To a first approximation the beam transport which follows the target will accept all kaons up to a semi-cone angle θ_o. The integral

$$\frac{dN}{dP} = \int_0^{\theta_o} \left(\frac{d^2N}{d\Omega dP}\right) 2\pi \sin\theta \, d\theta$$

has been evaluated numerically for various values of θ_o, P_o, and P. The results are given in Table 1.

A few remarks of this production spectrum and of the production target are useful.

(a) The spectrum is fitted only to p-Be data for proton momenta between 10 and 35 GeV/c. Other available data indicate that the kaon production spectrum is rather insensitive to the target material. There is a slight reduction for higher Z material, due presumably to absorption by the nuclear matter. But this amounts to only 10-15% going from Be to Pb. For the purpose of our crude estimating we can very well consider the spectrum to be independent of the target material.

(b) Although Table 1 contains P_o values from 1.5 to 100 GeV/c,

the validity of the Sanford-Wang formula is questionable much below 8 GeV/c or much above 50 GeV/c. For high P_o the Hagedorn-Ranft[2] spectrum is more accurate.

(c) The target should be 2-3 collision lengths long so that more than 90% of the incident protons will undergo interaction. For a longer target the number of kaons produced per interacting proton will decrease due to target absorption and increase due to cascade production. But generally the decrease outweighs the increase.

(d) Also, if the target is too long the kaons can no longer be considered as produced from a point source. This will generally decrease the effective acceptance of the beam transport.

3. Acceptance of kaon beam

The solid-angle acceptance of the beam transport depends on the f-number or the aperture/focal-length of the first element - the field lens. For a properly designed beam there should be no further aperture limitation downstream. The most efficient field lens is a quadrupole doublet. We give a simple derivation of the acceptance based on the thin-lens approximation. This approximation, crude as it is, nevertheless gives a clear guidance for design and a ball-park estimate.

For a pair of thin lens separated by a distance d the focal length of each lens should not be much less than 2d so that the astigmatism is not excessive. Taking the focal lengths of the lenses to be ±2d we get for the combined focal length F and the positions of principal planes in the horizontal and the vertical planes as shown in Fig. 1. Because of astigmatism the acceptance angle θ_o of the doublet is different in the two spatial planes, but an average value between the two planes is roughly

$$\theta_o = \frac{R}{F} = \frac{R}{4d}$$

where R = aperture radius of quadrupole. With the point kaon source placed at a distance F = 4d in front of the first lens the acceptance solid angle is, then, roughly a cone with semi-vertex angle θ_o. The mimimum lens separation is $d = \ell$ = length of each quadrupole. With pole tip field B, the quadrupole field gradient is B' = B/R and the focal length is

$$f = \frac{(B\rho)}{B'\ell} = \frac{(B\rho)}{Bd} R$$

where $(B\rho)$ is the rigidity of the kaon given approximately by

$$(B\rho) = \left(\frac{100}{3} \frac{\text{kGm}}{\text{GeV/c}}\right) P.$$

Setting f = 2d and solving for d we get

$$d = \ell = \sqrt{\frac{(B\rho)}{2B}} \sqrt{R}$$

and

$$\theta_o = \sqrt{\frac{B}{8(B\rho)}} \sqrt{R} .$$

In the high radiation environment immediately after the target only conventional iron-copper quadrupoles are applicable. Superconducting quadrupoles will quench when irradiated by the beam. The pole-tip field is, therefore, limited to ~15 kG. For kaon momenta P = 1 GeV/c and 2 GeV/c and various values of R we get

	P = 1 GeV/c			P = 2 GeV/c		
R(cm)	θ_o	B'(kG/cm)	ℓ(m)	θ_o	B'(kG/cm)	ℓ(m)
5	3.0°	3	0.24	2.1°	3	0.33
10	4.3°	1.5	0.33	3.0°	1.5	0.47
15	5.3°	1	0.41	3.7°	1	0.58

	P = 1 GeV/c			P = 2 GeV/c		
R(cm)	θ_o	B'(kG/cm)	ℓ(m)	θ_o	B'(kG/cm)	ℓ(m)
20	6.1°	0.75	0.47	4.3°	0.75	0.67
25	6.8°	0.6	0.53	4.8°	0.6	0.75

We probably do not want to use quadrupoles with aperture diameter larger than 30 cm. Hence we take the semi-vertex angle of the acceptance cone $\theta_o = 5°$ for P = 1 GeV/c and $\theta_o = 3.5°$ for P = 2 GeV/c. Even these numbers already represent optimistic upper limits. The total distance from the target to the end of the doublet is approximately $5\ell \sim 2$ m for 1 GeV/c and ~ 3 m for 2 GeV/c and is short compared to the kaon decay length.

The momentum acceptance of the beam transport depends on the detailed design of the whole beam, but is dictated principally by the requirement of the specific experiment for which the kaon beam is used. For a relative comparison, we will simply standardize on a $\Delta P/P = 1\%$, or $\Delta P = 10^{-2}$ GeV/c for P = 1 GeV/c and $\Delta P = 2\times10^{-2}$ GeV/c for P = 2 GeV/c.

We considered only the acceptance of the beam transport. But to obtain a reasonably pure kaon beam, shielding must be provided in the beam line to attenuate all other hadrons produced in the target. This favors a lower incident proton energy, because it is easier to shield against the lower energy unwanted secondary hadrons.

4. Energy and intensity of existing proton synchrotrons

These are given in the "Catalogue of High-Energy Accelerators" published for the IXth International Conference on High-Energy Accelerators by SLAC in May, 1974.

The peak field, pulse period, and flat-top duration of a

synchrotron are all variable, hence the energy, average intensity, and duty factor are all adjustable. We will compare these machines at a uniform standardized mode of operation, then make some general comments on scaling to other modes. As the standard mode, we assume all machines to be operated at their peak energies with zero flat-top. Operating synchrotrons with a longer flat-top increases the duty factor, but at a heavy expense of power consumption and average beam intensity. In a factory-type accelerator the machine itself should be relegated to producing as many particles per unit time as possible. The duty factor should be gained through a separate device such as a storage ring. Separating these functions one can then obtain the maximum average intensity and the maximum duty factor of close to 100%.

In Table 2 we list all proton synchrotrons from 3 to 100 GeV. Some numbers are modified by improvements carried out since 1974, and some pulse period at zero flat-top are extrapolations from values given in the Catalogue. In all cases the numbers listed are optimistic "record" values. Stable and practical operating conditions may be somewhat below those given.

One outstanding feature revealed by Table 2 is that a rapid-cycling synchrotron such as the Fermilab booster compares very favorably with much higher energy machines in the rate of manufacturing kaons, while creating much less of a shielding problem in the beam because of the much lower energy. This leads directly to the conclusion that the best kaon factory is a rapid-cycling synchrotron with a storage-ring - the subject of the next talk by Fred Mills.

The rate of momentum-gain $\dot{P}$, hence the rate of rise of the magnetic field $\dot{B}$ during acceleration, is limited by the available

rf accelerating voltage. The electrical power consumption during a pulse is generally dominated by the i^2r loss in the magnet coil. With $P \propto B \propto i$ we see that the energy consumption per pulse is proportional to P_o^3 (P_o = final momentum). When the operation of the synchrotron is power-consumption limited (whether by the capacity of the motor-generator set or by the power bill) there is a finite dwell-time between pulses and the pulse rate would be proportional to P_o^3. From Table 1 we see that over most of the range of P_o the kaon yield increases with P_o slower than P_o^3. In this regime, therefore, it is advantageous to reduce the final momentum of the protons from the accelerator and increase the repetition rate.

On the other hand, without flat-top the operation of most synchrotrons is not power-consumption limited. This means that no dwell-time is required between pulses and that as soon as the field is brought down to the injection value one can inject protons and start the next pulse. This regime of operation may be called $\dot{B}$- or rf-limited. In this regime the repetition rate is clearly proportional to P_o^{-1}. From Table 1 we see that the increase of kaon yield is slower than P_o only over a limited range of P_o. In this case, whether a synchrotron should be operated at the peak or a reduced P_o would depend on where P_o falls on the kaon-yield spectrum.

It is not possible to explore all the detailed optimal operating modes here, synchrotron by synchrotron. But the simple recipe and tables contained in this paper should be adequate to enable one to play the game for any specific synchrotron of interest.

The author wishes to thank Dave Johnson for performing the numerical integration needed for Table 1.

References

1. J.R. Sanford and C.L. Wang, "Empirical Formulas for Particle Production in P-Be Collision Between 10 and 35 BeV/c", BNL-11299 and BNL-11479 (1967)

2. H. Grote, R. Hagedorn, and J. Ranft, "Atlas of Particle Production Spectra", CERN publication (1970)

Table 1. Number of K^+ and K^- per GeV/c per interacting proton at momenta P = 1 GeV/c and 2 GeV/c for various values of the incident proton momentum P_o and the acceptance semi-cone angle θ_o of the beam transport.

P = 1.0 GEV/C

P0(GEV/C)		θ_o 1.0 DEG	2.0 DEG	3.0 DEG	4.0 DEG	5.0 DEG	6.0 DEG	7.0 DEG	8.0 DEG	9.0 DEG	10.0 DEG
1.5	K+	0.91951E-05	0.35125E-04	0.75673E-04	0.12879E-03	0.19247E-03	0.26476E-03	0.34376E-03	0.42769E-03	0.51492E-03	0.60401E-03
	K-	0.47569E-07	0.18172E-06	0.39144E-06	0.66593E-06	0.99453E-06	0.13668E-05	0.17730E-05	0.22038E-05	0.26513E-05	0.31085E-05
2.0	K+	0.22403E-04	0.86032E-04	0.18626E-03	0.31842E-03	0.47772E-03	0.65931E-03	0.85831E-03	0.10700E-02	0.12901E-02	0.15146E-02
	K-	0.46560E-06	0.17863E-05	0.38629E-05	0.65938E-05	0.98744E-05	0.13599E-04	0.17665E-04	0.21976E-04	0.26445E-04	0.31001E-04
3.0	K+	0.35712E-04	0.13859E-03	0.30303E-03	0.52271E-03	0.79042E-03	0.10981E-02	0.14372E-02	0.17989E-02	0.21750E-02	0.25578E-02
	K-	0.28245E-05	0.10930E-04	0.23822E-04	0.40938E-04	0.61645E-04	0.85252E-04	0.11105E-03	0.13838E-03	0.16661E-03	0.19526E-03
4.0	K+	0.41702E-04	0.16356E-03	0.36120E-03	0.62870E-03	0.95824E-03	0.13402E-02	0.17634E-02	0.22162E-02	0.26869E-02	0.31650E-02
	K-	0.58378E-05	0.22788E-04	0.50056E-04	0.86607E-04	0.13113E-03	0.18211E-03	0.23790E-03	0.29691E-03	0.35769E-03	0.41904E-03
5.0	K+	0.45283E-04	0.17950E-03	0.40039E-03	0.70328E-03	0.10805E-02	0.15213E-02	0.20126E-02	0.25396E-02	0.30875E-02	0.36429E-02
	K-	0.85194E-05	0.33545E-04	0.74269E-04	0.12938E-03	0.19699E-03	0.27471E-03	0.35988E-03	0.44985E-03	0.54223E-03	0.63503E-03
6.0	K+	0.47805E-04	0.19153E-03	0.43154E-03	0.76498E-03	0.11848E-02	0.16795E-02	0.22338E-02	0.28301E-02	0.34502E-02	0.40773E-02
	K-	0.10699E-04	0.42497E-04	0.94838E-04	0.16635E-03	0.25469E-03	0.35666E-03	0.46856E-03	0.58662E-03	0.70744E-03	0.82823E-03
7.0	K+	0.49766E-04	0.20154E-03	0.45870E-03	0.82066E-03	0.12814E-02	0.18288E-02	0.24455E-02	0.31109E-02	0.38029E-02	0.45011E-02
	K-	0.12447E-04	0.49871E-04	0.11218E-03	0.19814E-03	0.30505E-03	0.42898E-03	0.56516E-03	0.70867E-03	0.85503E-03	0.10007E-02
8.0	K+	0.51391E-04	0.21037E-03	0.48369E-03	0.87347E-03	0.13750E-02	0.19758E-02	0.26566E-02	0.33930E-02	0.41590E-02	0.49302E-02
	K-	0.13862E-04	0.56028E-04	0.12705E-03	0.22595E-03	0.34983E-03	0.49401E-03	0.65266E-03	0.81965E-03	0.98940E-03	0.11575E-02
10.0	K+	0.54064E-04	0.22615E-03	0.53076E-03	0.97663E-03	0.15629E-02	0.22771E-02	0.30952E-02	0.39851E-02	0.49111E-02	0.58392E-02
	K-	0.16008E-04	0.65853E-04	0.15175E-03	0.27367E-03	0.42852E-03	0.61025E-03	0.81074E-03	0.10213E-02	0.12339E-02	0.14424E-02
12.5	K+	0.56831E-04	0.24429E-03	0.58843E-03	0.11088E-02	0.18119E-02	0.26863E-02	0.37019E-02	0.48143E-02	0.59721E-02	0.71263E-02
	K-	0.17915E-04	0.75346E-04	0.17720E-03	0.32522E-03	0.51654E-03	0.74339E-03	0.99449E-03	0.12573E-02	0.15206E-02	0.17758E-02
15.0	K+	0.59288E-04	0.26196E-03	0.64782E-03	0.12506E-02	0.20874E-02	0.31498E-02	0.44009E-02	0.57806E-02	0.72173E-02	0.86416E-02
	K-	0.19348E-04	0.83208E-04	0.19975E-03	0.37319E-03	0.60128E-03	0.87454E-03	0.11780E-02	0.14947E-02	0.18094E-02	0.21106E-02
20.0	K+	0.63826E-04	0.29819E-03	0.77804E-03	0.15780E-02	0.27503E-02	0.43014E-02	0.61790E-02	0.82783E-02	0.10466E-01	0.12611E-01
	K-	0.21525E-04	0.96841E-04	0.24237E-03	0.46949E-03	0.77869E-03	0.11568E-02	0.15795E-02	0.20181E-02	0.24466E-02	0.28472E-02
25.0	K+	0.68240E-04	0.33741E-03	0.93009E-03	0.19844E-02	0.36152E-02	0.58636E-02	0.86612E-02	0.11832E-01	0.15139E-01	0.18346E-01
	K-	0.23287E-04	0.10967E-03	0.28640E-03	0.57558E-03	0.98311E-03	0.14918E-02	0.20643E-02	0.26546E-02	0.32221E-02	0.37400E-02
30.0	K+	0.72727E-04	0.38093E-03	0.11107E-02	0.24957E-02	0.47566E-02	0.80046E-02	0.12158E-01	0.16931E-01	0.21911E-01	0.26689E-01
	K-	0.24886E-04	0.12277E-03	0.33476E-03	0.69842E-03	0.12289E-02	0.19048E-02	0.26703E-02	0.34551E-02	0.41973E-02	0.48586E-02
40.0	K+	0.82258E-04	0.48478E-03	0.15865E-02	0.39663E-02	0.82917E-02	0.15036E-01	0.24146E-01	0.34902E-01	0.46141E-01	0.56692E-01
	K-	0.27965E-04	0.15163E-03	0.45177E-03	0.10175E-02	0.19014E-02	0.30745E-02	0.44209E-02	0.57845E-02	0.70329E-02	0.80929E-02
50.0	K+	0.92844E-04	0.61770E-03	0.22780E-02	0.63578E-02	0.14610E-01	0.28572E-01	0.48484E-01	0.72643E-01	0.97919E-01	0.12114E+00
	K-	0.31137E-04	0.18600E-03	0.60702E-03	0.14787E-02	0.29371E-02	0.49526E-02	0.72963E-02	0.96421E-02	0.11721E-01	0.13403E-01
70.0	K+	0.11823E-03	0.10115E-02	0.47832E-02	0.16762E-01	0.46741E-01	0.10638E+00	0.20114E+00	0.32247E+00	0.44976E+00	0.56174E+00
	K-	0.38262E-04	0.27915E-03	0.11003E-02	0.31492E-02	0.70762E-02	0.12959E-01	0.19979E-01	0.26841E-01	0.32540E-01	0.36733E-01
100.0	K+	0.17075E-03	0.21713E-02	0.15175E-01	0.75702E-01	0.28351E+00	0.80929E+00	0.17897E+01	0.31495E+01	0.45818E+01	0.57635E+01
	K-	0.51939E-04	0.51852E-03	0.27474E-02	0.10086E-01	0.27292E-01	0.56336E-01	0.92353E-01	0.12634E+00	0.15197E+00	0.16833E+00

Table 1. (Continued)

P = 2.0 GEV/C

P0(GEV/C)		1.0 DEG	2.0 DEG	3.0 DEG	4.0 DEG	5.0 DEG	6.0 DEG	7.0 DEG	8.0 DEG	9.0 DEG	10.0 DEG
		θ_0									
3.0	K+	0.11957E-04	0.43868E-04	0.90763E-04	0.14830E-03	0.21266E-03	0.28057E-03	0.34925E-03	0.41650E-03	0.48066E-03	0.54060E-03
	K-	0.89592E-07	0.32874E-06	0.67994E-06	0.11101E-05	0.15899E-05	0.20943E-05	0.26025E-05	0.30986E-05	0.35712E-05	0.40132E-05
4.0	K+	0.33133E-04	0.12283E-03	0.25657E-03	0.42282E-03	0.61085E-03	0.81088E-03	0.10143E-02	0.12141E-02	0.14047E-02	0.15824E-02
	K-	0.96543E-06	0.35726E-05	0.74456E-05	0.12235E-04	0.17614E-04	0.23293E-04	0.29022E-04	0.34609E-04	0.39913E-04	0.44849E-04
5.0	K+	0.47012E-04	0.17610E-03	0.37142E-03	0.61739E-03	0.89864E-03	0.12003E-02	0.15089E-02	0.18128E-02	0.21027E-02	0.23724E-02
	K-	0.31683E-05	0.11824E-04	0.24832E-04	0.41070E-04	0.59439E-04	0.78909E-04	0.98581E-04	0.11774E-03	0.13587E-03	0.15266E-03
6.0	K+	0.55745E-04	0.21100E-03	0.44937E-03	0.75351E-03	0.11051E-02	0.14854E-02	0.18765E-02	0.22627E-02	0.26312E-02	0.29733E-02
	K-	0.63623E-05	0.23948E-04	0.50679E-04	0.84369E-04	0.12275E-03	0.16360E-03	0.20492E-03	0.24512E-03	0.28304E-03	0.31799E-03
7.0	K+	0.61756E-04	0.23622E-03	0.50800E-03	0.85935E-03	0.12699E-02	0.17177E-02	0.21809E-02	0.26394E-02	0.30772E-02	0.34826E-02
	K-	0.99922E-05	0.37934E-04	0.80897E-04	0.13556E-03	0.19827E-03	0.26530E-03	0.33320E-03	0.39917E-03	0.46119E-03	0.51808E-03
8.0	K+	0.66257E-04	0.25612E-03	0.55624E-03	0.94931E-03	0.14137E-02	0.19244E-02	0.24555E-02	0.29829E-02	0.34864E-02	0.39516E-02
	K-	0.13665E-04	0.52324E-04	0.11245E-03	0.18969E-03	0.27892E-03	0.37468E-03	0.47183E-03	0.56610E-03	0.65444E-03	0.73509E-03
10.0	K+	0.72869E-04	0.28771E-03	0.63736E-03	0.11074E-02	0.16749E-02	0.23096E-02	0.29769E-02	0.36433E-02	0.42796E-02	0.48651E-02
	K-	0.20427E-04	0.79576E-04	0.17371E-03	0.29696E-03	0.44134E-03	0.59758E-03	0.75650E-03	0.91035E-03	0.10536E-02	0.11831E-02
12.5	K+	0.78919E-04	0.32002E-03	0.72699E-03	0.12922E-02	0.19935E-02	0.27941E-02	0.36477E-02	0.45060E-02	0.53260E-02	0.60762E-02
	K-	0.27378E-04	0.10900E-03	0.24266E-03	0.42190E-03	0.63554E-03	0.86915E-03	0.11076E-02	0.13377E-02	0.15502E-02	0.17400E-02
15.0	K+	0.83807E-04	0.34914E-03	0.81359E-03	0.14801E-02	0.23296E-02	0.33199E-02	0.43905E-02	0.54743E-02	0.65102E-02	0.74529E-02
	K-	0.32893E-04	0.13385E-03	0.30398E-03	0.53761E-03	0.82098E-03	0.11341E-02	0.14547E-02	0.17632E-02	0.20457E-02	0.22952E-02
20.0	K+	0.92076E-04	0.40514E-03	0.99454E-03	0.18971E-02	0.31118E-02	0.45875E-02	0.62276E-02	0.79111E-02	0.95215E-02	0.10971E-01
	K-	0.41120E-04	0.17490E-03	0.41363E-03	0.75740E-03	0.11891E-02	0.16760E-02	0.21780E-02	0.26582E-02	0.30909E-02	0.34637E-02
25.0	K+	0.99563E-04	0.46317E-03	0.11993E-02	0.24024E-02	0.41113E-02	0.62750E-02	0.87471E-02	0.11320E-01	0.13783E-01	0.15976E-01
	K-	0.47258E-04	0.21024E-03	0.51819E-03	0.98316E-03	0.15877E-02	0.22837E-02	0.30060E-02	0.36930E-02	0.43019E-02	0.48142E-02
30.0	K+	0.10687E-03	0.52612E-03	0.14389E-02	0.30306E-02	0.54169E-02	0.85654E-02	0.12263E-01	0.16164E-01	0.19902E-01	0.23195E-01
	K-	0.52351E-04	0.24376E-03	0.62665E-03	0.12328E-02	0.20485E-02	0.30073E-02	0.40090E-02	0.49560E-02	0.57819E-02	0.64602E-02
40.0	K+	0.12190E-03	0.67391E-03	0.20634E-02	0.48210E-02	0.94253E-02	0.16020E-01	0.24201E-01	0.33074E-01	0.41588E-01	0.48926E-01
	K-	0.61201E-04	0.31274E-03	0.87664E-03	0.18576E-02	0.32721E-02	0.50040E-02	0.68377E-02	0.85509E-02	0.99972E-02	0.11130E-01
50.0	K+	0.13823E-03	0.86114E-03	0.29643E-02	0.77132E-02	0.16539E-01	0.30258E-01	0.48231E-01	0.68264E-01	0.87516E-01	0.10376E+00
	K-	0.69557E-04	0.39100E-03	0.11983E-02	0.27412E-02	0.51238E-02	0.81625E-02	0.11423E-01	0.14433E-01	0.16893E-01	0.18730E-01
70.0	K+	0.17684E-03	0.14122E-02	0.62096E-02	0.20215E-01	0.52426E-01	0.11132E+00	0.19732E+00	0.29852E+00	0.39607E+00	0.47491E+00
	K-	0.87202E-04	0.59711E-03	0.22032E-02	0.59032E-02	0.12451E-01	0.21506E-01	0.31480E-01	0.40477E-01	0.47370E-01	0.52048E-01
100.0	K+	0.25604E-03	0.30272E-02	0.19596E-01	0.90503E-01	0.31434E+00	0.83513E+00	0.17281E+01	0.28670E+01	0.39698E+01	0.48060E+01
	K-	0.11977E-03	0.11187E-02	0.55283E-02	0.18940E-01	0.47986E-01	0.93276E-01	0.14514E+00	0.19034E+00	0.22179E+00	0.24031E+00

Table 2. Yield of K^+ and K^- from Existing Proton Synchrotrons

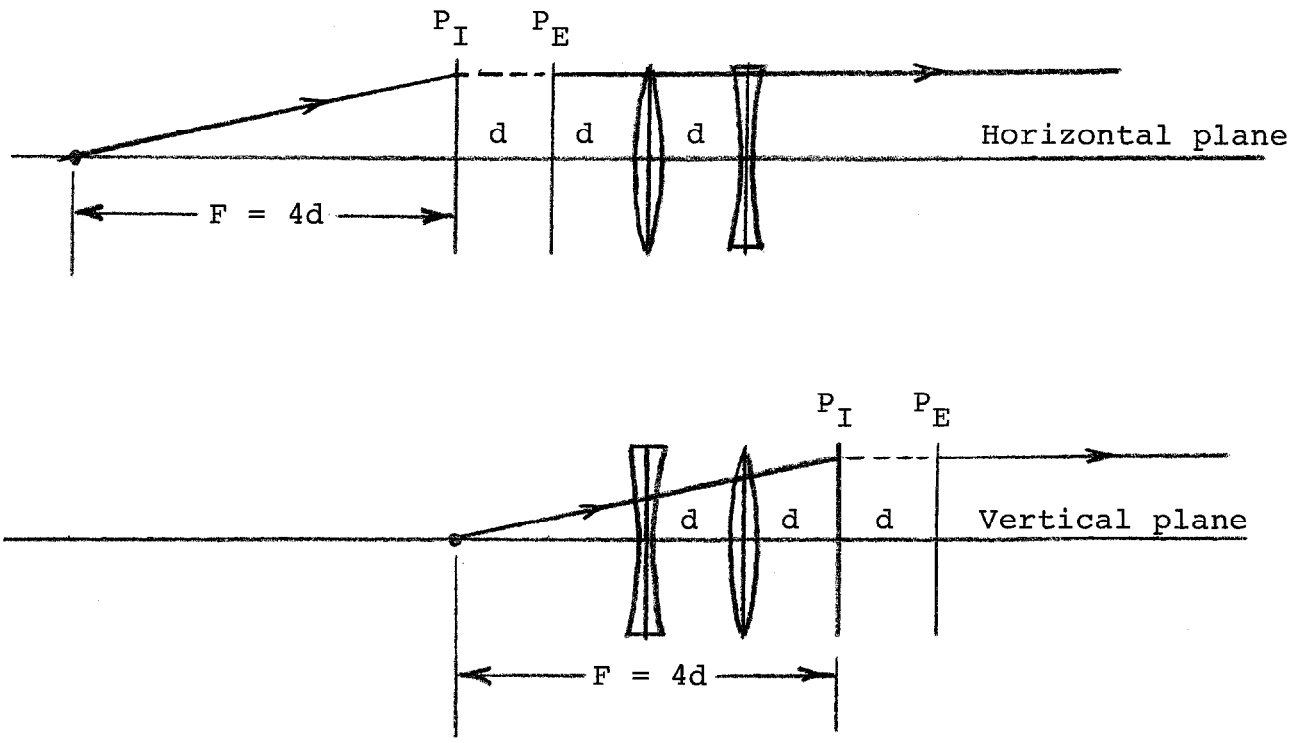
	Kinetic Energy (GeV)	p/pulse (10^{12})	Pulse Period (sec)	p/sec (10^{12})	P = 1 GeV/c[3]		P = 2 GeV/c[4]	
					K^+/sec (10^7)	K^-/sec (10^7)	K^+/sec (10^7)	K^-/sec (10^7)
Saturne[2]	3	1.6	4	0.4	0.37	0.048	0.24	0.006
Bevatron[2]	6.2	6	3.5	1.7	2.2	0.52	2.3	0.37
Nimrod	8	4.2	2.2	1.9	2.8	0.73	3.0	0.71
FANL-booster	8	2	1/15	30	44	12	48	11
Dubna	10	0.13	9	0.015	0.025	0.007	0.027	0.008
KEK[1]	12	10	1.6	6.2	11	3.3	13	4.2
ZGS	12.7	5	2	2.5	4.8	1.4	5.2	1.8
CPS	28	10	1.5	6.7	30	7.9	27	12
AGS	33	10	1.2	8.3	49	12	42	17
IHEP[2]	76	3	6	0.5	35	4.8	16	4.7

(1) Design performance

(2) Validity of the Sanford-Wang spectrum is questionable at these energies.

(3) Semi-vertex angle of acceptance cone $\theta_o = 5°$, momentum acceptance $\Delta p = 10^{-2}$ GeV/c, all protons interact.

(4) Semi-vertex angle of acceptance cone $\theta_o = 3.5°$, momentum acceptance $\Delta p = 2 \times 10^{-2}$ GeV/c, all protons interact.



P_I: Incident principal plane

P_E: Exit principal plane

Figure 1. Optics of the quadrupole-doublet field lens of the kaon beam