national accelerator laboratory

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THIRD-INTEGER RESONANT EXTRACTION FROM THE ENERGY DOUBLER/SAVER

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A. Beam Geometry at the Septum

For all resonant extraction schemes during extraction the cross-sectional geometry of the beam at the septum looks like that shown in Fig. 1.

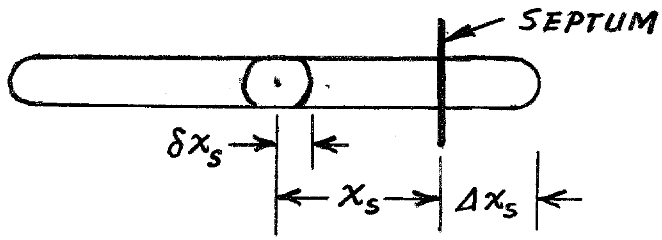

Figure 1

There is a stable beam core with width $\pm\ \delta x_s$. During extraction δx_s is slowly reduced and beam is squeezed out of the stable region until at the end of extraction $\delta x_s = 0$.

There is an unstable transitional part of the beam between δx_s and x_s in which the beam squeezed out of δx_s grows in oscillation amplitude coherently in steps until reaching the value x_s. The last step takes the beam across the septum to enter the extraction channel. The last step-size Δx_s and the septum thickness d determines the extraction efficiency. The fraction of beam hitting the septum is somewhat greater than $d/\Delta x_s$ because the density of the beam is higher at the septum. If the septum thickness is d = 0.05 mm a step-size of Δx_s = 10 mm will assure a beam loss of no more than 1% or an extraction efficiency of about 99%.

B. Comparison of Half-integer and Third-integer Schemes

1. For the $\frac{1}{3}$ - integer scheme the phase-space separatrices are formed by a sextupole field only, whereas for the $\frac{1}{2}$ - integer scheme one needs both a quadrupole field and an octupole field. Therefore the $\frac{1}{3}$ - integer scheme is conceptually simpler.

2. For the $\frac{1}{3}$ - integer scheme the size of the stable part of the beam is not increased by the extraction sextupole and δx_s at the beginning of extraction is roughly the same as the normal half-width of the beam. But for the $\frac{1}{2}$ - integer scheme because of the action of the extraction quadrupole the beam width at the beginning of extraction is considerably larger than normal. Hence a larger aperture is required for the $\frac{1}{2}$ - integer scheme.

3. With a given x_s it is easier to get a larger step-size Δx_s in the $\frac{1}{3}$ - integer scheme. Conversely to obtain a desired Δx_s a smaller x_s, hence a smaller aperture of the ring magnet is needed for the $\frac{1}{3}$ - integer scheme.

4. There is no finite-width stop-band on a $\frac{1}{3}$ - integer resonance. Hence the required field tolerances in allowable ripple and higher multipoles are tighter. These errors may cause some fraction of the beam to be left unextracted in

the machine. But presumably the superconducting ring magnets of the doubler/saver are very nearly ripple-free. Higher multipole field must, however, be controlled or compensated.

C. Lattice and Tune

We assume a lattice for the energy doubler/saver identical to that of the main ring except with the long straight sections rematched for a tune of $\sim$19.4. With F and D cell quadrupoles identical in strength the tune split will be $\sim$0.04. This gives the operating point $\nu_x = 19.40$, $\nu_y = 19.44$ as shown in Fig. 2. For extraction the strength of the cell

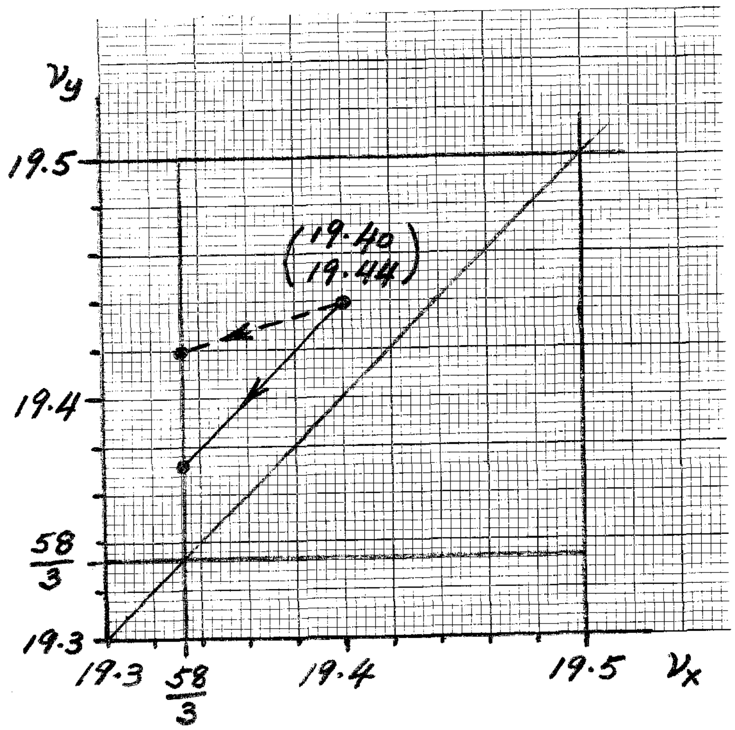

Figure 2

quadrupoles is reduced so that the operating point moves "south-west" to hit the $\frac{1}{3}$ - integer resonance $\nu_x = \frac{58}{3}$. It is also possible to move the operating point along other paths onto the resonance by using additional trim quadrupoles. One such path is shown as dashed line in Fig. 2. The precise value of ν_y during extraction is, however, immaterial for this approximate calculation.

D. Orbit Dynamics Near the $\nu = \frac{58}{3}$ Resonance

The formulas for third-integer resonance were given in TM-271. But since they are not exactly applicable to our case, we have to rederive them here. We shall write the equations of motion as

$$\begin{cases} \dfrac{d\phi}{d\theta} = \varepsilon + AR\cos(3\phi-\psi) + BR^2 \\ \dfrac{dR}{d\theta} = AR^2 \sin(3\phi-\psi) \end{cases} \tag{1}$$

where the polar coordinates (ϕ, R) in the rotation frame (angular velocity $= \frac{58}{3}\theta$) in the Floquet plane (u, p_u plane) are related to x and x' (or p_x) by

$$\begin{cases} x = \sqrt{\beta}\, R\cos(\phi + \frac{58}{3}\theta) = \sqrt{\beta}\, u \\ x' = -\dfrac{R}{\sqrt{\beta}}\left[\sin(\phi+\frac{58}{3}\theta) + \alpha\cos(\phi+\frac{58}{3}\theta)\right] \\ \quad = -\dfrac{1}{\sqrt{\beta}}(p_u + \alpha u) \end{cases} \tag{2}$$

or

$$\begin{cases} R = (\gamma x^2+2\alpha xx'+\beta x'^2)^{1/2} \\ \tan\ (\phi + \frac{58}{3}\ \theta) = -\ (\alpha+\beta\frac{x'}{x}) \end{cases} \quad (3)$$

and

$$\left\{ \begin{array}{l} \theta = \int \frac{ds}{\nu\beta} = \text{linear betatron oscillation phase advance normalized to } 2\pi \text{ per revolution} \\ \varepsilon = \nu - \frac{58}{3} = \text{deviation of } \nu \text{ from resonant value } \frac{58}{3} \\ A\cos\ (58\theta+\psi) = \text{58th harmonic of } \left[\frac{\nu}{16}\ \beta^{5/2}\ \frac{B''}{B\rho}\right] \\ B = \text{0th harmonic (average) of } \left[\frac{\nu}{16}\ \beta^{3}\ \frac{B'''}{B\rho}\right] \\ \alpha,\ \beta,\ \gamma = \text{linear betatron oscillation functions} \\ B'' = \frac{\partial^2 B_y}{\partial x^2} \text{ sextupole field} \\ B''' = \frac{\partial^3 B_y}{\partial x^3} = \text{octupole field} \\ B\rho = \text{rigidity of particle} = \frac{pc}{e} \end{array} \right. \quad (4)$$

In Equ. 1 a small kinematic term which slightly modifies B is neglected.

1. <u>Location of sextupole</u>

The fixed points are given by $\frac{d\phi}{d\theta} = \frac{dR}{d\theta} = 0$

and are

Unstable fixed points

$$\begin{cases} R = \dfrac{\varepsilon}{A} \\ \phi = \dfrac{\pi+\psi}{3}, \dfrac{\pi+\psi}{3} + \dfrac{2\pi}{3}, \dfrac{\pi+\psi}{3} + \dfrac{4\pi}{3} \end{cases} \tag{5}$$

Outboard stable fixed points

$$\begin{cases} R = \dfrac{A}{2B}\left(1+\sqrt{1-\dfrac{4B}{A^2}\varepsilon}\right) \cong \dfrac{A}{B} - \dfrac{\varepsilon}{A} \\ \phi = \dfrac{\pi+\psi}{3}, \dfrac{\pi+\psi}{3} + \dfrac{2\pi}{3}, \dfrac{\pi+\psi}{3} + \dfrac{4\pi}{3} \end{cases} \quad \text{if } B>0 \tag{6}$$

$$\begin{cases} R = \dfrac{A}{2B}\left(1+\sqrt{1-\dfrac{4B}{A^2}\varepsilon}\right) \cong -\dfrac{A}{B} + \dfrac{\varepsilon}{A} \\ \phi = \dfrac{\psi}{3}, \dfrac{\psi}{3} + \dfrac{2\pi}{3}, \dfrac{\psi}{3} + \dfrac{4\pi}{3} \end{cases} \quad \text{if } B<0 \tag{7}$$

With no octupole (B=0) the directions of the out-streaming separatrices are $\phi = \frac{\psi}{3} + \frac{\pi}{6}, \frac{\psi}{3} + \frac{\pi}{6} + \frac{2\pi}{3}, \frac{\psi}{3} + \frac{\pi}{6} + \frac{4\pi}{3}$ and those of the in-streaming separatrices are

$$\phi = \frac{\psi}{3} + \frac{\pi}{2}, \frac{\psi}{3} + \frac{\pi}{2} + \frac{2\pi}{3}, \frac{\psi}{3} + \frac{\pi}{2} + \frac{4\pi}{3}.$$

At the septum (θ=0) if α is near zero we can locate the sextupole such that x'=0 at the septum (as in TM-271), but if α is very different from zero we should set ϕ = 0 at the septum so that the beam reaches the septum $x = x_s$ at the smallest value of R, hence the beam is smallest all around the ring.

Putting an outstreaming separatrices at $\phi = 0$, namely $\frac{\psi}{3} + \frac{\pi}{6} = 0$, gives $\psi = -\frac{\pi}{2}$. The sextupole should be placed at an amplitude of A cos $(58\theta+\psi)$ or at $58\theta+\psi = n\pi$ (n=integer). This gives the possible locations $(\theta=\theta_A)$ of the sextupole as

$$\boxed{\theta_A = \frac{\pi}{58}\left(n+\frac{1}{2}\right)} \tag{8}$$

At n = even the sextupole should be positive and at n = odd the sextupole should be negative.

2. <u>Strength of sextupole</u>

At a fixed θ-location we have from Equ. 2

$$\begin{cases} x = \sqrt{\beta}\, R \cos\left(\phi + \frac{58}{3}\theta\right) \\ \frac{dx}{d\theta} = \sqrt{\beta}\left[\frac{dR}{d\theta}\cos\left(\phi + \frac{58}{3}\theta\right) - R\frac{d\phi}{d\theta}\sin\left(\phi + \frac{58}{3}\theta\right)\right] \end{cases} \tag{9}$$

At the septum $\theta=0$, $\phi=0$, $\beta=\beta_s$ (subscript s means septum) and we have from Equs 9 and 1

$$\begin{cases} x_s = \sqrt{\beta_s}\, R \\ \left(\frac{dx}{d\theta}\right)_s = \sqrt{\beta_s}\, AR^2 = \frac{A}{\sqrt{\beta_s}}\, x_s^2 \end{cases} \tag{10}$$

If the desired step-size is Δx_s in 3 revolutions $(\Delta\theta=6\pi)$ the necessary sextupole is given by

$$\boxed{A = \frac{\sqrt{\beta_s}}{6\pi}\frac{\Delta x_s}{x_s^2}} \tag{11}$$

It is interesting to note that with the choice of location of sextupole to give $\phi=0$ at the septum, the step-size is independent of ε, namely constant throughout the duration of extraction. In addition the step-size is not affected by the octupole error B.

3. Onset of extraction

The distance to the unstable fixed points is given by Equ. 5 to be $R=\frac{\varepsilon}{A}$. The area of the stable triangle is, therefore

$$\text{Area} = 3\left(\frac{\varepsilon}{A}\sin\frac{\pi}{6}\right)\left(\frac{\varepsilon}{A}\cos\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{4}\left(\frac{\varepsilon}{A}\right)^2$$

Extraction starts ($\varepsilon=\varepsilon_o$) when this area equals the emittance E of the beam. This gives

$$\boxed{\varepsilon_o = \left(\frac{4E}{3\sqrt{3}}\right)^{1/2} A} \qquad (12)$$

Extraction ends when $\varepsilon=0$.

4. Tolerance on octupole

The octupole field curves the separatrices at large R. We require that the outboard stable fixed points (at $R \cong \frac{A}{B}$) be farther out than the septum. (At $R = \frac{x_s}{\sqrt{\beta_s}}$ from Eq. 10).

This gives

$$\boxed{B < \frac{\sqrt{\beta_s}}{x_s} A} \qquad (13)$$

5. Relation to hardware

With one sextupole of length ℓ located at $\theta = \theta_A = \frac{1}{58}\frac{\pi}{2}$ and $\beta = \beta_A$ we have

$$B''(\theta) = \frac{B''\ell}{\nu\beta_A}\,\delta\left(\theta - \frac{1}{58}\frac{\pi}{2}\right) = \frac{1}{2\pi}\frac{B''\ell}{\nu\beta_A}\sum_n e^{in\left(\theta - \frac{1}{58}\frac{\pi}{2}\right)}$$

Hence

$$A\cos\left(58\theta - \frac{\pi}{2}\right) = \frac{\nu}{16}\beta_A^{5/2}\,\frac{1}{2\pi}\,\frac{1}{\nu\beta_A}\,\frac{B''\ell}{B\rho}\left[e^{i58\left(\theta - \frac{1}{58}\frac{\pi}{2}\right)} + e^{-i58\left(\theta - \frac{1}{58}\frac{\pi}{2}\right)}\right]$$

$$= \frac{1}{16\pi}\,\beta_A^{3/2}\,\frac{B''\ell}{B\rho}\cos\left(58\theta - \frac{\pi}{2}\right)$$

or

$$\boxed{A = \frac{1}{16\pi}\,\beta_A^{3/2}\,\frac{B''\ell}{B\rho}} \qquad (14)$$

With scattered octupole of length ℓ located at $\beta = \beta_B$ we have

$$\boxed{B = \frac{1}{32\pi}\sum \beta_B^2\,\frac{B'''\ell}{B\rho}} \qquad (15)$$

E. Numerical Results

1. Location of sextupole

We assume that in the doubler/saver the dipole F47-5 is moved to the downstream end of the long straight section to make room for an additional 6m of electrostatic septum. Hence $\theta = 0$ at the upstream end of F47-5. Looking through the lattice at focusing stations (large β_x) we found the following stations to have the appropriate phase

$$\theta = \theta_A = \frac{\pi}{58} \left(n+\frac{1}{2}\right) .$$

In order of exactness of phase-advance they are

Station	n	Sextupole Polarity
A13, D13	2, 60	Positive
B44, E44	36, 94	Positive
A32, D32	11, 69	Negative
B24, E24	27, 85	Negative

It is desirable to locate the sextupole just upstream of the septum. Therefore the most desirable location is E44. Of course, it is also possible to place two sextupoles at arbitrary phase locations and adjust their relative strength to produce a 58th harmonic of the sextupole field with the proper phase.

2. Strength of sextupole

We shall take

$$\begin{cases} \Delta x_s = 0.01\text{m}, & x_s = 0.02\text{m} \\ \beta_s = 75\text{m} & \end{cases}$$

and get from Equs. 11 and 14

$$\beta_A^{3/2} \ \frac{B''\ell}{B\rho} = \frac{8}{3} \sqrt{\beta_s} \frac{\Delta x_s}{x_s^2} = 577 \ \text{m}^{-\frac{1}{2}}$$

At 1000 GeV, $B\rho$ = 33388 kG m and if the sextupole is located at a high β value of β_A = 90m we obtain

$$B''\ell = 2.26 \times 10^4 \ \text{kG/m}$$

This is a very strong sextupole. To reduce the strength required one has to either reduce Δx_s or increase x_s. For the above values the required good-field aperture is $2(x_s+\Delta x_s)$ = 6 cm = 2.36 in. Keeping the same good-field aperture but reducing Δx_s to 0.5 cm and increasing x_s to 2.5 cm will reduce $B''\ell$ by about a factor of 3. However, this will increase the amount of beam hitting the septum by a factor of 2.

3. Onset of extraction

The measured emittance of the main ring beam at 300 GeV is $\sim\pi \times 10^{-7}$ m. Theoretically the emittance should vary inversely as momentum, but there are also many mechanisms acting to dilute the phase-space density, hence to increase the emittance as the beam is transferred to and accelerated in the doubler/saver. So we shall assume this same value of $E = \pi \times 10^{-7}$m at extraction from the doubler/saver. Equ. 12 then gives

$$\varepsilon_o = \left(\frac{4E}{3\sqrt{3}}\right)^{\frac{1}{2}} \frac{1}{16\pi} \left(\beta_A^{3/2} \frac{B''\ell}{B\rho}\right) = 0.0056$$

Thus, during extraction ν_x should be reduced smoothly from $\frac{58}{3} + 0.0056$ to $\frac{58}{3}$.

4. Tolerance on octupole

Equs. 13, 14, and 15 give

$$\sum \beta_B^2 \frac{B'''\ell}{B\rho} < 2 \frac{\sqrt{B_s}}{x_s} \left(\beta_A^{3/2} \frac{B''\ell}{B\rho}\right) = 5 \times 10^5 \text{ m}^{-1}$$

If random octupole errors exist in the 774 dipoles we should put β_B = average $\beta \cong$ 50m and $\ell = 2\pi\rho$ in the left-hand-side, and multiply the right-hand-side by $\sqrt{774}$ to give

$$\left(\frac{B'''}{B}\right)_{\text{random}} < \frac{\sqrt{774}}{2\pi \times 50^2} \times 5 \times 10^5 \text{ m}^{-3} = 886 \text{ m}^{-3}$$

$$= 0.0145 \text{ in}^{-3}$$

which is very easy to achieve. This implies that as far as random octupole error in the dipoles is concerned if it meets the requirement for containing the beam during acceleration it will not cause any trouble for extraction. Even if the octupole error is systematic the required tolerance of

$$\left(\frac{B'''}{B}\right)_{syst} < \frac{1}{\sqrt{774}} \left(\frac{B'''}{B}\right)_{random} = 5.2 \times 10^{-4} \text{ in}^{-3}$$

is still not difficult to meet.