national accelerator laboratory

TM-352
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TRANSFER FUNCTION AND COIL IMPEDANCE FOR MAIN-RING QUADRUPOLE MAGNETS

S. C. Snowdon

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SUMMARY

The transfer function between the Fourier amplitudes of magnetic gradient and the excitation current has been calculated in a manner similar to that given in TM-325. To reduce the complexity only the lowest mode is calculated thereby restricting the validity to frequencies below 1kHz. The coil impedance is also given for a similar frequency range. Computer results are given for the 7 ft. and the 4 ft. quadrupoles.

MAGNETIC CIRCUIT

Fig. 1 indicates the cross section of the quadrupole under consideration. Let a designate the aperture radius, b the horizontal half aperture of the vacuum chamber, d the distance from the quadrupole center to the inner edge of the coil in the horizontal slot, h the half height of the coil in the horizontal slot, ℓ the length of the mean flux path in the horizontal octant of the yoke, w the average width of the horizontal octant, and N

the number of coil turns in the full slot. For simplicity the quadrupole is considered symmetrical with d representing the mean of the horizontal and vertical coil distances.

If U(x) designates the flux per unit length of the quadrupole passing through the median plane from x = 0, the quadrupole center, to x = x, then the average flux density B_o in the horizontal octant of the yoke is to be determined by

$$U(d) + U_{coil} = wB_o, \tag{1}$$

where Appendix IV of TM-325 may be used to give the flux passing through the coil. Thus

$$U_{coil} = \frac{\cosh Pc - 1}{P \sinh Pc} \cdot \left[NH(d) + \frac{\pi N^2}{h} I \right], \quad \text{(emu)} \tag{2}$$

where

$$P^2 = 4\pi j\omega\sigma_{coil}, \quad \text{(emu)} \tag{3}$$

c is the horizontal width of the conductor in the horizontal slot, and H(d) is the magnetic field at the edge of the horizontal coil. Although easily modified for other cases, it is assumed that only one layer of conductors exists between the median plane and the pole.

To complete the magnetic circuit description one evaluates the Ampere integral along the flux line beginning at x = x, y = 0 through the vacuum chamber. In the iron the mean flux path will be used. Thus, remembering that in the ideal quadrupole the potential on the iron is given by $\frac{1}{2} B' a^2$ where B'

is the gradient, one has approximately

$$\frac{1}{2}a^2H'(x)+\frac{\ell}{\mu}B_o = 2\pi NI-4\pi \int_{(1)}^{(2)} i_c dt-4\pi \int_{(2)}^{(3)} i_c dt-4\pi i_I \ell, \tag{4}$$

where from TM-325 the surface current density of eddy currents in the vacuum chamber is

$$i_c = j\omega sU(x), \tag{5}$$

s being the surface conductivity, and the effective surface current density due to eddy currents in the iron is

$$4\pi i_I = \frac{k}{\mu} B_o, \tag{6}$$

where μ is the iron permeability and

$$k = \frac{\lambda\delta}{2} \cdot \frac{\cosh \lambda\frac{\delta}{2}-1}{\sinh \lambda\frac{\delta}{2}} . \tag{7}$$

Here δ is the lamination thickness and

$$\lambda^2 = 4\pi\mu\sigma j\omega \qquad \text{(emu)} \tag{8}$$

where σ is the volume conductivity of the iron.

From the geometry of the ideal quadrupole dt along the iron surface (assumed to be coincident with the vacuum chamber surface) is

$$dt = \frac{x}{\sqrt[4]{x^4+a^4}}\, dx. \tag{9}$$

Along the flux line at x = b one has approximately

$$\int_{(2)}^{(3)} dt = \frac{1}{2}\frac{a^2}{b} . \tag{10}$$

Thus, remembering that

$$U(x) = \int_0^x H(x)\,dx \tag{11}$$

one has from Eq. (4)

$$\frac{1}{2}a^2U''(x) + 4\pi j\omega s \int_x^b \frac{x}{\sqrt[4]{x^4+a^4}}\,U(x)\,dx$$

$$+ 4\pi j\omega s\frac{a^2}{b}U(b) + \frac{(k+1)}{\mu}\,\ell B_o = 2\pi NI. \text{ (emu)} \tag{12}$$

Differentiating Eq. (12) gives

$$\frac{1}{2}a^2U'''(x) - 4\pi j\omega s\,\frac{x}{\sqrt[4]{x^4+a^4}}\,U(x) = 0. \qquad \text{(emu)} \tag{13}$$

SOLUTION

By definition the flux line passing through the origin may be designated by $U(0) = 0$. A quadrupole field is further designated by $U'(0) = 0$, and $U''(0) = H_0'$. Thus if, for convenience,

$$U(x) = H_0'u(x), \tag{14}$$

then the conditions on $U(x)$ are

$$u(0) = 0, \qquad u'(0) = 0, \qquad u''(0) = 1 \tag{15}$$

and the differential equation is

$$u'''(x) = \begin{cases} Q(x)u(x) & 0 < x < b \\ 0 \quad , & b < x < d \end{cases} \tag{16}$$

where

$$Q(x) = \frac{8\pi j\omega s}{a^2} \frac{x}{\sqrt[4]{x^4+a^4}} \; . \tag{17}$$

Numerical integration using a Runge-Kutta procedure yields u(x) in the interval $0 < x < b$. Outside b Eq. (16) yields directly

$$u(x) = u(b) + \frac{1}{2}\alpha(x^2-b^2) . \tag{18}$$

The constant α may be found using Eq. (12) once for $x = b$ and once for $x = 0$. Subtraction yields

$$\alpha = 1 + \int_0^b Q(x)u(x)dx. \tag{19}$$

TRANSFER FUNCTION AND COIL IMPEDANCE

Having found u(b) and α one may write Eq. (1) and Eq. (12) for $x = 0$ in the form

$$\left[u(b)+\frac{1}{2}\alpha(d^2-b^2)+\alpha cdN\frac{(\cosh Pc-1)}{Pc \sinh Pc}\right]H_o'-wB_o = \frac{\pi cN^2}{h}\cdot\frac{(\cosh Pc-1)}{Pc \sinh Pc} I \tag{20}$$

and

$$\left[\frac{1}{2}a^2\alpha + \frac{4\pi j\omega sa^2}{b} u(b)\right]H_o' + \frac{k+1}{\mu}\ell B_o = 2\pi NI. \tag{21}$$

Eqs (20 and (21) may be solved simultaneously for H_o' and B_o. The transfer function is then given by

$$Y = \frac{H_o'}{I} \quad \text{(emu)} \tag{22}$$

and the complex inductance by

$$L = 4Nw\frac{B_o}{I} \times \text{(quadrupole length)}. \quad \text{(emu)} \tag{23}$$

From this inductance and the ac resistance R of the conductors paired by twos in the coil slot one has for the coil impedance

$$Z = R + j\omega L. \tag{24}$$

The inductance, of course, will have an imaginary part and contribute to the actual resistance seen between the coil terminals.

NUMERICAL RESULTS

The relations necessary to solve Eqs. (20) and (21) have been coded for the CDC6600 in a program GITRANS. The complex values of Y in Gauss/cm/A and Z in Ohms is given for each assumed frequency in the range 0 to 1000 Hz. A second transfer function is produced from that of Eq. (22) if each quadrupole is paralleled by a resistor R_o. The effective transfer function is

$$YE = \frac{R_o}{Z+R_o} \cdot Y \tag{25}$$

and the effective impedance is

$$ZE = \frac{R_o Z}{Z+R_o} . \tag{26}$$

For each frequency Y, Z, YE, and ZE are given. Results for both the 7 ft. and the 4 ft. quadrupoles are included. See Figs. 1 and 2 and attached computer output.

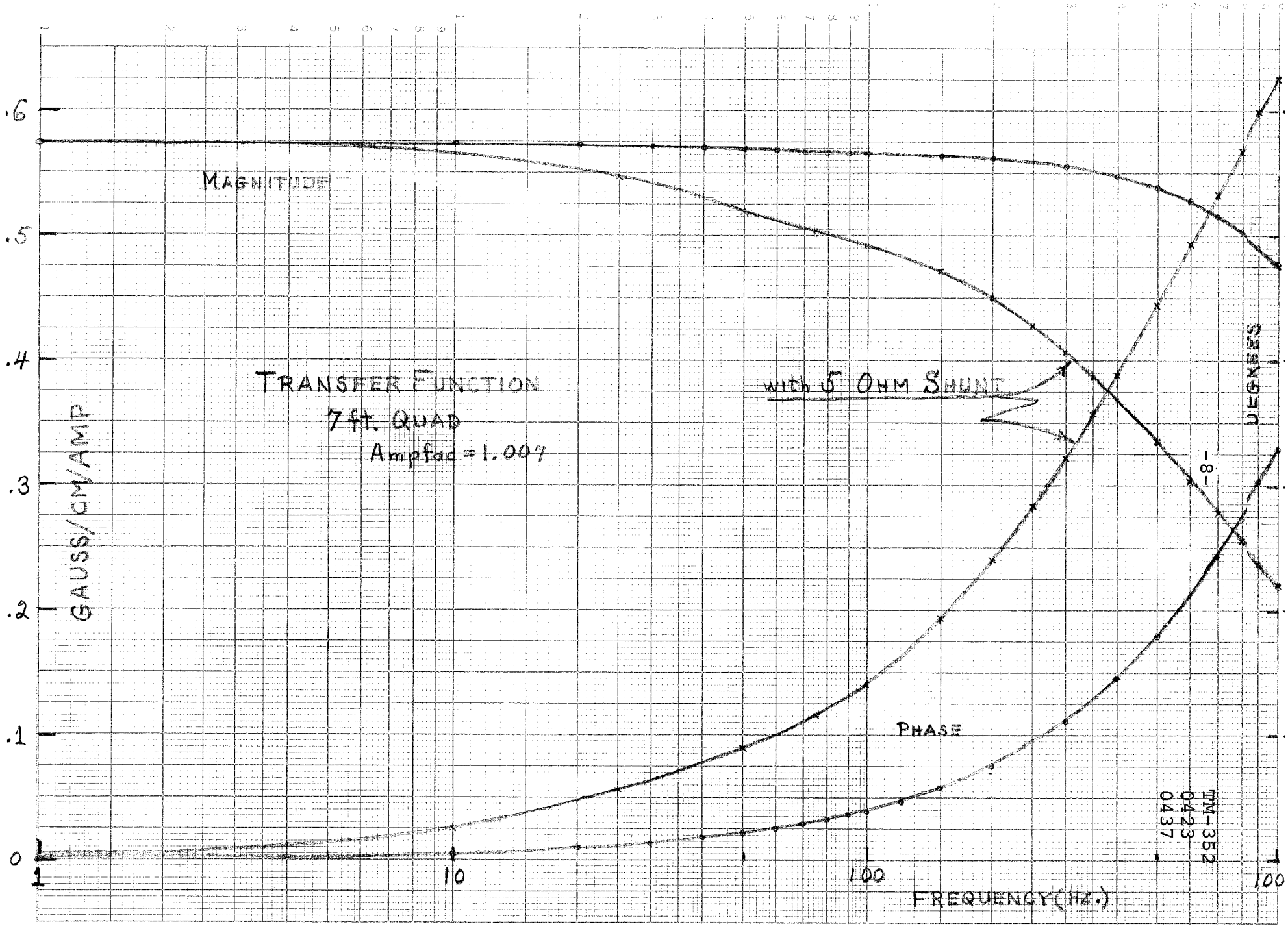
.6
.5
.4
.3
.2
.1
0
GAUSS/CM/AMP
MAGNITUDE
TRANSFER FUNCTION
7 ft. QUAD
Ampfac = 1.007
with 5 OHM SHUNT
PHASE
DEGREES
-8-
TM-352
0423
0437
1
10
100
100
FREQUENCY (HZ.)

FIG. 1

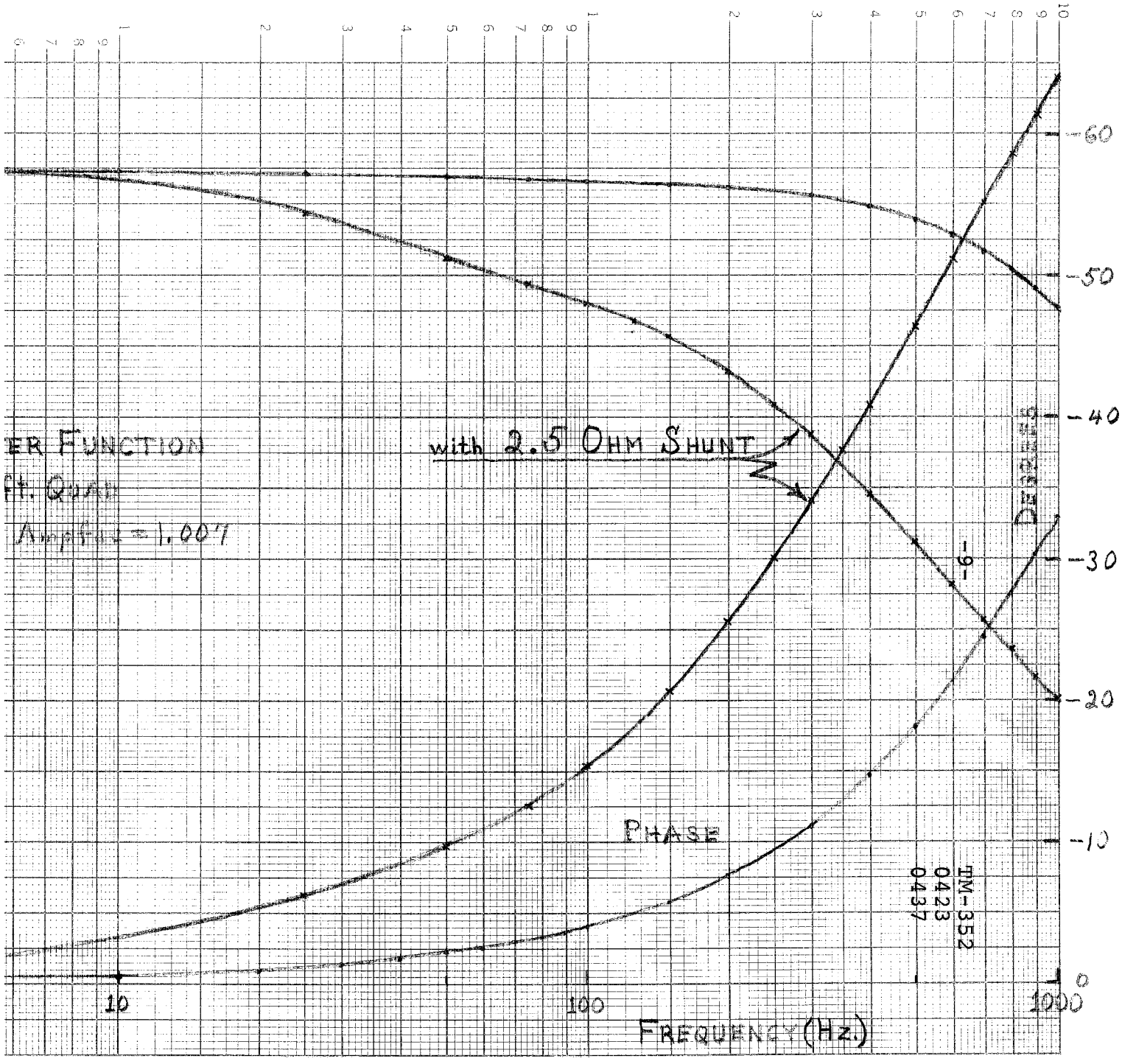
ER FUNCTION
ft. QUAD
Amplitude = 1.007
with 2.5 OHM SHUNT
PHASE
DEGREES
-60
-50
-40
-30
-20
-10
0
10
100
1000
FREQUENCY (Hz.)
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FIG. 2

TRANSFER FUNCTION BETWEEN GRADIENT AND CURRENT IN MAIN RING QUADRUPOLE(7FT)

APER(IN)	= 3.286	YOKE WIDTH(IN)	=23.270	YOKE HEIGHT(IN)	=17.624 HOR. YOKE TI
VER. YOKE THICKNESS(IN)	= 2.582	HOR. COIL APERTURE(IN)	= 8.000	VER. COIL APERTURE(IN)	= 4.000 HOR. VAC. CI
VER. VAC. CH. APER(IN)	= 3.500	COIL SLOT THICKNESS(IN)	= 1.370	LAMINATION THICKNESS(IN)	= .0615 VAC. CHAMB.
IRON REST.(MUOHM-CM)	=12.000	VAC. CH. REST.(MUOHM-CM)	=75.500	PERMEABILITY OF IRON	=3000.0 AMPFAC
DC INDUCTANCE(HY)	= .0032	COIL RESIST.(MUOHM-IN)	= .772	COIL CORNER RADIUS(IN)	= .0625 MAGNET LENG
NO. TURNS IN COIL SLOT	= 8	CONDUCTOR WIDTH(IN)	= 1.000	CONDUCTOR HEIGHT(IN)	= .565 COND. HOLE I

	MAGNET ALONE				MAGNET WITH 5 I		
FREQ (HZ)	TRANSFER FCT. (G/CM/AMP)	TRANSFER FCT. (DEGREES)	AC IMPEDANCE (OHMS)		TRANSFER FCT. (G/CM/AMP)	TRANSFER FCT. (DEGREES)	
0.0000	.5733	0.0000	.0049	0.0000	.5728	0.0000	
25.0000	.5714	-1.2326	.2001	.4131	.5476	-5.7747	
50.0000	.5690	-2.1923	.4257	.6442	.5207	-8.9639	
75.0000	.5676	-3.0840	.5597	.8325	.5048	-11.6003	
100.0000	.5665	-3.9747	.6549	1.0201	.4929	-14.2003	
125.0000	.5655	-4.8719	.7386	1.2095	.4821	-16.7738	
150.0000	.5644	-5.7738	.8214	1.3986	.4714	-19.2829	
175.0000	.5633	-6.6779	.9063	1.5851	.4606	-21.7010	1
200.0000	.5621	-7.5820	.9944	1.7680	.4497	-24.0146	1
225.0000	.5608	-8.4847	1.0859	1.9463	.4388	-26.2194	1
250.0000	.5593	-9.3845	1.1805	2.1199	.4280	-28.3165	1
275.0000	.5578	-10.2807	1.2777	2.2885	.4174	-30.3098	1
300.0000	.5561	-11.1722	1.3772	2.4522	.4069	-32.2050	1
325.0000	.5543	-12.0586	1.4787	2.6109	.3968	-34.0082	1
350.0000	.5524	-12.9391	1.5819	2.7649	.3869	-35.7252	1
375.0000	.5503	-13.8134	1.6866	2.9142	.3773	-37.3620	1
400.0000	.5482	-14.6811	1.7926	3.0588	.3680	-38.9240	1
425.0000	.5460	-15.5418	1.8998	3.1990	.3590	-40.4161	2
450.0000	.5437	-16.3952	2.0080	3.3348	.3503	-41.8429	2
475.0000	.5413	-17.2410	2.1172	3.4663	.3419	-43.2085	2
500.0000	.5388	-18.0790	2.2271	3.5935	.3338	-44.5168	2
525.0000	.5362	-18.9091	2.3376	3.7165	.3260	-45.7714	2
550.0000	.5336	-19.7310	2.4487	3.8354	.3184	-46.9756	2
575.0000	.5308	-20.5446	2.5602	3.9503	.3112	-48.1323	2
600.0000	.5280	-21.3497	2.6720	4.0612	.3041	-49.2445	2
625.0000	.5252	-22.1463	2.7840	4.1682	.2974	-50.3146	2
650.0000	.5223	-22.9342	2.8960	4.2713	.2909	-51.3452	2
675.0000	.5193	-23.7133	3.0080	4.3707	.2846	-52.3386	2
700.0000	.5163	-24.4836	3.1199	4.4664	.2785	-53.2969	2
725.0000	.5132	-25.2451	3.2314	4.5584	.2727	-54.2220	2
750.0000	.5101	-25.9975	3.3427	4.6470	.2671	-55.1158	2
775.0000	.5069	-26.7410	3.4534	4.7320	.2616	-55.9802	2
800.0000	.5038	-27.4754	3.5636	4.8138	.2564	-56.8166	2
825.0000	.5005	-28.2008	3.6731	4.8922	.2513	-57.6266	2
850.0000	.4973	-28.9172	3.7819	4.9674	.2464	-58.4116	2
875.0000	.4940	-29.6244	3.8899	5.0396	.2417	-59.1730	2
900.0000	.4907	-30.3227	3.9969	5.1088	.2371	-59.9121	2
925.0000	.4874	-31.0119	4.1030	5.1750	.2327	-60.6299	2
950.0000	.4840	-31.6920	4.2080	5.2384	.2285	-61.3275	2
975.0000	.4807	-32.3632	4.3120	5.2992	.2243	-62.0061	2
1000.0000	.4773	-33.0255	4.4147	5.3572	.2203	-62.6665	2

TRANSFER FUNCTION BETWEEN GRADIENT AND CURRENT IN MAIN RING QUADRUPOLE(4FT)

APER(IN)	= 3.286	YOKE WIDTH(IN)	=23.270	YOKE HEIGHT(IN)	=17.624 HOR. YOKE TH
VER. YOKE THICKNESS(IN)	= 2.582	HOR. COIL APERTURE(IN)	= 8.000	VER. COIL APERTURE(IN)	= 4.000 HOR. VAC. CH
VER. VAC. CH. APER(IN)	= 3.500	COIL SLOT THICKNESS(IN)	= 1.370	LAMINATION THICKNESS(IN)	= .0615 VAC. CHAMB.
IRON REST.(MUOHM-CM)	=12.000	VAC. CH. REST.(MUOHM-CM)	=75.500	PERMEABILITY OF IRON	=3000.0 AMPFAC
DC INDUCTANCE(HY)	= .0018	COIL RESIST.(MUOHM-IN)	= .772	COIL CORNER RADIUS(IN)	= .0625 MAGNET LENGT
NO. TURNS IN COIL SLOT	= 8	CONDUCTOR WIDTH(IN)	= 1.000	CONDUCTOR HEIGHT(IN)	= .565 COND. HOLE D

	MAGNET ALONE				MAGNET WITH 2.5		
FREQ (HZ)	TRANSFER FCT. (G/CM/AMP)	(DEGREES)	AC IMPEDANCE (OHMS)		TRANSFER FCT. (G/CM/AMP)	(DEGREES)	
0.0000	.5753	0.0000	.0032	0.0000	.5726	0.0000	.
25.0000	.5714	-1.2326	.1196	.2361	.5431	-6.3819	.
50.0000	.5690	-2.1923	.2540	.3681	.5120	-9.8062	.
75.0000	.5676	-3.0840	.3336	.4757	.4938	-12.6142	.
100.0000	.5665	-3.9747	.3900	.5829	.4804	-15.3782	.
125.0000	.5655	-4.8719	.4394	.6912	.4682	-18.1040	.
150.0000	.5644	-5.7738	.4879	.7992	.4562	-20.7482	.
175.0000	.5633	-6.6779	.5377	.9058	.4443	-23.2817	.
200.0000	.5621	-7.5820	.5892	1.0103	.4324	-25.6912	.
225.0000	.5608	-8.4847	.6427	1.1122	.4205	-27.9735	.
250.0000	.5593	-9.3845	.6978	1.2114	.4089	-30.1317	.
275.0000	.5578	-10.2807	.7545	1.3077	.3976	-32.1719	.
300.0000	.5561	-11.1722	.8124	1.4012	.3865	-34.1018	.
325.0000	.5543	-12.0586	.8715	1.4920	.3759	-35.9292	.
350.0000	.5524	-12.9391	.9314	1.5799	.3655	-37.6618	.
375.0000	.5503	-13.8134	.9923	1.6652	.3556	-39.3069	1.
400.0000	.5482	-14.6811	1.0538	1.7479	.3461	-40.8710	1.
425.0000	.5460	-15.5418	1.1160	1.8280	.3369	-42.3601	1.
450.0000	.5437	-16.3952	1.1787	1.9056	.3281	-43.7797	1.
475.0000	.5413	-17.2410	1.2419	1.9807	.3196	-45.1347	1.
500.0000	.5388	-18.0790	1.3056	2.0534	.3115	-46.4295	1.
525.0000	.5362	-18.9091	1.3696	2.1237	.3037	-47.6683	1.
550.0000	.5336	-19.7310	1.4338	2.1917	.2962	-48.8547	1.
575.0000	.5308	-20.5446	1.4983	2.2573	.2890	-49.9921	1.
600.0000	.5280	-21.3497	1.5630	2.3207	.2821	-51.0838	1.
625.0000	.5252	-22.1463	1.6277	2.3818	.2755	-52.1325	1.
650.0000	.5223	-22.9342	1.6925	2.4408	.2691	-53.1411	1.
675.0000	.5193	-23.7133	1.7572	2.4975	.2630	-54.1119	1.
700.0000	.5163	-24.4836	1.8218	2.5522	.2572	-55.0473	1.
725.0000	.5132	-25.2451	1.8863	2.6048	.2515	-55.9493	1.
750.0000	.5101	-25.9975	1.9505	2.6554	.2461	-56.8201	1.
775.0000	.5069	-26.7410	2.0145	2.7040	.2408	-57.6613	1.
800.0000	.5038	-27.4754	2.0781	2.7507	.2358	-58.4748	1.
825.0000	.5005	-28.2008	2.1413	2.7955	.2309	-59.2621	1.
850.0000	.4973	-28.9172	2.2041	2.8385	.2263	-60.0247	1.
875.0000	.4940	-29.6244	2.2664	2.8798	.2218	-60.7639	1.
900.0000	.4907	-30.3227	2.3282	2.9193	.2174	-61.4812	1.
925.0000	.4874	-31.0119	2.3894	2.9571	.2132	-62.1775	1.
950.0000	.4840	-31.6920	2.4501	2.9934	.2092	-62.8542	1.
975.0000	.4807	-32.3632	2.5100	3.0281	.2053	-63.5121	1.
1000.0000	.4773	-33.0255	2.5694	3.0613	.2015	-64.1524	1.