national accelerator laboratory

TM-282
0100

# LINEAR BEAM OPTICS PROGRAM FOR TIME SHARING USE

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ILBOP is a FORTRAN coded interactive beam optics program written for small time sharing computers. It now runs on the NCN[1] system (Honeywell 1648 computer) in less than 16K. Although this code has many features in common with BEAMTRC,[2] it is considerably more powerful and usually more convenient to use. The principal features are:

1. 5 x 5 transfer matrices.
2. Tracing of up to ten rays.
3. Calculation of beam envelopes with several options for input and output formats.
4. Projection of element apertures back to the start of the system.
5. Bending magnet edge effects.
6. Skew elements.
7. Easy change of units.
8. Built in editing for calculating from an existing data file.
9. Updating of only variable elements for repeated executions.

If the new features are not needed, BEAMTRC may be easier to use for data entered directly from the teletype during execution because there are fewer parameters for each element.

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For work on a prepared file, however, ILBOP is much easier to use. The details of all features are given in the step by step instructions for use. The notation used in the examples is summarized by four conventions:

1. Items which are to appear exactly as shown are written in capitals.
2. An item for which a substitution is required is described in a lower case entry.
3. Items typed by the user are underlined.
4. An optional item or a set of items from which a single choice is made is enclosed in square brackets [ ].

All numbers are entered with decimal points and may either be in F10.n format or in NCN "free" format where each number is separated by commas and uses ten or fewer positions. A zero entry can be represented by a null response for the item, viz., just a comma. In this manner fields which are not relevant can be simply filled. An entry is terminated by a carriage return denoted (CR). Units for the program as compiled are transverse millimeters and milliradians, longitudinal meters, rotations in degrees, fields in kG and momentum in BeV/c. These units may be changed by changing the assignment of conversion factors in the subroutine READIN (see Appendix II for program listing). An example of program use appears as Appendix I.

I. SIGN ON

ILBOP is stored in executable form on the NCN system. Sign on under 146 or your section ID if 146 is busy. The execution command is:

?LRUN ILBOP [/146] (CR)

where /146 is used from other ID's.

II. DATA SOURCE

A. The data source may be either a data file (DSK) prepared in advance or the teletype keyboard (TTY). The prompt and response are:

ENTER DATA SOURCE (DSK OR TTY)! [DSK,TTY] (CR)

B. If the DSK option is chosen, one must give the name of a file containing the data and control information in the same form given below for TTY input:

DEFINE FILE 1 = file name (CR)

The information read from the file is printed after the relevant prompt.

III. NUMBER OF TRANSPORT ELEMENTS AND CENTRAL MOMENTUM

The prompt and response are:

THE NUMBER OF ELEMENTS AND CENTRAL MOMENTUM! $n,p_0$ (CR)

The response is n, the number of elements, and $p_0$, the reference momentum.

IV. CONTROL OPTIONS

The four quantities OPTION (I), I = 1,4 activate program features and choose input and ouput options. A value of zero

for any OPTION (I) suppresses calculation and output relevant to a corresponding program feature.

OPTIONS FOR MATRIX, TRACE, ENVELOPE, AND APERTURES!

$n_1$, $n_2$, $n_3$, $n_4$ (CR)

Particular attention is called to the response

,,11.,,

which specifies a beam envelope trace with input and output variables chosen to be closely related to quantities measured in a transport system adjustment.

A. Transfer matrix output.

1. $n_1$ = 1. 5 x 5 output.

2. $n_1$ = 2. Betatron function and equilibrium orbit. This output is similar to SYNCH[3] output.

B. Ray trace output.

$n_2$ is the number of rays to be traced.

INITIAL X, X', Y, Y', DP/P FOR TRACE OF $n_2$ RAYS

There are required $n_2$ entries of the form:

!x, x', y, y', Δp/p (CR)

C. Beam envelope output. $n_3 = \ell m$, a two-digit number, where the first digit governs output form and the second governs the input form.

1. Input

a. m = 1

INITIAL BEAM PARAMETERS WX, RX, EX, WY, RY, EY, WP

!$w_x$, $r_x$, $e_x$, $w_x$, $r_y$, $e_y$, $w_p$ (CR)

where

$w_x$ is beam half width in mm,

$r_x$ is the correlation in x, x' ($-1 \le r_x \le 1$),

$e_x$ is the area of the x, x' ellipse in mm mrad,

$w_y$ is the beam half height in mm,

$r_y$ is the correlation in y, y',

$e_y$ is the area of the y, y' ellipse,

$w_p$ is the momentum half width $\Delta p/p_0$ in %,

$p_0$ is the momentum of the reference orbit.

There is also a request for the beam centroid. The divergence displacements $c_x$ and $c_y$ are assumed to be zero.

BEAM CENTER CX, CY, CP! $\underline{c_x,\ c_y,\ c_p}$

where

$c_x$ is the horizontal displacement of the beam center from the reference orbit,

$c_y$ is the vertical displacement of the beam center from the reference orbit,

$c_p$ is the momentum difference from the reference momentum in %.

b. m = 2

BEAM HALF WIDTHS WX, WX', WY, WY', WP

!$\underline{w_x,\ w_x',\ w_y,\ w_y',\ w_p\ \text{(CR)}}$

BEAM CENTER CX, CX', CY, CY', CP

!$\underline{e_x,\ e_x',\ c_y,\ c_y',\ c_p\ \text{(CR)}}$

c. $m = 3$

INITIAL BEAM MATRIX IN INTERNAL FORM

$!\sigma_{ij}$ $(i=1,2,3,4,5;\quad j=1,i)$ (CR)

The elements of the beam matrix $\sigma^4$ are entered in lower triangular order one row per line.

BEAM CENTER CX, CX', CY, CY', CP

$!c_x,\ c_x',\ c_y,\ c_y',\ c_p$ (CR)

where the c's are the components of the displacement of the phase ellipsoid from the reference trajectory.

d. $m = 4$

INITIAL BEAM MATRIX IN TRANSPORT FORM

$!w_i, r_{ij} (i=1,2,3,4,5;\quad j=1,i-1)$

where the $r_{ij}$ are the beam correlations[4]

$r_{ij} = \sigma_{ij} / \sqrt{\sigma_{ii}\sigma_{jj}}$

and $w_i$ is the half width in the ith coordinate. This form of input can be taken directly from a TRANSPORT output.

2. Output

a. $\ell = 1$

The beam envelope is output in a form considered closely related to measurable quantities. The printed quantities are $w_x$, $c_x$, $r_x$, $w_x'$, $w_y$ $c_y$, $r_y$, $w_y'$ where

$w_x$ is the horizontal beam half width in mm,

$c_x$ is the horizontal displacement of the beam center from the reference orbit in mm,

$r_x$ is the correlation of x, x' ($-1 \leq r_x \leq 1$),

$w_x'$ is the horizontal divergence half width in mrad,

$w_y$ is vertical beam half width in mm,

$c_y$ is the vertical displacement of the beam center from the reference orbit in mm,

$r_y$ is the correlation of y, y',

$w_y'$ is the vertical divergence half width in mrad.

b. $\ell = 2$

The beam matrix $\sigma$ is output in TRANSPORT[4] form, i.e., it is lower triangular with half widths $w_i$ on the diagonal and correlations $r_{ij}$ off the diagonal (see IV-V.1(d) for definitions). The last row contains the displacement of the center of the beam ellipsoid from the reference orbit.

c. $\ell = 3$

The beam matrix elements, $\sigma_{ij}$, are given in lower triangular form. The last row printed is the displacement of the center of the beam ellipsoid from the reference orbit.

V. PARAMETERIZATION OF THE TRANSPORT ELEMENTS

If the elements are being read from a data file, the user is asked if he wishes to see a listing as they are read.

LIST ELEMENTS! [YES, NO] (CR)

If a YES is given, the elements are listed as they are read. The file is read using the same formats used for TTY data. A beam element has the nine parameters described in the paragraphs A through I below. Each of these parameters has a name which is used in data modification as described in Section VI-B.3.

A. LABL a four-character label; it can be blank but cannot be omitted.

B. T, the element type.

The type codes given below may be prefixed with a minus sign; when this is done the element is designated as a changing element, and new values are prompted for each recalculation from the data (see VI Data modification).

1. T = 1. Drift space.

2. T = 2. Bending magnet.

The bending magnet can include no focusing at all, at entrance only, at exit only, or at both entrance and exit. It is, therefore, possible to represent a real magnet by several type 2 elements in sequence to get results inside the magnet and still treat the edge focusing correctly.

a. T = 2. No edge focusing.

b. T = 2.n. Entrance edge focusing included,

where n is the number of 2. elements used to represent the magnet.

c. T = 2.0n. Exit edge focusing included, where n is the number of 2. elements used to represent the magnet.

d. T = 2.11. Entrance and exit focusing included.

3. T = 3. Quadrupole.

4. T = 4. Coordinate rotation.

This element is used to represent a skew element by a three-element sequence with the element to be skewed sandwiched between the skewing rotation and its negative.

a. A downward bend is formed from a 2. element between a $+90^{\circ}$ and a $-90^{\circ}$ rotation.

b. An upward bend is formed from a 2. element between a $-90^{\circ}$ rotation and a $+90^{\circ}$ rotation.

c. A bend to the right is formed from a 2. element between a $+180^{\circ}$ rotation and a $-180^{\circ}$ rotation.

C. L, the length of the element in meters.

D. S, the strength of the element.

1. For a drift this field can be null (zero).

2. For bending magnets S is the field in kG.

3. For quadrupoles S is the gradient in kG/mm with S > 0 horizontal focusing.

4. For rotations this is the rotation in degrees.

E. K, the profile parameter $B'/B_0$ in $(mm)^{-1}$ for bending magnets. This parameter is not used for other elements.

F. AH, the horizontal half aperture in mm.

If the aperture option is zero, this element is not used.

G. AV, the vertical half aperture in mm used only when the aperture trace option is active.

H. C, a conversion factor between arbitrary units of excitation (e.g., amperes) and the proper units of S. This feature is to assist in transport adjustment.

I. P, the output switch.

A non-zero P indicates that the output specified by the options is desired for this element.

## VI. DATA MODIFICATION AND REPETITIVE CALCULATIONS

When data have been entered from a disk file, they are subject to modification before the calculation begins. Data entered from the TTY are processed once and then, like disk data, are subject to modification for further calculations.

A. Variable elements.

Elements which have been flagged by a negative type code are considered changing elements and require redefinition for every calculation.

CHANGE EL#4

| labl | t | ℓ | k | ah | av | c | p |
|---|---|---|---|---|---|---|---|
| !label, | type, | length, | profile, | $a_h$, | $a_v$, | c, | print (CR) |

The new values may be entered using commas to separate numerical fields as usual or, for ease in verification, they may be entered directly under the old values without commas.

B. Changes indicated by code words.

After any variable elements have been redefined, the user may change any program parameters by properly naming what he wishes to change.

CHANGES BY NAME

ITEM #1! code word (CR)

Code word is a four-letter code. When the change corresponding to a code has been effected, the "ITEM" prompt is repeated with an incremented count. Code words recognized by ILBOP are

1. DONE - No more changes; perform new calculation.
2. STOP - Leave the program.
3. label - The LABL parameter of any element. The prompt and response for this code are:

   PARAMETER! [T, L, S, K, AV, AH, C, P] (CR)

   where the parameter to be changed is specified by one of the codes defined in Section V. The new value of the parameter is entered after the old is printed out:

   numerical value NEW VALUE = ! new value (CR)

4. ELMN - A transport element (possibly blank labeled) is to be changed. One must know the parameter number (from 1 to 8 for T to P) and the element's serial position in the system.

INDICES! parameter number, element number (CR)

The changed value is then entered after the old value.

numerical value NEW VALUE = ! new value (CR)

The INDICES prompt will be given for further element changes. One signifies completion by null entries

INDICES!,,(CR)

and returns to the ITEM prompt for a code word.

5. BEAM - Changes to be made in the beam envelope. The form of prompts for the beam envelope changes is determined by the current input option for the beam envelope, the units digit, m, of the ENVELOPE option, $\ell m$.

a. m = 1

PARAMETER![DONE, WX, RX, EX, WY, AY, RY, WP] (CR)

where these parameters are defined in IV-C.1(a) and DONE means no more changes, return to ITEM prompt.

b. m = 2

INDEX![i = 1,2,3,4,5] (CR)

numerical value NEW VALUE = !$w_i$

where these numbers label $w_x$ through $w_p$, respectively. A zero or null entry gives a return to the ITEM prompt

c. m = 3

INDICES! i, j (CR)

numerical value NEW VALUE

= ![$r_{ij}$($i \neq j$), $w_i$($i=j$)](CR)

where

$r_{ij}$ is a correlation and $w_i$ is a beam half width,

$i=j=0$ returns program to the ITEM prompt.

d. $m = 4$

INDICES !i, j (CR)

numerical value NEW VALUE = $\sigma_{ij}$ (CR)

where $\sigma_{ij}$ is the beam envelope matrix element in internal form. The symmetric element is automatically changed, $i=j=0$ returns the program to the ITEM prompt.

6. XVEC - Changes to be made in the rays for tracing.

INDICES !coordinate number, vector number (CR)

numerical value NEW VALUE = !coordinate (CR)

The coordinates are numbered from 1 to 5 for x, x', y, y', and $\Delta p/p$, respectively. A zero or null coordinate number terminates the ray trace updates.

7. NELM - Change the number of elements to be included.

value NEW VALUE = !number of elements (CR)

8. PZER - Change the momentum of the reference trajectory.

value NEW VALUE = !momentum (CR)

9. DP/P - Change $\Delta p/p_0$ for aperture trace.

value NEW VALUE = !$\Delta p/p_0$ (CR)

10. OPTN - Change program control options.

$value_1$ $value_2$ $value_3$ $value_4$ NEW VALUE
= !$n_1$, $n_2$, $n_3$, $n_4$ (CR)

11. DATA - Change a known storage location.

INDEX !relative storage location (CR)

value NEW VALUE = !new value (CR)

Any item in storage may be changed by figuring from the program the address of that item relative to the beginning of blank common. See the program listing in Appendix II.

## GENERAL HINTS

1. Preparation of data base: The NCN[1] context editor may be used to rework the data base for permanent changes. The data will be easier to manipulate if numerical values are in F10.n format without the commas used in free format.

2. Satisfying repeated prompts. Various places in the data modification routine prompts are repeated until the user indicates he has completed all updating of a given type. There are two different cases:

a. Prompts for code word or parameter:

ITEM #n !DONE (CR)

PARAMETER !DONE (CR)

b. Prompts for numerical indices:

INDEX !, (CR)

INDICES !,, (CR)

3. Getting out of the program:

ITEM #n !STOP (CR)

or anywhere in the program hold the control key down and strike P. Then hit K to kill the job or C to continue from where you left off.

## REFERENCES

1. National Computer Network of Chicago, Inc., 1929 North Harlem Avenue, Chicago, Illinois 60635.
2. J. A. MacLachlan, TM-193, NAL.
3. A. A. Garren and J. W. Eusebio, UCID-10153, UCLRL.
4. K. L. Brow and S. K. Howry, SLAC-91.

## APPENDIX I--EXAMPLE OF PROGRAM USE

There follows below an example of the use of ILBOP for the 200 MeV transport system. One finds here the use of a 5. element not described in the body of this note. This element is a thin lens set in the middle of a drift space and is used here to represent the RF defocussing of the final few gaps of the LINAC. The parameters are

| | |
|---|---|
| LAB | a four character label |
| T | 5. |
| L | the length of the entire section |
| S | the 2,1 element of the thin lens transfer matrix |
| K | the 4,3 element of the thin lens transfer matrix |

```
H-1648 TIME-SHARING
PLEASE IDENTIFY YØUR PRØJECT
MACLACHLAN
ØN AT 16:34  03/08/71


!9:08...PLEASE LIST INFØ/1 FØR A VERY IMPØRTANT MESSAGE!

?LRUN ILBØP/146

ENTER DATA SØURCE (DSK ØR TTY)! DSK

THE NUMBER ØF ELEMENTS AND CENTRAL MØMENTUM
DEFINE FILE(S)
1=JM200

       31.    0.64400
ØPTIØNS FØR MATRIX, TRACE, ENVELØPE, AND APERTURES
        0.        0.        11.        0.
INITIAL BEAM PARAMETERS WX,RX,EX,WY,RY,EY,WP

    6.8900    0.0000   25.1310    5.7400    0.0000   25.1200    0.1000
BEAM CENTER CX,CY,CP
   0.00000   0.00000   0.00000
ELEMENTS: LAB,T,L,S,K,AH,AV,C,P

LIST ELEMENTS?! YES

TRANSPØRT ELEMENTS EL(I),I=1,9
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| DT16 | 3.00 | 0.0820 | 0.0634 | 0.0000 | 20.0000 | 20.0000 | 1.00000 | 0. |
| | 5.00 | 0.6800 | 0.0437 | 0.0437 | 100.0000 | 100.0000 | 1.00000 | 0. |
| DT17 | 3.00 | 0.1640 | -0.0418 | 0.0000 | 20.0000 | 20.0000 | 1.00000 | 0. |
| | 5.00 | 0.6920 | 0.0437 | 0.0437 | 100.0000 | 100.0000 | 1.00000 | 0. |
| DT18 | 1.00 | 0.1520 | 0.0000 | 0.0000 | 20.0000 | 20.0000 | 1.00000 | 0. |
| | 5.00 | 0.6920 | 0.0437 | 0.0437 | 100.0000 | 100.0000 | 1.00000 | 0. |
| DT19 | 3.00 | 0.1520 | -0.0644 | 0.0000 | 20.0000 | 20.0000 | 1.00000 | 0. |
| | 5.00 | 0.6920 | 0.0437 | 0.0437 | 100.0000 | 100.0000 | 1.00000 | 0. |
| DT20 | 3.00 | 0.1520 | 0.1125 | 0.0000 | 20.0000 | 20.0000 | 1.00000 | 0. |
| | 1.00 | 0.7490 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| LBEK | 1.00 | 0.8500 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| | 1.00 | 0.5730 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| Q1 | 3.00 | 0.3000 | -18.6000 | 0.0000 | 41.2700 | 41.2700 | 0.00182 | 0. |
| BPW1 | 1.00 | 0.3000 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 1. |
| | 1.00 | 0.3450 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| LBMS | 2.00 | 1.0920 | 868.0000 | 0.0000 | 100.0000 | 100.0000 | 0.00368 | 0. |
| | 1.00 | 1.3340 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| Q6 | 3.00 | 0.3000 | 32.9100 | 0.0000 | 41.2700 | 41.2700 | 0.00182 | 0. |
| | 1.00 | 3.0957 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| LBMH | 2.11 | 0.4950 | 362.0000 | 0.0000 | 100.0000 | 100.0000 | 0.01240 | 0. |
| | 1.00 | 0.4460 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| Q7 | 3.00 | 0.3000 | -34.5600 | 0.0000 | 41.2700 | 41.2700 | 0.00182 | 0. |
| | 1.00 | 0.3250 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| Q8 | 3.00 | 0.3000 | 31.2400 | 0.0000 | 41.2700 | 41.2700 | 0.00182 | 0. |
| BPM9 | 1.00 | 0.4500 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 1. |
| | 1.00 | 0.7710 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| | 4.00 | 0.0000 | 90.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| LBMV | 2.11 | 0.3795 | 229.0000 | 0.0000 | 100.0000 | 100.0000 | 0.05515 | 0. |
| | 4.00 | 0.0000 | -90.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| | 1.00 | 4.9500 | 0.0000 | 0.0000 | 100.0000 | 100.0000 | 1.00000 | 0. |
| Q9 | 3.00 | 0.3000 | -26.4400 | 0.0000 | 41.2700 | 41.2700 | 0.00182 | 1. |

```
CHANGES BY NAME
ITEM # 1! BEAM

PARAMETER! RX

    0.0000 NEW VALUE=! .2

PARAMETER! DØNE

ØVFL, (SQRT)
ITEM # 2! DØNE


RESULTS FØR EL= 14 LABELD BPW1

ENVELØPE IN WX,CX,RX,WXP/WY,CY,RY,WYP
   2.98982E 00   0.00000E-01   5.93587E-01   3.47063E 00
   9.84801E 00   0.00000E-01  -9.48181E-01   2.87479E 00

RESULTS FØR EL= 25 LABELD BPM9

ENVELØPE IN WX,CX,RX,WXP/WY,CY,RY,WYP
   2.79016E 01   0.00000E-01  -9.67340E-01   1.18105E 00
   8.16281E 00   0.00000E-01  -9.28557E-01   2.96880E 00

RESULTS FØR EL= 31 LABELD Q9

ENVELØPE IN WX,CX,RX,WXP/WY,CY,RY,WYP
   2.65629E 00   0.00000E-01  -3.33903E-01   3.33608E 00
   1.07493E 01   0.00000E-01  -9.83333E-01   4.64753E 00

CHANGES BY NAME
ITEM # 1! STØP

STØP,
?ØFF
ØFF AT 16:41  03/08/71
CØMPUTE SEC. - 10.7
CØNNECT MIN. - 07
```

ILBØP INTERACTIVE BEAM ØPTICS 3/7/71

```
      CØMMØN // EL(9,100),TL(5,5),T(5,5),BEAM(5,5),CENTER(5),
     +     X(5,10),XNELM,PO,DP,ØPTIØN(4)
      DIMENSIØN LABEL(2)
      EQUIVALENCE (LABEL(1),XLABEL)
      WRITE(9,1010)
      READ(9,1020) INPUT
      NPUT=1
      IF(INPUT.EQ.'TT') NPUT=9
      CALL READIN(NPUT)
      IF(NPUT.EQ.9) GØ TØ 110
  100 CALL MØDS
  110 DØ 115 I=1,5
      DØ 114 J=1,5
  114 T(I,J)=0.
  115 T(I,I)=1.
      NELM=XNELM
      DØ 200 NL=1,NELM
      XLABEL=EL(1,NL)
      IF(EL(9,NL).GT.0.)WRITE(9,1030) NL,LABEL
      NØPT4 =ØPTIØN(4)*EL(9,NL)
      IF(NØPT4.GT.0) CALL APERT(EL(1,NL),DP,T,ØPTIØN(4))
      CALL MATRIX(EL(1,NL),TL,PO)
      CALL MATMUL(TL,T,T)
      IF(NØPT4.GT.0) CALL APERT(EL(1,NL),DP,T,ØPTIØN(4))
      IF(ØPTIØN(2)*EL(9,NL).GT.0.) CALL TRACE(X,T,ØPTIØN(2))
      IF(ØPTIØN(3)*EL(9,NL).GT.0.)
     +     CALL ENVEL(BEAM,CENTER,T,ØPTIØN(3))
      IF(ØPTIØN(1)*EL(9,NL).GT.0.) CALL MATØUT(T,ØPTIØN(1))
  200 CØNTINUE
      GØ TØ 100
 1010 FØRMAT('ENTER DATA SØURCE (DSK ØR TTY)')
 1020 FØRMAT(A2)
 1030 FØRMAT(/,'RESULTS FØR EL=',I3,' LABELD',1X2A2/)
      END
      SUBRØUTINE READIN(NPUT)
      CØMMØN // EL(9,100),TL(5,5),T(5,5),BEAM(5,5),CENTER(5),
     +     X(5,10),XNELM,PO,DP,ØPTIØN(4)
      CØMMØN /UNITS/ UNIT(6)
      DIMENSIØN BELIPS(7),LABEL(2)
      EQUIVALENCE (XLABEL,LABEL(1))
      UNIT(1)=.001
      UNIT(2)=.001
      UNIT(3)=1.
      UNIT(4)=.01
      UNIT(5)=.0174573
      UNIT(6)=1.
      WRITE(9,1010)
      READ(NPUT,1020) XNELM,PO
      IF(NPUT.NE.9) WRITE(9,1020) XNELM,PO
      WRITE(9,1030)
      READ(NPUT,1040) ØPTIØN
      IF(NPUT.NE.9) WRITE(9,1040) ØPTIØN
      IF(ØPTIØN(3).EQ.0.) GØ TØ 50
      NEXT=MØD(IFIX(ØPTIØN(3)),10)
      GØ TØ (10,20,30,40),NEXT
   10 WRITE(9,1042)
```

```
      READ(NPUT,1045) BELIPS
        CALL SIGMA(BELIPS,BEAM)
        IF(NPUT.NE.9) WRITE(9,1045) BELIPS
        WRITE(9,1047)
        READ(NPUT,1060) CENTER(1),CENTER(3),CENTER(5)
        IF(NPUT.NE.9) WRITE(9,1060) CENTER(1),CENTER(3),CENTER(5)
        CENTER(2)=0.
        CENTER(4)=0.
        GØ TØ 50
 20     WRITE(9,1050)
        DØ 23 I=1,5
        DØ 23 J=1,5
 23     BEAM(I,J)=0.
        READ(NPUT,1060) (BEAM(I,I), I=1,5)
        IF(NPUT.NE.9) WRITE(9,1060) (BEAM(I,I), I=1,5)
        WRITE(9,1055)
        READ(NPUT,1060) CENTER
        DØ 25 I=1,5
   25   BEAM(I,I)=BEAM(I,I)**2
        GØ TØ 50
 30     WRITE(9,1070)
        DØ 35 I=1,5
        READ(NPUT,1060) (BEAM(I,J), J=1,I)
        IF(NPUT.NE.9) WRITE(9,1060) (BEAM(I,J), J=1,I)
        IM1=I-1
        IF(IM1.EQ.0) GØ TØ 35
        DØ 34 J=1,IM1
   34   BEAM(J,I)=BEAM(I,J)
   35   CØNTINUE
        WRITE(9,1055)
        READ(NPUT,1060) CENTER
        IF(NPUT.NE.9) WRITE(9,1060) CENTER
        GØ TØ 50
 40     WRITE(9,1080)
        READ(NPUT,1060) BEAM(1,1)
        IF(NPUT.NE.9) WRITE(9,1060) BEAM(1,1)
        DØ 42 I=2,5
        IM1=I-1
         READ(NPUT,1060) BEAM(I,I),(BEAM(I,J), J=1,IM1)
   42   IF(NPUT.NE.9) WRITE(9,1060) BEAM(I,I),(BEAM(I,J), J=1,IM1)
        DØ 44 I=2,5
        IM1=I-1
        DØ 44 J=1,IM1
        BEAM(I,J)=BEAM(I,J)*BEAM(I,I)*BEAM(J,J)
   44   BEAM(J,I)=BEAM(I,J)
        DØ 46 I=1,5
   46   BEAM(I,I)=BEAM(I,I)**2
        WRITE(9,1055)
        READ(NPUT,1060) CENTER
        IF(NPUT.NE.9) WRITE(9,1060) CENTER
 50     IF(ØPTIØN(2).LE.0) GØ TØ 60
        NØPT2=ØPTIØN(2)
        WRITE(9,1090) NØPT2
        DØ 55 J=1,NØPT2
        READ(NPUT,1060) (X(I,J), I=1,5)
 55     IF(NPUT.NE.9) WRITE(9,1060) (X(I,J), I=1,5)
```

```
 60    IF(ØPTIØN(4).LE.0.) GØ TØ 70
       WRITE(9,1100)
       READ(NPUT,1110) DP
       IF(NPUT.NE.9) WRITE(9,1110) DP
 70    NELM=XNELM
       WRITE(9,1115)
       DØ 75 NL=1,NELM
       READ(NPUT,1120) LABEL,(EL(I,NL), I=2,9)
       EL(1,NL)=XLABEL
    75 CØNTINUE
       IF(NPUT.EQ.9) GØ TØ 110
       WRITE(9,1125)
       READ(9,1130) LIST
       IF(LIST.EQ.'NØ') GØ TØ 110
       WRITE(9,1140)
       DØ 80 NL=1,NELM
       XLABEL=EL(1,NL)
    80 WRITE(9,1120) LABEL,(EL(I,NL), I=2,9)
   110 RETURN
  1010 FØRMAT('THE NUMBER ØF ELEMENTS AND CENTRAL MØMENTUM'/)
  1020 FØRMAT(F10.0,F10.5)
  1030 FØRMAT('ØPTIØNS FØR MATRIX, TRACE, ENVELØPE, AND APERTURES')
  1040 FØRMAT(4F10.0)
  1042 FØRMAT('INITIAL BEAM PARAMETERS WX,RX,EX,WY,RY,EY,WP',/)
  1045 FØRMAT(7F10.4)
  1047 FØRMAT('BEAM CENTER CX,CY,CP')
  1050 FØRMAT('INITIAL BEAM HALF WIDTHS AND CENTER,'/)
  1055 FØRMAT('BEAM CENTER CX,CX'',CY,CY'',CP',/)
  1060 FØRMAT(5F10.5)
  1070 FØRMAT('INITIAL BEAM MATRIX IN INTERNAL FØRM,'/)
  1080 FØRMAT('INITIAL BEAM MATRIX IN TRANSPØRT FØRM',/)
  1090 FØRMAT('INITIAL X,X'',Y,Y'',DP/P FØR',I2,' RAYS',/)
  1100 FØRMAT('DELTAP/P FØR APERTURE PRØJECTIØN')
  1110 FØRMAT(F10.7)
  1115 FØRMAT('ELEMENTS: LAB,T,L,S,K,AH,AV,C,P',/)
  1120 FØRMAT(2A2,F6.2,5F9.4,F9.5,F6.0)
  1125 FØRMAT('LIST ELEMENTS?')
  1140 FØRMAT('TRANSPØRT ELEMENTS EL(I),I=1,9',/)
  1130 FØRMAT(A2)
       END
SUBRØUTINE SIGMA(BELIPS,BEAM)
CØMMØN /UNITS/ UNIT(6)
DIMENSIØN BEAM(5,5),BELIPS(7)
DATA PI,HALFPI /3.141593,1.570796/
DØ 50 I=1,5
DØ 50 J=1,5
50 BEAM(I,J)=0.
. TRACE 200
BEAM(1,1)=BELIPS(1)**2
BEAM(3,3)=BELIPS(4)**2
BEAM(5,5)=BELIPS(7)**2
       EPSXSQ=(BELIPS(3)/PI)**2
       BEAM(2,2)=EPSXSQ/((1.-BELIPS(2)**2)*BEAM(1,1))
       BEAM(1,2)=SIGN(SQRT(BEAM(1,1)*BEAM(2,2)-EPSXSQ),BELIPS(2))
       BEAM(2,1)=BEAM(1,2)
       EPSYSQ=(BELIPS(6)/PI)**2
```

```
      BEAM(4,4)=EPSYSQ/((1.-BELIPS(5)**2)*BEAM(3,3))
      BEAM(3,4)=SIGN(SQRT(BEAM(3,3)*BEAM(4,4)-EPSYSQ),BELIPS(5))
      BEAM(4,3)=BEAM(3,4)
200 RETURN
END
      SUBRØUTINE MØDS
      CØMMØN // EL(9,100),TL(5,5),T(5,5),BEAM(5,5),CENTER(5),
+         X(5,10),XNELM,PO,DP,ØPTIØN(4)
      DIMENSIØN XLABL(14),LABL(28),LABEL(2),DAT(1037),BELIPS(7)
      INTEGER PARM(7),PMTR(8),PARAM
      EQUIVALENCE (EL(1,1),DAT(1)),(XLABEL,LABEL(1)),
     +     (LABL(1),XLABL(1))
      DATA LABL /'EL','MN','BE','AM','CN','TR','XV','EC','NE','LM'
     +     'PZ','ER','DP','/P','ØP','TN','NØ','  ','NØ','NE',
     +     'DØ','NE','QU','IT','ST','ØP','DA','TA'/
      DATA PMTR /'T','L','S','K','AH','AV','C','P'/
      DATA PARM /'WX','RX','EX','WY','RY','EY','WP'/
      NELM=XNELM
      ITEM=0
      DØ 25 NL=1,NELM
      IF(EL(2,NL).GT.0.) GØ TØ 25
      XLABEL=EL(1,NL)
      WRITE(9,1010) NL,LABEL,(EL(I,NL), I=2,9)
      READ(9,1020) LABEL,(EL(I,NL), I=2,9)
      EL(1,NL)=XLABEL
25    CØNTINUE
30    WRITE(9,1030)
   35 ITEM=ITEM+1
      WRITE(9,1035) ITEM
      READ(9,1040) LABEL
      DØ 50 NL=1,NELM
      IF(XLABEL.EQ.EL(1,NL)) GØ TØ 100
50    CØNTINUE
      DØ 60 IQ=1,14
      IF(XLABEL.EQ.XLABL(IQ)) GØ TØ 200
60    CØNTINUE
      WRITE(9,1050) LABEL
      GØ TØ 35
  100 WRITE(9,1055)
      READ(9,1045) PARAM
     DØ 110 I=1,8
     IF(PARAM.EQ.PMTR(I)) GØ TØ 115
110  CØNTINUE
     WRITE(9,1060) PARAM
     GØ TØ 100
115  I=I+1
     WRITE(9,1080) EL(I,NL)
     READ(9,1070) EL(I,NL)
     GØ TØ 35
  200 GØ TØ (250,350,400,450,500,550,600,650,700,
+          700,700,750,750,800), IQ
  250 WRITE(9,1072)
      READ(9,1070) XI,XJ
      I=XI
      IF(I.EQ.0) GØ TØ 30
      I=I+1
```

```
      J=XJ
      WRITE(9,1080) EL(I,J)
      READ(9,1070) EL(I,J)
      GØ TØ 250
  350 NØPT3=MØD(IFIX(ØPTIØN(3)),10)
      GØ TØ (351,360,370,380), NØPT3
  351 CALL ELIPSE(BEAM,BELIPS)
  352 WRITE(9,1055)
      READ(9,1045) PARAM
      DØ 353 I=1,7
      IF(PARAM.EQ.'DØ') GØ TØ 355
      IF(PARAM.EQ.PARM(I)) GØ TØ 354
  353 CØNTINUE
      WRITE(9,1060) PARAM
      GØ TØ 352
  354 WRITE(9,1080) BELIPS(I)
      READ(9,1070) BELIPS(I)
      GØ TØ 352
  355 CALL SIGMA(BELIPS,BEAM)
      GØ TØ35
  360 WRITE(9,1074)
      READ(9,1070) XI
      I=XI
      IF(I.EQ.0) GØ TØ 35
      W=SQRT(BEAM(I,I))
      WRITE(9,1080) W
      READ(9,1070) W
      BEAM(I,I)=W**2
      GØ TØ 360
  370 WRITE(9,1072)
      READ(9,1070) XI,XJ
      I=XI
      IF(I.EQ.0) GØ TØ 35
      J=XJ
      WRITE(9,1080) BEAM(I,J)
      READ(9,1070) BEAM(I,J)
      BEAM(J,I)=BEAM(I,J)
      GØ TØ 370
  380 WRITE(9,1072)
      READ(9,1070) XI,XJ
      I=XI
      IF(I.EQ.0) GØ TØ 35
       J=XJ-1
       IF(J.EQ.0) GØ TØ 385
      SQRRT=SQRT(BEAM(I,I)*BEAM(J,J))
      CØRIJ=BEAM(I,J)/SQRRT
      WRITE(9,1080) CØRIJ
      READ(9,1070) CØRIJ
      BEAM(I,J)=CØRIJ*SQRRT
      BEAM(J,I)=BEAM(I,J)
      GØ TØ 380
  385 W=SQRT(BEAM(I,I))
      WRITE(9,1080) W
      READ(9,1070) W
      BEAM(I,I)=W**2
      GØ TØ 380
```

```
  400 WRITE(9,1074)
      READ(9,1070) XI
      I=XI
      IF(I.EQ.0) GØ TØ 35
      WRITE(9,1080) CENTER(I)
      READ(9,1070) CENTER(I)
      GØ TØ 400
  450 WRITE(9,1072)
      READ(9,1070) XI,XJ
      I=XI
      IF(I.EQ.0) GØ TØ 35
      J=XJ
      WRITE(9,1080) X(I,J)
      READ(9,1070) X(I,J)
      GØ TØ 450
  500 WRITE(9,1080) XNELM
      READ(9,1070) XNELM
      GØ TØ 35
  550 WRITE(9,1080) PO
      READ(9,1070) PO
      GØ TØ 35
  600 WRITE(9,1080) DP
      READ(9,1070) DP
      GØ TØ 35
  650 WRITE(9,1085) ØPTIØN
      READ(9,1070) ØPTIØN
      GØ TØ 35
 700  RETURN
 750  STØP
  800 WRITE(9,1074)
      READ(9,1070) XI
      I=XI
      IF(I.EQ.0) GØ TØ 30
      WRITE(9,1080) DAT(I)
      READ(9,1070) DAT(I)
      GØ TØ 800
 1010 FØRMAT('CHANGE EL=',I3/2X2A2,F6.2,5F9.4,F9.5,F6.0/)
 1020 FØRMAT(2A2,F6.2,5F9.4,F9.5,F6.0)
 1030 FØRMAT(/,'CHANGES BY NAME')
 1035 FØRMAT('ITEM #',I2)
 1040 FØRMAT(2A2)
 1045 FØRMAT(A2)
 1050 FØRMAT('LABEL= ',2A2,' NØT RECØGNIZED. TRY AGAIN.')
 1055 FØRMAT('PARAMETER')
 1060 FØRMAT('PARAM=',A2,' NØT RECØGNIZED. TRY AGAIN.')
 1070 FØRMAT(7F10.4)
 1072 FØRMAT('INDICES')
 1074 FØRMAT('INDEX')
 1080 FØRMAT(F10.4,' NEW VALUE=')
 1085 FØRMAT(4F5.0,' NEW VALUE=')
      END
SUBRØUTINE ELIPSE(BEAM,BELIPS)
CØMMØN /UNITS/ UNIT(6)
DIMENSIØN BEAM(5,5),BELIPS(7)
DATA PI /3.14159/
BELIPS(1)=SQRT(BEAM(1,1))
```

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```
 3460 BELIPS(4)=SQRT(BEAM(3,3))
 3470 BELIPS(7)=SQRT(BEAM(5,5))
 3480 B1B2=BEAM(1,1)*BEAM(2,2)
 3490 BELIPS(2)=BEAM(1,2)/SQRT(B1B2)
 3500 BELIPS(3)=PI*SQRT(B1B2-BEAM(1,2)**2)
 3510 B3B4=BEAM(3,3)*BEAM(4,4)
 3520 BELIPS(5)=BEAM(3,4)/SQRT(B3B4)
 3530 BELIPS(6)=PI*SQRT(B3B4-BEAM(3,4)**2)
 3540 RETURN
 3550 END
 3560        SUBRØUTINE APERT(EL,DP,T,ØPTIØN)
 3570        DIMENSIØN EL(9),T(5,5),R(5,4),TINV(5,5),A(4),B(4)
 3580        CØMMØN /UNITS/ UNIT(6)
 3590        DATA ITIMES /-1/
 3600        ITIMES=-1*ITIMES
 3610        IF(ITIMES.LT.0) GØ TØ 5
 3620        WRITE(9,1010)
 3630        WRITE(9,1014)
 3640        GØ TØ 6
 3650      5 WRITE(9,1018)
 3660      6 DXP=.001/UNIT(2)
 3670        R(1,1)=EL(6)
 3680        R(2,1)=0.
 3690        R(3,1)=EL(7)
 3700        R(4,1)=0.
 3710        R(5,1)=DP
 3720        R(1,2)=EL(6)
 3730        R(2,2)=DXP
 3740        R(3,2)=EL(7)
 3750        R(4,2)=DXP
 3760        R(5,2)=DP
 3770        R(1,3)=-EL(6)
 3780        R(2,3)=0.
 3790        R(3,3)=-EL(7)
 3800        R(4,3)=0.
 3810        R(5,3)=DP
 3820        R(1,4)=-EL(6)
 3830        R(2,4)=DXP
 3840        R(3,4)=-EL(7)
 3850        R(4,4)=DXP
 3860        R(5,4)=DP
 3870        DØ 10 I=1,5
 3880        DØ 10 J=1,5
 3890   10   TINV(I,J)=T(I,J)
 3900        CALL MATINV(TINV,R,4)
 3910        IF(ØPTIØN.GT.1.) GØ TØ 25
 3920        WRITE(9,1020) ((R(I,J),R(I+1,J), J=1,4), I=1,3,2)
 3930        RETURN
 3940   25   INDEX=0
 3950        DØ 50 I=1,3,2
 3960        DØ 50 J=1,3,2
 3970        INDEX=INDEX+1
 3980        DYDX=(R(I+1,J+1)-R(I+1,J))/(R(I,J+1)-R(I,J))
 3990        A(INDEX)=R(I,J+1)-R(I+1,J+1)/DYDX
 4000     50 B(INDEX)=R(I+1,J+1)-R(I,J+1)*DYDX
 4010        WRITE(9,1030) (A(I),B(I), I=1,4)
```

```
      RETURN
 1010 FØRMAT('APERTURE PRØJECTIØN')
 1014 FØRMAT('ENTERANCE')
 1018 FØRMAT('EXIT')
 1020 FØRMAT('H',1P4E15.6/1X4E15.6/'V',4E15.6/1X4E15.6)
 1030 FØRMAT('H INT',1P4E15.6/'V INT',4E15.6)
      END
      SUBRØUTINE MATINV(A,R,N)
      DIMENSIØN A(5,5),R(5,N),STØRE(5)
      SAVE=A(1,5)
      A(1,5)=A(1,2)*A(2,5)-SAVE*A(2,2)
      A(2,5)=SAVE*A(2,1)-A(2,5)*A(1,1)
      SAVE=A(3,5)
      A(3,5)=A(3,4)*A(4,5)-SAVE*A(4,4)
      A(4,5)=SAVE*A(4,3)-A(4,5)*A(3,3)
      SAVE=A(1,1)
      A(1,1)=A(2,2)
      A(2,2)=SAVE
      A(1,2)=-A(1,2)
      A(2,1)=-A(2,1)
      SAVE=A(3,3)
      A(3,3)=A(4,4)
      A(4,4)=SAVE
      A(3,4)=-A(3,4)
      A(4,3)=-A(4,3)
      IF(N.EQ.0)RETURN
      DØ 20 I=1,N
      DØ 15 J=1,5
      STØRE(J)=0.
      DØ 15 K=1,5
15    STØRE(J)=STØRE(J)+A(J,K)*R(K,I)
      DØ 20 J=1,5
20    R(J,I)=STØRE(J)
      RETURN
      END
      SUBRØUTINE MATRIX(EL,T,P)
      CØMMØN /UNITS/ UNIT(6)
      DIMENSIØN EL(9),T(5,5)
      KIND=IABS(IFIX(EL(2)))
      CP=.02997925/(P*UNIT(6))
      DØ 50 I=1,5
      DØ 49 J=1,5
49    T(I,J)=0.
50    T(I,I)=1.
      GØ TØ (100,200,300,400,500),KIND
100   ZL=EL(3)*UNIT(3)
      T(1,2)=ZL
      T(3,4)=ZL
      GØ TØ 600
200   ZL=EL(3)*UNIT(3)
      H=CP*EL(4)*EL(8)
      TEMP=H+EL(5)/UNIT(1)
      XK=SQRT(H*ABS(TEMP))
      XX=ZL*XK
      IF(TEMP) 222,224,226
222   EX=.5*EXP(XX)
```

```
      REX=.25/EX
      C=EX+REX
      S=EX-REX
      SIGN=1.
      GØ TØ 228
  224 T(1,2)=ZL
      T(2,5)=T(1,2)*H
      T(1,5)=T(2,5)*.5*H
      GØ TØ 230
  226 C=CØS(XX)
      S=SIN(XX)
      SIGN=-1.
  228 T(1,1)=C
      T(1,2)=S/XK
      T(1,5)=(1.-C)/TEMP
      T(2,1)=SIGN*XK*S
      T(2,2)=C
      T(2,5)=H*T(1,2)
  230 TEMP=-EL(5)/UNIT(1)
      XK=SQRT(ABS(H*TEMP))
      XX=ZL*XK
      IF(TEMP) 232,234,236
  232 EX=.5*EXP(XX)
      REX=.25/EX
      C=EX+REX
      S=EX-REX
      SIGN=1.
      GØ TØ 237
  234 T(3,4)=ZL
      GØ TØ 238
  236 C=CØS(XX)
      S=SIN(XX)
      SIGN=-1.
  237 T(3,3)=C
      T(3,4)=S/XK
      T(4,4)=C
      T(4,3)=SIGN*S*XK
  238 TEMP=ABS(EL(2))-FLØAT(KIND)+.001
      IF(TEMP.LT..1) GØ TØ 250
      ITEMP=TEMP*10.
      ZL=0.
      DØ 240 I=1,ITEMP
  240 ZL=ZL+EL(9*I-6)
      ARG=.5*H*ZL
      EDGE=H*SIN(ARG)/CØS(ARG)
      T(1,1)=T(1,1)+EDGE*T(1,2)
      T(2,1)=T(2,1)+EDGE*T(2,2)
      T(3,3)=T(3,3)-EDGE*T(3,4)
      T(4,3)=T(4,3)-EDGE*T(4,4)
      TEMP=TEMP-.1
  250 IF(TEMP.LT..009) GØ TØ 600
      ITEMP=100.*TEMP
      ZL=0.
      DØ 260 I=1,ITEMP
      NEG=12-9*I
  260 ZL=ZL+EL(NEG)
```

```
      ARG=.5*H*ZL
      EDGE=H*SIN(ARG)/CØS(ARG)
      T(2,1)=T(2,1)+EDGE*T(1,1)
      T(2,2)=T(2,2)+EDGE*T(1,2)
      T(2,5)=T(2,5)+EDGE*T(1,5)
      T(4,3)=T(4,3)-EDGE*T(3,3)
      T(4,4)=T(4,4)-EDGE*T(3,4)
      GØ TØ 600
 300  TEMP=CP*EL(4)*EL(8)/UNIT(1)
      XK=SQRT(ABS(TEMP))
      XX=EL(3)*UNIT(3)*XK
      EX=.5*EXP(XX)
      REX=.25/EX
      IF(TEMP.GT.0.) GØ TØ 345
      SX=EX-REX
      CX=EX+REX
      CY=CØS(XX)
      SY=SIN(XX)
      SIGN=1.
      GØ TØ 350
 345  CX=CØS(XX)
      SX=SIN(XX)
      CY=EX+REX
      SY=EX-REX
      SIGN=-1.
 350  T(1,1)=CX
      T(1,2)=SX/XK
      T(2,1)=SIGN*XK*SX
      T(2,2)=CX
      T(3,3)=CY
      T(3,4)=SY/XK
      T(4,3)=-SIGN*XK*SY
      T(4,4)=CY
      GØ TØ 600
  400 XX=EL(4)*UNIT(5)
      C=CØS(XX)
      S=SIN(XX)
      DØ 410 I=1,4
 410  T(I,I)=C
      T(1,3)=S
      T(2,4)=S
      T(3,1)=-S
      T(4,2)=-S
      RETURN
* THIS ELEMENT NØT DESCRIBED IN TM-282 IS A THIN LENS IN THE MIDDL
* ØF ADRIFT TØ PUT IN RF DEFØCUSSING
* THE PARAMETERS ARE T,L,R(2,1),R(4,3) ASSUMED IN CØRRECT UNITS
 500  ZL=EL(3)/2.
      ZH=ZL*EL(4)
       ZV=ZL*EL(5)
     T(1,1)=1.+ZH
      T(1,2)=ZL*(2.+ZH*ZL)
      T(2,1)=ZH
      T(2,2)=T(1,1)
      T(3,3)=1.+ZV
      T(3,4)=ZV*(2.+ZV*ZL)
```

```
      T(4,3)=ZV
      T(4,4)=T(3,3)
      RETURN
600   UN21=UNIT(2)/UNIT(1)
      UN41=UNIT(4)/UNIT(1)
      UN42=UNIT(4)/UNIT(2)
      T(1,2)=T(1,2)*UN21
      T(1,5)=T(1,5)*UN41
      T(2,1)=T(2,1)/UN21
      T(2,5)=T(2,5)*UN42
      T(3,4)=T(3,4)*UN21
      T(4,3)=T(4,3)/UN21
      RETURN
      END
      SUBRØUTINE MATMUL(A,B,C)
      DIMENSIØN A(5,5),B(5,5),C(5,5),D(5,5)
      DØ 100 I=1,5
      DØ 100 J=1,5
      D(I,J)=0.
      DØ 100 K=1,5
100   D(I,J)=D(I,J)+A(I,K)*B(K,J)
      DØ 150 I=1,5
      DØ 150 J=1,5
150   C(I,J)=D(I,J)
      RETURN
      END
      SUBRØUTINE TRACE(X,T,ØPTIØN)
      DIMENSIØN T(5,5),X(5,1),STØRE(5)
      NRAY=ØPTIØN
      WRITE(9,1010)
      DØ 100 NR=1,NRAY
      DØ 10 I=1,5
      STØRE(I)=0.
      DØ 10 J=1,5
10    STØRE(I)=STØRE(I)+T(I,J)*X(J,NR)
100   WRITE(9,1020) NR,(STØRE(I),I=1,5)
      RETURN
1010  FØRMAT('RAY TRACE (X,X'',Y,Y'',DP/P)')
1020  FØRMAT(I5,5F10.3)
      END
      SUBRØUTINE ENVEL(BEAM,CENTER,T,ØPTIØN)
      DIMENSIØN BEAM(5,5),CENTER(5),T(5,5),B(5,5),A(5,5),C(5),
     +     BELIPS(7)
      NØPT=ØPTIØN/10.
      DØ 100 I=1,5
      DØ 100 J=1,5
      A(I,J)=0.
      DØ 100 K=1,5
100   A(I,J)=A(I,J)+T(I,K)*BEAM(K,J)
      DØ 150 I=1,5
      DØ 150 J=1,5
      B(I,J)=0.
      DØ 150 K=1,5
150   B(I,J)=A(I,K)*T(J,K)+B(I,J)
      DØ 175 I=1,5
      C(I)=0.
```

```
      DØ 175 J=1,5
  175 C(I)=C(I)+T(I,J)*CENTER(J)
      GØ TØ (200,300,400),NØPT
  200 CALL ELIPSE(B,BELIPS)
      WRITE(9,1010)
      WXP=SQRT(B(2,2))
      WYP=SQRT(B(4,4))
      WRITE(9,1015) BELIPS(1),C(1),BELIPS(2),WXP,
     +          BELIPS(4),C(3),BELIPS(5),WYP
      RETURN
  300 DØ 325 I=1,5
  325 A(I,I)=SQRT(B(I,I))
      DØ 327 I=2,5
      IM1=I-1
      DØ 327 J=1,IM1
  327 A(I,J)=B(I,J)/(A(I,I)*A(J,J))
      WRITE(9,1030)
      DØ 330 I=1,5
  330 WRITE(9,1020) (A(I,J), J=1,I)
      WRITE(9,1020) C
      RETURN
  400 WRITE(9,1040)
      DØ 410 I=1,5
  410 WRITE(9,1020) (B(I,J), J=1,I)
      WRITE(9,1020) C
      RETURN
 1010 FØRMAT('ENVELØPE IN WX,CX,RX,WXP/WY,CY,RY,WYP')
 1015 FØRMAT(1P4E14.5)
 1020 FØRMAT(1P5E14.5)
 1030 FØRMAT('ENVELØPE IN TRANSPØRT FØRM')
 1040 FØRMAT('ENVELØPE IN SIGMA FØRM')
      END
      SUBRØUTINE MATØUT(T,ØPTIØN)
      DIMENSIØN T(5,5)
      NØPT=ØPTIØN
      GØ TØ (100,200),NØPT
  100 WRITE(9,1010)
      WRITE(9,1020)((T(I,J),J=1,5),I=1,5)
      RETURN
  200 WRITE(9,1030)
      CØSMUX=(T(1,1)+T(2,2))/2.
      MUX=ATAN2(SQRT(1.-CØSMUX**2),CØSMUX)
      SINMUX=SIN(MUX)
      ALPHAX=(T(1,1)-CØSMUX)/SINMUX
      BETAX=T(1,2)/SINMUX
      GAMMAX=-T(2,1)/SINMUX
      XEQ=(T(1,5)+T(1,2)*T(2,5)-T(2,2)*T(1,5))/(1.-CØSMUX)
      XPEQ=(T(2,5)+T(2,1)*T(1,5)-T(1,1)*T(2,5))/(1.-CØSMUX)
      CØSMUY=(T(3,3)+T(4,4))/2.
      MUY=ATAN2(SQRT(1.-CØSMUY**2),CØSMUY)
      SINMUY=SIN(MUY)
      ALPHAY=(T(3,3)-CØSMUY)/SINMUY
      GAMMAY=-T(4,3)/SINMUY
      YEQ=(T(3,5)+  T(3,4)*T(4,5)-T(4,4)*T(3,5))/(1.-CØSMUY)
      YPEQ=(T(4,5)+ T(4,1)*T(3,5)-T(3,3)*T(4,5))/(1.-CØSMUY)
      WRITE(9,1040)BETAX,ALPHAX,GAMMAX,MUX,XEQ,XPEQ
```

```
      WRITE(9,1040)BETAY,ALPHAY,GAMMAY,MUY,YEQ,YPEQ
      RETURN
 1010 FØRMAT('TRANSFER MATRIX')
 1020 FØRMAT(1P5E14.5)
 1030 FØRMAT('BETATRØN FUNCTIØNS ETC.')
 1040 FØRMAT(1P6E11.3)
      END
```