INELASTIC MUON + PROTON EXPERIMENTS AT NAL

M. L. Perl
Stanford Linear Accelerator Center

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SUMMARY

For two types of muon + proton inelastic-scattering experiments we find:

1. A muon flux of 5×10^5 per pulse is required, which is of the order of 1/100 of the maximum flux available.

2. One experiment and part of another can be done with an analyzing magnet of 1 m^2 aperture, 1 to 1.5 m deep, and 15-kG field.

3. Complete identification of all types of muon + proton inelastic events in general requires a very large detector system of the magnitude of that discussed in NAL Summer Study Report A. 3-68-12.

Introduction

In this note we outline two muon-proton inelastic scattering experiments for NAL with two objects in mind. First, we wish to see what muon flux is needed. Second, we wish to see what size analyzing magnets would be needed.

Some examples of inelastic muon + proton reactions are

$$\mu + p \rightarrow \mu + n + \pi^+ \tag{1}$$

$$\mu + p \rightarrow \mu + N^* \tag{2}$$

$$\mu + p \rightarrow \mu + p + \rho^o \tag{3}$$

$$\mu + p \rightarrow \mu + K^+ + \Lambda^o \tag{4}$$

$$\mu + p \rightarrow \mu + n + \rho^{\pm}. \tag{5}$$

The ultimate goal of studying muon + proton inelastic scattering is to examine these kinds of reactions in detail. But there is a basic simplicity in muon + proton inelastic scattering which occurs when all the reactions are summed over all the outgoing particles except the muon. Namely, this summarization can be expressed in terms of just two inelastic form factors σ_T and σ_L. This result is stated in the following equation with some approximations (see Appendix A) for NAL energies and using only one-photon exchange.

$$\frac{d^2\sigma}{d\Omega_2 dp_2} = \frac{\alpha}{2\pi q^2}\left(\frac{E_2 q_o}{E_1}\right)\left(1 - \frac{2m_\mu^2}{q^2} + \frac{2E_1E_2}{q_o^2}\right) \times$$

$$\times \left[\sigma_T(q^2, q_o) + \epsilon\, \sigma_L(q^2, q_o)\right].$$

Thus the differential cross section $d^2\sigma/d\Omega_2 dp_2$ of the scattered muon may be expressed as a function of the two inelastic form factors σ_T and σ_L, which are functions of q^2 and q_o. As $q^2 \rightarrow 0$, $\sigma_T \rightarrow \sigma$ real photons and $\sigma_L \rightarrow 0$. Here

E_1 is the incident muon energy in the lab system,

E_2 is the final muon energy in the lab system,

$q_o = E_1 - E_2$ is the energy of the virtual photon in the lab system,

$d\Omega_2$ is the differential solid angle of the final muon $= d\phi_2 d\cos\theta_2$,

q^2 is the absolute value of the square of the four-momentum transfer from the muons,

m_μ is the muon mass,

ϵ varies from 1 to almost 0.

We note that for muons in the lab system:

$$q^2 = 2(E_1E_2 - p_1p_2 - m_\mu^2) + 2p_1p_2(1 - \cos\theta_2),$$

where θ_2 is the muon-scattering angle. Even for $\theta_2 = 0$, $q^2 > 0$ because there is a q^2_{min} where

$$q^2_{min} = 2(E_1E_2 - p_1p_2 - m_\mu^2) \approx m_\mu^2 \left[\frac{(E_1 - E_2)^2}{E_1E_2}\right].$$

For example q^2_{min} may be appreciable if $E_1 = 100$ GeV and $E_2 = 10$ GeV then $q^2_{min} \approx 0.08\ (\mathrm{GeV}/c)^2$.

The simplest experiment is to measure σ_T and σ_L as functions of q^2 and q_o. We then are measuring the two total cross sections for a virtual photon. The most meaningful way to look at this at NAL energies is to think of fixing q^2 and then looking at the variation of σ_T and σ_L over a q_o range from 10 to 80 or 100 GeV/c. Conversely at a fixed q_o we may be interested in how σ_T and σ_L behave as $q^2 \to q^2_{min}$. Further discussion on the significance of these virtual total cross sections is given by Bjorken[4] and Hand.[5] This first experiment I call the σ_T, σ_L experiment.

A more complicated but also richer experiment is to detect not only the outgoing muon, but also some or all of the particles from the nuclear vertex. For example one would detect the pion in forward single-pion production in reaction (1); the pions from the ρ^o and the recoil proton in reaction (3); and so forth. This second experiment I shall refer to as the multibody-detection experiment.

Next I shall discuss the two experiments separately.

The σ_T, σ_L Experiment

Figure 2 shows the experiment. It is necessary to measure the momentum and angle of the muons before the target, using spark chambers 1, 2, 3, and 4. After the target come chambers 5 and 6 (to determine the scattering angle θ_2), then the analyzing magnet A, and then chambers 7 and 8 to obtain the muon momentum. Chambers 9, 10, and 11 are separated from 8 and each other by iron and are used to show the detected particle is a muon. The purpose of this note is simply to look at the required muon flux and the required size of A. Therefore, we will not go into many experimental design details. We use a liquid-hydrogen target 40 in. long. We would trigger, as in the present SLAC muon experiment, on all muons which scatter above a certain minimum angle $\theta_{2\,min}$. This requires a hodoscope system similar to the one used in the SLAC experiment, but will not be discussed further here. Preliminary considerations show that at NAL, for 50 to 150 GeV initial muon energies, $\theta_{2\,min} = 4$ mrads is attainable, and we take the hodoscope system (not shown in Fig. 2) to be capable of doing this.

Next we consider the event rate at present for $15 > q_o > 2$ GeV. We have only crude knowledge of the sum $(\sigma_T + \epsilon\, \sigma_L)$, where ϵ is close to 1, from electron experiments at CEA, DESY, and SLAC and muon experiments at SLAC. Our object is to determine σ_T and σ_L separately and at much higher q_o. For total event rate estimates we use the approximation, based partly on experiments, and partly on vector dominance ideas

$$(\sigma_T + \epsilon\,\sigma_L) \approx 200\,\mu b\,(1 + q^2/m^2\rho)^{-2},$$

where m_ρ is the ρ mass and $\epsilon > 0.2$. Thus according to the value of ϵ, we will under- or overestimate by a factor of 2 or 3, but we are only guessing at $\sigma_T\,(q^2, q_o)$ and $\sigma_L\,(q^2, q_o)$ at the q_o for NAL, and the simplicity is worthwhile. Also to see what precision is required to separate σ_T and σ_L we shall take σ_T and σ_L to be the same order of magnitude for different q^2 intervals; Fig. 3 gives the events per pulse per GeV/c of p_2 for 10^6 muons per pulse and a 40-in. long hydrogen target.

As shown in Fig. 2, it is necessary to have spark chambers in the beam. Therefore, we feel to avoid confusion from stray tracks in event measurement, a flux of 5×10^5 muons per pulse is the maximum one can use. The event rate per pulse and per hour is given below and is satisfactory. While special devices may allow some higher flux, the difficulties of turn-on argue against such devices in the first muon experiments.

The event rate on the following page is just possible with optical chambers taking several pictures per pulse. Wire chambers would allow a higher rate. But all experience with muon experiments show

Event Rate for σ_T, σ_L Experiment.

100 GeV incident muons, 90 to 10 GeV, Final Muons

Final Muons	Event/Pulse	Event/Hour
$\theta_2 > 4$ mrads $q^2 < 0.2\,(\mathrm{GeV})^2$	0.55	330
$0.2 < q^2 < 0.5\,(\mathrm{GeV})^2$	0.70	420
$0.5 < q^2 < 1.0\,(\mathrm{GeV})^2$	0.25	150
$1.0 < q^2 < 2.0\,(\mathrm{GeV})^2$	0.12	70
$2.0 < q^2$	0.06	40
Total	1.40	1010

it is hard to get a near 100% useful trigger. In fact, usually only 1/3 to 1/10 of the triggers are the desired events. There is no reason for a substantial improvement at NAL. Therefore, a trigger rate of about 5 per pulse for 5×10^5 muons per pulse should be expected. This is a little high for optical chambers, but it is easily handled by wire chambers.

Yamanouchi (B. 2-68-38) has calculated that one can obtain μ^+ (μ^-) beams of about $4\,(0.8) \times 10^7$ muons of energy 100 GeV for 5×10^{12} protons. If we use μ^+ we need only 1/80 of this flux; allowing a factor of about 2 for estimate errors, 10^{11} protons per pulse would seem sufficient. Thus it would seem that for this experiment the external-beam power and associated heating and radiation problems are in the range of similar problems with high-intensity AGS beams.

Next, we look at the required size of the analyzing magnet A. First what length along the particle trajectory is required? This depends on how well we wish to measure p_2. I see no great value in the first experiment in trying to determine the recoil muons sufficiently well to see isobar peaks as in reaction (2). Also then p_1 must be measured as precisely as p_2. Let us see what precision would be required. If M_r is the recoil mass

$$M_r^2 = (E_1 + M - E_2)^2 - (\vec{p}_1 - \vec{p}_2)^2 = -q^2 + 2M(E_1 - E_2) + M^2.$$

Here M is the proton mass.

The crude mass error for θ_2 fixed and q^2 small is

$$\Delta M_r \approx \frac{M}{M_r} \Delta E \approx \frac{1}{2} \Delta E.$$

Therefore, to measure M_r to ± 0.05 GeV, to get inside the isobar width, requires $\Delta E = \pm 0.1$ GeV or 0.1% measurement of p_1 and p_2. This is a difficult job, especially for p_1.

Therefore, we only measure p_2 (and p_1) well enough to fix q_o to about 2%. The angle of deflection in the magnet (α) is given by

$$\alpha \approx \frac{0.03\,BL}{p_2},$$

where B is the field in kilogauss,

L is the length along the field in meters,

p_2 is in GeV/c.

We take $B \approx 16$ kG, and as we shall show next the measurement uncertainty in α (called $\Delta\alpha$) can be kept to 10^{-4} rad. Then $\alpha \approx 50\,\Delta\alpha = 5\times 10^{-3}$ rad, and $L \approx 10^{-2}\, p_2$. To use p_1 values up to 150 GeV/c requires an L of 1.5 m. But an L as little as 1 m might be used.

The aperture of the magnet A limits the angles θ_2 accepted by the system and hence the maximum q^2. In this experiment it is difficult to get many events with $q^2 > 6$ or 8 $(\mathrm{GeV}/c)^2$ without substantially increasing the muon flux. When the $p_1\, p_2$ product is a minimum we shall require the maximum θ_2. Now it is not sufficient to run at one high energy only. For example, if the highest energy is 100 GeV, it is necessary to run at lower energies to separate σ_T and σ_L. To see how this must be done, take $\sigma_L = r\sigma_T$; then for a particular q_o and q_2 at two different E_1 we measure

$$N = \sigma_T (1 + r\epsilon),$$

$$N' = \sigma_T (1 + r\epsilon'),$$

where

$$\epsilon = \frac{2E_1(E_1 - q_o)}{q_o^2 + 2E_1(E_1 - q_o)},$$

$$\epsilon' = \frac{2E_1'(E_1' - q_o)}{q_o^2 + 2E_1'(E_1' - q_o)}.$$

For $q_o = 60$, $E_1 = 100$, $E_1' = 80$, we get $\epsilon = 0.69$, $\epsilon' = 0.44$ and the separation can be done. But for $q_o = 40$, $E_1 = 100$, $E_1' = 80$ we get $\epsilon = 0.88$, $\epsilon' = 0.80$, and if $r = 0.5$,

$$\frac{N - N'}{N} = 0.03,$$

which makes separation difficult. Therefore, we must expect to use p_1 as low as 50 GeV and p_2 as low as 10 GeV/c.

The aperture of A is also set by distance from the hydrogen target to A, which is mostly the required spacing between chamber 5 and 6. We shall set this spacing by requiring that q^2 be measured to at least ±5% accuracy.

The smallest angle we intend to trigger on is 4 mrads. Since for small angles $\Delta q^2/q^2 = 2\Delta\theta/\theta$, an error of ±5% in q^2 requires $\Delta\theta = \pm 0.1$ mrad. If the total error in transverse measurement in the two spark chambers 5 and 6 is ±0.5 mm then a spacing of 5 m is required between 5 and 6. If the half-aperture of the magnet is (a/2) m then the maximum angle of scatter ($\theta_{2\,max}$) is $\theta_{2\,max}$ (rad) = 0.1 a. For large q^2 we can neglect q^2_{min} and take $q^2 = p_1 p_2 \theta^2$. If we wish to have $q^2_{max} = 8\,(\mathrm{GeV/c})^2$ in this first experiment, then θ_{max} occurs when p_1 and p_2 are smallest, namely 50 and 10 GeV/c respectively. Then $\theta_{max} = 0.13$ rad and a = 1.3 m.

Now it is possible to make a magnet with a square aperture about 1 m on a side but this leads to a nonuniform field. Such magnets are very useful for other experiments if they also have removable pole. There are two such magnets at SLAC now: both have circular poles. One has a 54-in. pole diameter and a 36-in. gap. The other has a 2-m diameter pole and a 1-m gap. Therefore, one type of magnet which would fit this experiment is one with circular poles of 1.5-m diameter. A 1.3-m gap is excessive, but the experiment loses little if the gap is reduced to 0.8 to 1.0 m. All that is lost is some high q^2 events at low p_1p_2.

Another magnet design would be a uniform-field type with square poles and "window frame" coils. Here a smaller gap would be necessary. The magnet gap might be 1 to 1.5 m along the particle path, 0.6 to 0.8 m high and about 2 m wide. The greater width is for other experiments. For this experiment the 0.6 to 0.8-m width hurts the high q^2, but not too much. Even at $q^2 = 8(\mathrm{GeV}/c)^2$ there is about half acceptance for low p_1p_2 values.

The Multibody-Detection Experiment

The flux in this experiment must be 5×10^5/pulse, and not much larger, because as in the σ_T, σ_L experiment, there are spark chambers in the beams.

There are now three new questions:

1. Can the chamber 5 through 8 and magnet A encompass a good fraction of the other particles produced in the inelastic interaction?

2. Can the vector momenta of these particles be measured well enough to detect forward production of boson resonancies?

3. Can the event be completely identified?

To answer these questions we can use the study of Fields et al. (A. 3-68-12) on "High-Accuracy, Large Solid-Angle Detector for Multiparticle Final States at 100 GeV." The problems with a multibody reaction from 100 GeV muons are the same as with 100-GeV pions in general. The same solid angles and the same precision are needed.

Of course, we cannot use a small bubble chamber for a target; we must use a liquid-hydrogen target and spark chambers even in the vertex region. Therefore, the answers to questions 1, 2, and 3 lie in the study of a large, purely spark-chamber detector of the size of that in the Field report. I do not want to go into that design problem here. I think for the first muon experiments it would be actually unwise to attempt to use a very large and complicated detector. Therefore, I shall only consider here what can be done with the system designed for the σ_T, σ_L experiment.

The half-aperture of the magnet is about 0.5 m. Then the maximum laboratory angle accepted is about 100 mrads. Using the Field report the angle covered in the center-of-mass is $\theta^* \leq 2\gamma(0.1) \approx 0.14\sqrt{p_1}$ where p_1 is the initial muon momentum in GeV/c. Thus for $p_1 = 50$ GeV/c, $\theta^* \leq 1$ rad and for $p_1 = 100$ GeV/c, $\theta^* \leq 1/4$ rad. This is quite good; it means we can do a lot of work on the forward particles. But it also means that if the sum of the total number of neutral particles and of the charged particles with too large a θ^* is two or greater, we cannot do complete event identification at high energies; this will be the most common case. Therefore, the answer to question 1 is yes but to question 3 is generally no.

However, one way to help somewhat is to put chambers above and below the hydrogen target. It is particularly useful to pick up the angle of the recoil proton for the low q^2 events. These might be range chambers without too much expense. Figure 2 shows a vertical view of the apparatus with these chambers (U and D) added. This I regard as the most probable setup for this experiment.

We next consider the reaction $\mu + p \rightarrow \mu + p + \rho^o$. For real photons $\gamma + p \rightarrow \rho^o + p$ is about 10% of the total photon + proton cross section. If this ratio continues at high q_o and $q^2 = 0$, then 10% of the inelastic events will be $\mu + p \rightarrow \mu + p + \rho^o$. The reason for expecting this ratio to hold up at NAL energies is that it is a diffraction process. Further at $q^2 > 0$ but for q^2 not much larger than m_ρ^2, it is not expected that $q^2 = 0$ will reduce a diffraction process any faster than the total cross section.

To give an example, consider a 100-GeV muon producing a 50-GeV virtual photon, that is $E_1 = 100$, $E_2 = 50$, $q_o = 50$ GeV/c, for $q^2 < 5\,(\mathrm{GeV/c})^2$, θ_2 (the final muon angle with the beam axis) < 32 mrads and θ_γ (the angle of the virtual photon with the beam) is also < 32 mrads. The photon changes to a ρ^o on the target proton with small four-momentum transfer ($\sqrt{|t|}$) to the proton. Generally we can take $|t| < 1\,(\mathrm{GeV/c})^2$ and hence the ρ^o can diverge in angle from the virtual photon by up to 20 mrads.

Finally the pions from the ρ^o decay can have a maximum angle of divergence from the ρ^o direction of $m_\rho^2 / (2m_\pi E_\rho) \approx 30$ mrads. The sum of these angles gives the extreme angle of acceptance needed, this is $\approx$ 80 mrads. Since the acceptance is $\approx$ 100 mrads this is good. But if q_o decreases the angles all increase. Reducing the maximum q^2 will help. But for $q_o < 25$ GeV/c the acceptance will become poor.

Since it is important to see almost the complete decay angular distribution of the ρ^o, information from $q_o < 25$ GeV/c will probably not be useful. However, it is not worthwhile to increase the A magnet aperture just to cover the low q_o range.

Next can we measure the ρ^o mass? For a magnet 1 m long, we showed in II that $\Delta p/p = 0.02\ (p/100)$ and that the uncertainty in an angle θ is $\Delta\theta = 10^{-4}$ rad. Then for a ρ^o of momentum p, the mass uncertainty ΔM due to angle uncertainty $\Delta\theta$ is

$$\Delta M = (p/2)\Delta\theta = 5 \times 10^{-5} p,$$

for $p \leq 100$ GeV/c, $\Delta M \leq 5$ MeV.

For a momentum uncertainty Δp_π of a decay pion, $\Delta p_\pi = 2 \times 10^{-4}$ p_π.

$$\Delta M = (M/2)(2 \times 10^{-4})(p_\pi) \leq 10^{-2} M = 7 \text{ MeV}.$$

Therefore, the ρ^o mass is quite well measured.

With the additional chambers (U and D) the reaction $\mu + p \rightarrow \mu + \rho^o + p$ would be a three- or four-constraint fit (depending on the proton range) if the momenta were well measured. But there will be total longitudinal momentum uncertainties of about 2 GeV/c. Therefore, a slow forward π^o could not be detected. Transverse momentum balance may help, but in general we shall have to depend on the strength of this reaction to separate it out.

Finally, let us look at a reaction which we cannot do with this system; this is $\mu + p \rightarrow \mu + n + \pi^+$. First the cross section is small, about 10^{-3} to 5×10^{-3} of the total inelastic cross section. Second, there is no hope of separating it from additional π^0 production. Either special detection of the neutron must be carried out, or one must go to a large, high-accuracy detection system.

Summary of Magnet Designs

Figure 4 shows the two possible magnet shapes. The removable pole in the circular pole design is not for this experiment but is generally useful.

REFERENCES

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[3] F. S. Gilman, Phys. Rev. 167, 1365 (1968).

[4] J. D. Bjorken, Proc. 1967 Intl. Symposium on Electron and Photon Interactions at High Energies, Stanford, 1967.

[5] L. Hand, ibid.

APPENDIX A

The equation for inelastic muon-proton scattering may be written in several ways which differ only in the method of definition of the two inelastic form factors 1, 2, 3. We make the following definitions (see Fig. 1).

$\vec{p}_1$, E_1 = initial muon momentum, energy

$\vec{p}_2$, E_2 = final muon momentum, energy

$\vec{q}_2 q_o$ = virtual photon momentum, energy

$\vec{q} = \vec{p}_1 - \vec{p}_2$

$q_o = E_1 - E_2$

$q^2 = \left| q_o^2 - (\vec{q})^2 \right|$ = absolute value of square of four-momentum transfer from muon.

For NAL we shall see that

$$q^2 << E_1^2, \qquad q^2 << E_2^2, \qquad q^2 < q_o^2.$$

In this case all the different form-factor definitions become the same. The exact equation we use is

$$\frac{d^2\sigma}{d\Omega_2 dp_2} = \frac{\alpha}{2\pi^2 q^2}\left(\frac{p_2^2 \,|\vec{q}\,|}{E_2 p_1}\right)\left[\left(1 - \frac{2m_\mu^2}{q^2} + \frac{2E_1E_2 - q^2/2}{|\vec{q}\,|^2}\right)\right.$$

$$\left. \times \; \sigma_T(q^2, q_o) + \left(\frac{2E_1E_2 - q^2/2}{|\vec{q}|^2}\right)\sigma_L\,(q^2, q_o)\right] .$$

Here Ω_2 is the solid angle of the final muon, m_μ is the muon mass and σ_T and σ_L (both functions of q^2 and q_o) are the inelastic form factors separated according to Hand-Wilson.[1] But they are slightly different than those of Hand[1] because of the use of $(\vec{q})$ in the first parenthesis on the right side of Eq. (1). This change is due to Gilman[3] and seems to us more meaningful at low energies. σ_T and σ_L have the units of micro-barns. As $q^2 \rightarrow 0$, σ_T becomes the real photon total cross section on protons and $\sigma_L \rightarrow 0$.

We restrict ourselves for NAL to $q_o \geq 10$ GeV/c, $q^2 < 4(\mathrm{GeV/c})^2$ $E_2 \geq 10$ GeV. Then,

$$\frac{d^2\sigma}{d\Omega_2 dE_2} = \frac{\alpha}{2\pi^2 q^2}\left(\frac{E_2 q_o}{E_1}\right)\left(1 - \frac{2m_\mu^2}{q^2} + \frac{2E_1E_2}{q_o^2}\right)(\sigma_T + \epsilon\ \sigma_L),$$

where
$$\epsilon = \frac{(2E_1E_2/q_o^2)}{(1 - 2m_\mu^2/q^2 + 2E_1E_2/q_2)} .$$

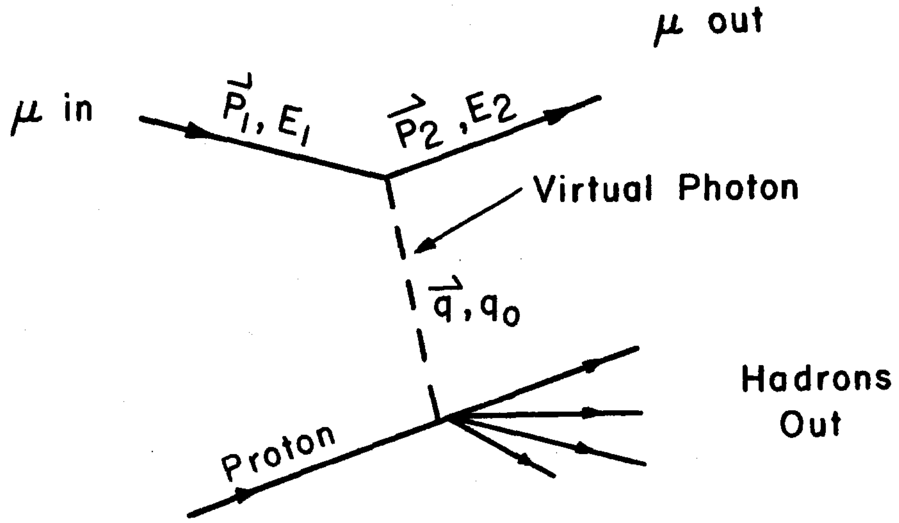

Fig. 1. Muon scattering diagram.

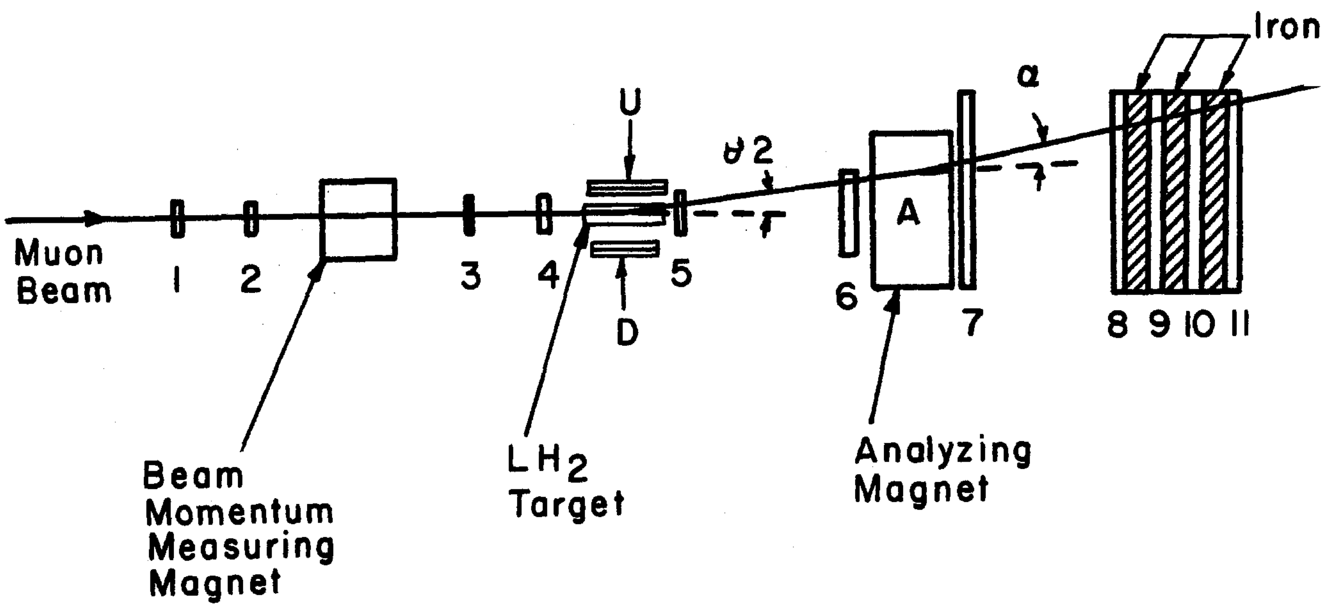

Fig. 2. Schematic of muon experiments. 1 to 11 are spark chambers for cross-section measurements; U and D are spark chambers for multibody detection experiment.

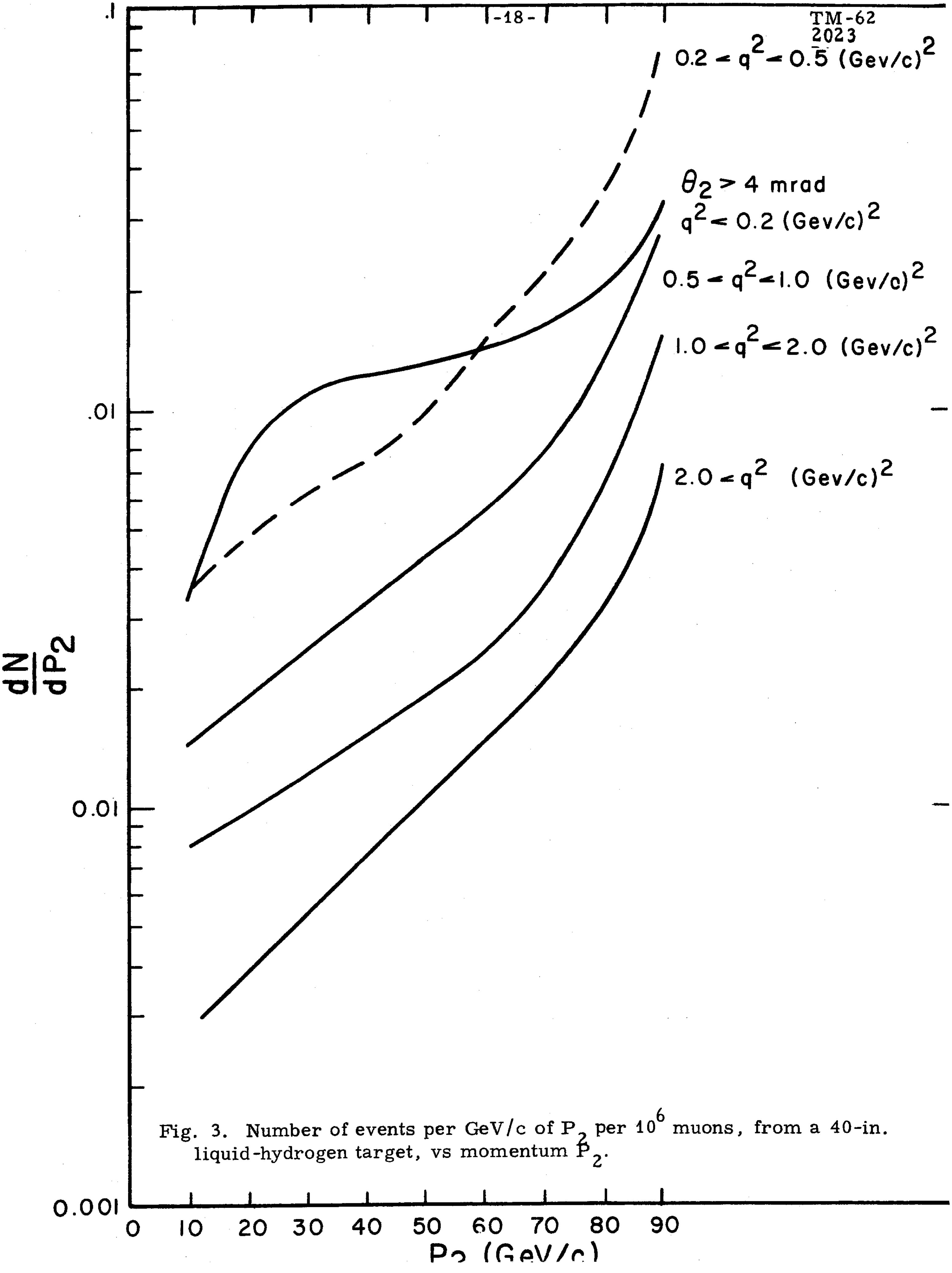

Fig. 3. Number of events per GeV/c of P_2 per 10^6 muons, from a 40-in. liquid-hydrogen target, vs momentum P_2.

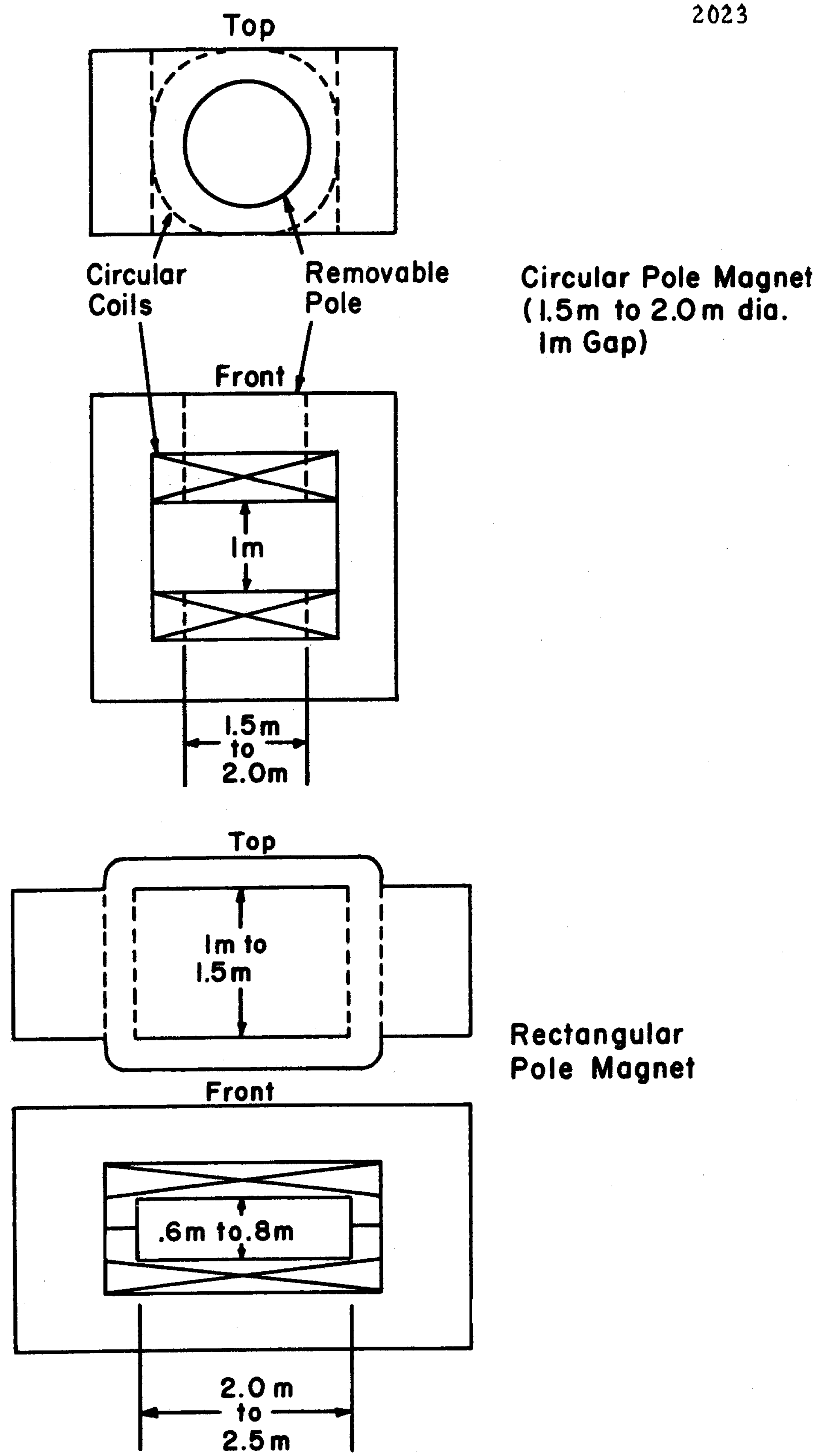

Fig. 4. Sketch of magnet construction.