

FURTHER COMMENTS ON THE INFLUENCE  
OF DETECTOR SPATIAL RESOLUTION ON SPECTROMETER SCALINGL. Rosenson  
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It was pointed out by Lederman in C. 3-68-65 that great cost savings and possible experimental simplicity would be achieved if one could improve the spatial resolution ( $\delta x$ ) of the detectors used in high-energy experiments. In particular, one finds a scaling of magnetic field dimensions (horizontal and vertical apertures and required length of field for a given H) like  $\sqrt{\delta x}$  if one seeks to optimize the required field volume. The volume (and presumably the cost) scales as  $(\delta x)^{3/2}$ , and this scaling is roughly true as long as the main errors in the momentum determination arise from spatial resolution only. If multiple-scattering errors are important in the determination of the bending angle in the spectrometer then the scaling of volume is not as strong as  $(\delta x)^{3/2}$ . Lederman also points out the possible application of the technique developed by Charpak using wires operating at dc as proportional counters to fill this need of high spatial resolution. In addition one seems to get the bonus of high rate capacity with the devices (see C. 3-68-65).

The purpose of this note is to point out some details and complications of such scaling of magnets using the Charpak technique to obtain the resolution. We will take as an example the spectrometer magnets required for the backward  $\pi^-p$  scattering experiment discussed by White

(C. 1-68-11). The forward spectrometer magnet is a large one by any standards and is a good candidate for scaling if possible. White required a spectrometer with  $\delta p/p = 10^{-3}$  at 100 GeV/c to measure the recoil protons going forward into a solid angle defined by  $\theta_{\min} = 2$  mrad,  $\theta_{\max} = 12$  mrad and  $\Delta\phi = 2\pi/10$ . He defined the beam direction with parallel beam optics and measures the portion of particles emerging from the target with a coarse-grain counter hodoscope (20 elements, 0.1 in. wide). He requires a counter hodoscope, rather than wire chambers for example, because of the high beam intensities required ( $\sim 10^7$  particles/sec) to get acceptable counting rate. White sets the scale for his magnet by requiring its effective field volume to start 6 in. from the beam line. In this note we will relax this condition--since we shall soon see that scaling the system down will quickly bring any magnets into the beam region--and compare various possible systems.

### Volume Optimization of Spectrometer Magnets

Consider the spectrometer shown in Fig. 1. The uncertainty in momentum arising from a determination with such a spectrometer is

$$\frac{\delta p}{p} = \frac{\delta\theta}{\theta} = \frac{p\delta\theta}{0.3} \int H dl, \quad (1)$$

where  $\delta\theta$  is the root mean square uncertainty in the turning angle and may be taken as:

$$\delta\theta = \sqrt{(\delta\theta)_{\text{meas}}^2 + (\delta\theta)_{\text{mult. scat.}}^2} . \quad (2)$$

One then has:

$$\int H dl = \frac{1}{0.3 \delta p/p} \sqrt{\frac{4 (P \delta x)^2}{D^2} + (0.015)^2 t} , \quad (3)$$

for an equal arm spectrometer with all position determinations having the same spatial resolution,  $\delta x$ .

Figure 2 is a plot of formula (3) for some relevant values of the parameters.

For the forward spectrometers of White it is assumed that there is only one significant spatial uncertainty, due to the coarse-grained hodoscope mentioned above. For this case the first term under the square root in (3) does not have the factor 4. Also  $\delta x$  is so large that the multiple-scattering term is negligible for sensible choices of the parameters. This term has been unimportant for many spectrometers used in high-energy experiments but will not continue to be so if  $\delta x$  is driven down by a factor of 10 below the presently attainable 0.2-0.3 mm.

Consider now the case of negligible multiple scattering. We will quickly see that it is possible to optimize the field volume.

If multiple scattering is negligible we have:

$$\int H dl \approx HL = \frac{2 \delta x p^2}{0.3 \delta p} \times \frac{1}{D} , \quad (4)$$

where  $L$  = length of the magnetic field in meters.

The field volume, ignoring the correction ( $\leq 20\%$ ) due to the sagitta of the particle in the field, is given by:  $V = LW_H W_V$  where:

$$W_H = \text{horizontal aperture} \approx (L + D) (\theta_2 - \theta_1)$$

$$W_V = \text{vertical aperture} \approx (L + D) \theta_2 \times \Delta\phi$$

$$\text{Thus: } V = L (L + D)^2 (\theta_2 - \theta_1) \theta_2 \Delta\phi, \quad (5)$$

and from (4) we have:

$$D = \gamma/L, \quad (6)$$

with

$$\gamma = 2 \delta x p^2 / 0.3 H \delta p, \quad (7)$$

and

$$V \propto L (L + \gamma/L)^2,$$

with a minimum for  $V$  with  $L = \sqrt{\gamma/3}$  and  $D = \sqrt{3\gamma}$ . Note that for this optimization  $D = 3L$ . Also since  $\gamma \propto \delta x$  we have  $D$  and  $L$  with  $V_{\min} \sim (\delta x)^{3/2}$ .

For the magnet of White,  $\gamma$  is smaller than in (7) by a factor of 2 as observed before.

One can perform the optimization including the multiple-scattering term, but no longer in closed form, and the results for some relevant parameter choices are shown in Table I.

We observe several things from this table:

1. One can drop the field volume by a factor of 10 ( $\sim 5^{3/2}$ ) rather

than the factor  $10^{3/2}$  which one might first expect) by improving the spatial resolution by a factor of 10 [see cases (a) and (b)]. This is so because in case (a) the coarse-grained hodoscope determination of position may be made to dominate the momentum resolution. The other three position determinations required to determine the momentum can be made to  $\sim 0.2 - 0.3$  mm by the use of existing wire chamber techniques and will then give a negligible contribution to the error. In case (b), however, we assume all four position determinations made by elements having  $\delta x = 0.25$  mm. The first of these (that measuring  $x$  in Fig. 1), could be a Charpak device such as that described by Lederman in C. 3-68-65, since it will have to be in the beam and would require both high spatial and time resolution.

In fact, the second element (that measuring  $x'_1$ ) would probably have to be in the beam too (note that  $a \sim 4$  cm) and therefore would have to be a Charpak chamber also. This requirement of high counting rates and time resolution would seem to require some form of counter hodoscope along with the wire arrays to break down the large numbers of wires required into smaller groups consistent with the rate capacity of the individual proportional counter wires. Such scintillators mean the introduction of multiple scattering uncertainty into the angle determinations ( $1/8$  in. of scintillator  $\approx 0.5 \times 10^{-2}$  radiation lengths). Presumably it would be possible to make the resolution error after the magnet negligible by the use of large wire chamber arrays and a large lever arm.

It could seem that at best one might get a volume scaling factor

$$(10/\sqrt{2})^{1/2} \sim 19.$$

2. In order to achieve the cases c, d, e one must use devices such as the Charpak chamber since  $\delta x \leq 0.1$  mm is certainly beyond any present technique.

3. As evidenced by the systems b, c, d, e (note the value of a) one will frequently have the elements determining  $x_1$  and  $x_1'$  and very possibly  $x_2$ , also located in the main beam. In particular  $x_1'$  and  $x_2$  elements introduce relevant multiple scattering and this must be taken into account in any design.

4. There seems to be little mileage to be gained on magnets such as are required for the large angle spectrometer magnet (the one measuring the recoil momentum of the  $\pi^-$ ) by increasing the resolution. This magnet has its aperture mainly fixed by the large solid angle bite required.

Table I. Magnet Optimization for Various Parameter Values.

| Case | $\delta x$ (m)       | t         | D    | L    | a     | $W_H$ | $W_H'$ | $W_V$ | Field Volume } <sup>m</sup> 3 |
|------|----------------------|-----------|------|------|-------|-------|--------|-------|-------------------------------|
| a*   | $2.5 \times 10^{-3}$ | $10^{-2}$ | 34.6 | 11.5 | 0.092 | 0.46  | 0.56   | 0.35  | 2.260                         |
| b    | $2.5 \times 10^{-4}$ | $10^{-2}$ | 15.0 | 6.1  | 0.042 | 0.21  | 0.23   | 0.16  | 0.225                         |
| b'   | $2.5 \times 10^{-4}$ | 0         | 16.0 | 5.3  | 0.042 | 0.21  | 0.23   | 0.16  | 0.196                         |
| c    | $1.0 \times 10^{-4}$ | $10^{-2}$ | 8.0  | 5.0  | 0.026 | 0.13  | 0.15   | 0.098 | 0.0730                        |
| c'   | $1.0 \times 10^{-4}$ | 0         | 10.0 | 3.3  | 0.026 | 0.13  | 0.15   | 0.098 | 0.0488                        |
| d    | $3.0 \times 10^{-5}$ | $10^{-2}$ | 4.0  | 3.6  | 0.015 | 0.076 | 0.09   | 0.057 | 0.0184                        |
| d'   | $3.0 \times 10^{-5}$ | 0         | 5.5  | 1.8  | 0.015 | 0.073 | 0.08   | 0.055 | 0.0079                        |
| e    | $3.0 \times 10^{-5}$ | $10^{-3}$ | 5.0  | 2.2  | 0.014 | 0.072 | 0.08   | 0.054 | 0.0085                        |

For all cases:  $p = 100 \text{ GeV}/c$

$$\frac{\delta p}{p} = 10^{-3}$$

$$H = 2 \text{ w/m}^2$$

All column headings have been defined in the text except:

$a$  = displacement of beginning of active field region from center of beam line.

$W_H' = W_H + \text{correction for sagitta of detected 100-BeV particle.}$

Field Volume =  $LW_V W_H'$ .

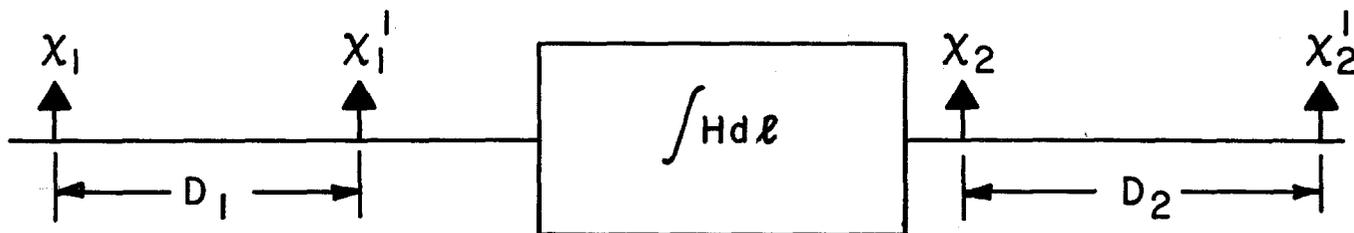


Fig. 1.

Relevant formulae:

$$\theta_{\text{meas}} = \frac{x_2' - x_2}{D_2} - \frac{x_1' - x_1}{D_1}$$

$$(\delta\theta)_{\text{meas}} = \left[ \frac{(\delta x_2')^2 + (\delta x_2)^2}{D_2^2} + \frac{(\delta x_1')^2 + (\delta x_1)^2}{D_1^2} \right]^{1/2}$$

For an equal arm spectrometer  $D_1 = D_2 = D$

$$\text{and } (\delta\theta)_{\text{meas}} = \frac{1}{D} \sqrt{\sum (\delta x)^2}$$

$$(\delta\theta)_{\text{mult. scat.}} = \frac{0.015}{p} \sqrt{t} ,$$

$$\theta = \frac{0.3 \int H dl}{p} ,$$

where:

$\theta_{\text{meas}}$  = the measured turning angle in radians,

$(\delta\theta)_{\text{meas}}$  = uncertainty in  $\theta_{\text{meas}}$  due to spatial resolution,

$(\delta\theta)_{\text{mult. scat.}}$  = uncertainty in turning angle due to multiple scattering (particularly in the position measuring devices at  $x'_1$  and  $x_2$ ),

$t$  = amount of material contributing to multiple scattering, in radiation lengths,

$\theta$  = turning angle due to spectrometer,

$H$  = magnetic field in  $w/m^2$ ,

$p$  = momentum of particle in  $\text{BeV}/c$ .

FIGURE CAPTIONS

Fig. 2. Field integral for an equal-arm spectrometer with error  $\Delta x$  in all position measurements and total material thickness  $t$ .

All curves are for  $P=100 \text{ GeV}/c$   
and  $\Delta p/p = 10^{-3}$

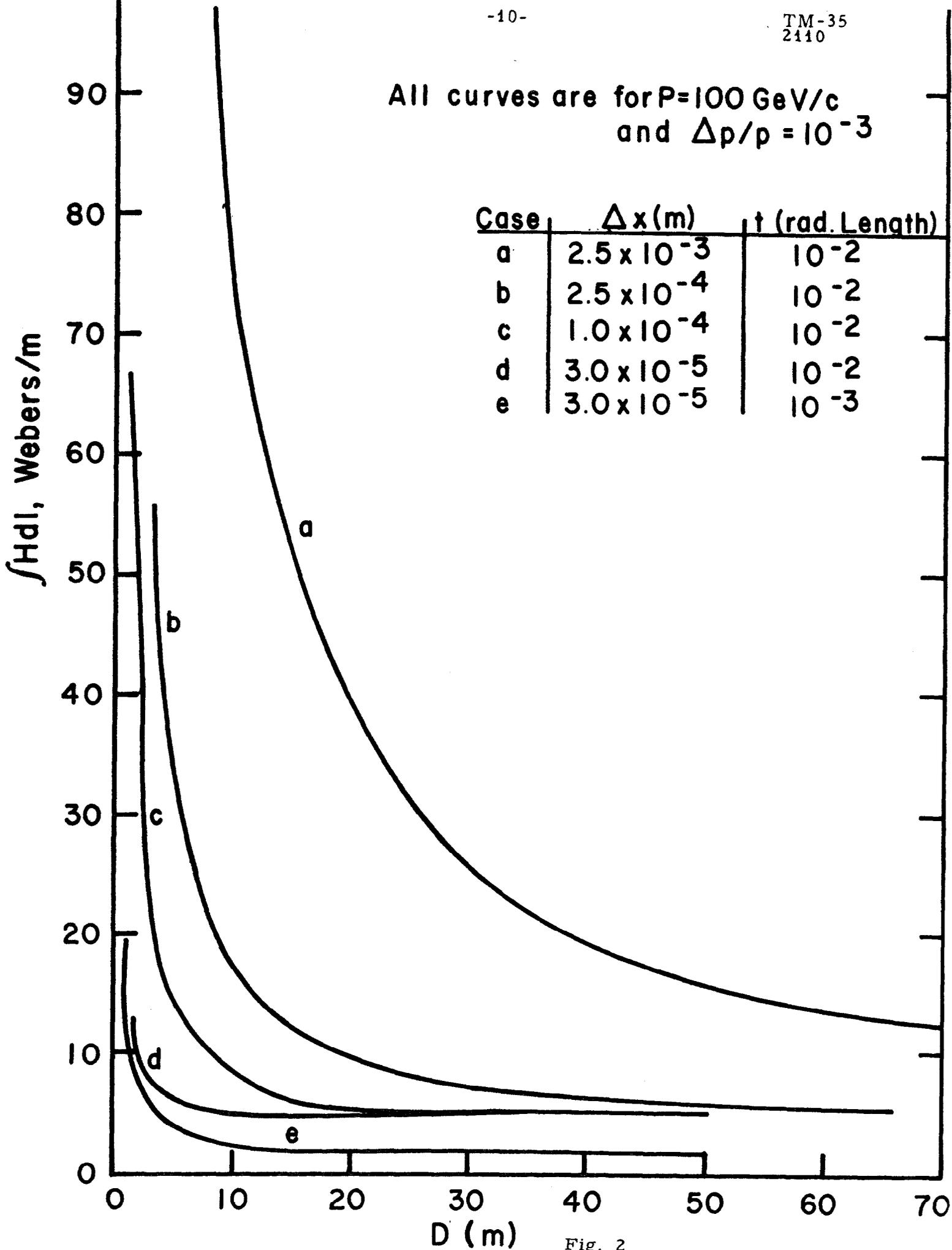


Fig. 2