MEASURING THE INCLUSIVE CENTRAL MUON CROSS SECTION IN
PROTON-ANTIPROTON COLLISIONS AT 1.8 TeV

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Appendix

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B.1 The three plots show the momentum distribution of $b$ quarks that produced muons in the bins $12 < p_t < 17$ GeV/c, $17 < p_t < 22$ GeV/c, and $p_t > 22$ GeV/c respectively.

B.2 Inclusive cross section for the production of $b$ quarks with $p_t > p_t^{\text{min}}$. 

High momentum muons and electrons are distinguishable from other charged particles in a colliding hadronic beam experiment. All the heavy quarks decay into leptons with significant branching fractions. It is for these reasons leptons are the source of information in heavy quark production at hadron colliders. Therefore, obtaining the heavy quark cross section requires the measurement of the lepton cross section.

This thesis describes a measurement of the cross section of central muons at the Collider Detector at Fermilab. The cross sections are $3.90 \pm 0.27 \pm 0.51$ nb for muon transverse momentum between 12 and 17 GeV/c; $0.52 \pm 0.13 \pm 0.08$ nb for $p_t$ between 17 and 22 GeV/c; and less than 0.17 nb for $p_t$ between 22 and 27 GeV/c at the 95% confidence level.
1. INTRODUCTION

The discovery of the $J/\Psi$ \cite{1, 2} in 1974 at Brookhaven and SLAC and the realization that this was a charm anti-charm bound state signaled the beginning of heavy flavor physics. This discovery also supported the theory of QCD and the parton model of hadrons.\cite{3, 4} In 1976 the first $D$ mesons were found.\cite{5, 6} The study of charm particles continues to this date.

The subsequent discovery of the $\Upsilon$ and its resonances identifying it as a $b\bar{b}$ state lent more support for the Standard Model.\cite{7} The discovery of the $B$ mesons in 1980 and their study occurred (and continues) at the $\Upsilon(4S)$ resonance.\cite{8, 9} The Standard Model says the $B$ mesons consist of a bound $b$ or $\bar{b}$ quark with a lighter partner. All types of $B$ mesons should be produced in $p-\bar{p}$ collisions though some states such as $B_s$ and $B_c$ have yet to be seen at hadron colliders.

The study of heavy quarks at $p-\bar{p}$ colliders provides tests of the parton model of strong interactions and permits the search for new physics over a spectrum of energies. The production of charm and bottom occurs at small longitudinal momentum fractions ($z$) but still in the realm where perturbative QCD should be applicable. There has been considerable interest in using the $b$ quark production cross section to fix the gluon structure functions at small $z$.

1.1 Theory of Heavy Quark Production

The process of heavy quark inclusive production from a $p-\bar{p}$ collision is shown schematically in Figure 1.1. The incoming proton and anti-proton are shown with
momenta $P_A$ and $P_B$. The incoming partons have momenta $z_1 P_A$ and $z_2 P_B$ and the outgoing heavy quark momentum $k$. $z$ is defined as the momentum fraction of the parton. The formula describing this process is:

$$
\frac{d^3 \sigma}{d^3 k} = \sum_{i,j} \int dz_1 dz_2 \left[ \frac{E d^3 \hat{\sigma}_{ij}(z_1 P_A, z_2 P_B, k, m, \mu)}{d^3 k} \right] F_i^A(z_1, \mu) F_j^B(z_2, \mu) \quad (1.1)
$$

where $F_{i,A,B}$ are the structure functions for the $i$th parton in the incoming hadrons $A$ and $B$. The short distance cross section is denoted by $\hat{\sigma}$ and the mass of the heavy quark by $m$. $\mu$ is the energy scale for the strong coupling constant. The theory of color interactions is perturbative at short distance scales allowing the solution of $d\hat{\sigma}$ as an expansion in powers of $\alpha_s(\mu^2)$. Figure 1.2 shows Feynman diagrams of the lowest order contributions to $\hat{\sigma}$.

There are several theoretical uncertainties that make this calculation inaccurate at the factor of two level. First is our knowledge of the proton structure functions. Most of our data comes from deep inelastic scattering of leptons from protons.[11] This occurs at small $\sqrt{S}$ and large $z$. At the Tevatron $z; \approx 10^{-2}$ for partons producing heavy quarks at transverse momenta in the tens of GeV/c and $\sqrt{S} = 1800$ GeV which is just the opposite from deep inelastic scattering measurements.

Additionally, the gluon structure function is not measured directly in this $z$ region. Since it is estimated that the three upper diagrams in Figure 1.2 dominate the cross section at small $z$, the unknown gluon structure function causes the calculations to be uncertain. Conversely the $b$ quark cross section measurements allow us to unfold the gluon structure function.

Finally, the calculation can be quite sensitive to higher orders in $\alpha_s(\mu^2)$. This is shown in Figure 1.3.[12] The solid line shows how the bottom quark cross section varies with $\mu$ in a calculation to order $\alpha_s^2$. The dashed line is the same curve but with only the leading order terms. The cross section is about a factor of two smaller
Figure 1.1

Illustration of the processes leading to equation (1.1).

Figure 1.2

Feynman diagrams for heavy quark production at lowest order ($\alpha_s^2(\mu^2)$).
Figure 1.3

Scale dependence of the bottom quark cross section in leading and leading+next-to-leading order.

when only terms of leading order in the strong coupling constant are included. Some examples of the type of diagrams that enter the $\alpha_s^3$ calculation are shown in Figure 1.4.

The results of the next-to-leading order calculation are shown in Figure 1.5 from [10] at $\sqrt{S} = 1.8$ TeV. This plot shows the value of the total b quark cross section for transverse momentum greater than $k_{\text{min}}$ and rapidity from $-1$ to $1$. The dotted
1.2 Tagging Heavy Quarks

The heavy quarks, charm, bottom, and top, all decay into leptons with significant branching ratios. The $B$ meson decays to muons with an average branching ratio of $10.8 \pm 0.6\%$. These decays have been studied extensively at $e^+e^-$ machines. $b$ or $c$ quarks are also so short lived that they are not detected before they decay in this data sample. If a $u,d$, or $s$ quark decays, the decay distance is measurable and the decay muon is not mistaken as coming from the primary collision. Since the production process for both heavy and light particles invariably produces hadronic
Figure 1.5

The result of the third order calculation by Nason, Dawson, and Ellis of the $b$ quark cross section for $b$ quarks with $p_t > k^\text{min}$. 

pp collisions, $\sqrt{S} = 1.8$ TeV, $|y|<1$, $k_T > k^\text{min}$

- $m_b=4.75$ GeV, $\Lambda=260$ MeV.
- DFLM, $\mu_0 = \sqrt{(m_b^2 + k_T^2)}$
- $4.5 < m_b < 5$ GeV, $160 < \Lambda < 360$ MeV
  $\mu_0/2 < \mu < 2\mu_0$
The result of the third order calculation by Nason, Dawson, and Ellis of the b quark cross section for b quarks with $p_t > k_{min}$. This plot is for $\sqrt{S} = 0.63$ TeV. The points are from the UA1 experiment.
jets, the detection of direct leptons and specifically muons is useful in distinguishing heavy quarks.

Muons are detected at the Collider Detector at Fermilab (CDF) by tracking chambers located behind the central calorimeters. This provides approximately 5 hadronic absorption lengths to stop most of the large number of hadrons produced and enables the identification of high momentum muons. This thesis will explain the procedure needed to extract a muon signal. The charm quark background is subtracted and the cross section of muons originating from b quarks is presented.
2. THE COLLIDER DETECTOR

The Collider Detector at Fermilab (CDF) is a cylindrically symmetric, layered detector consisting of tracking, calorimeter, and muon detectors with muon coverage for pseudorapidity less than \(|\eta|<0.6\). CDF also has forward-backward muon coverage but since it is not used in this work it will not be described here.\(^{[16]}\) The main sub-detectors employed in this analysis are the vertex time projection chamber (VTPC), the central tracking chamber (CTC), and the central muon chambers. This chapter will discuss each of these detectors in turn and explain their purpose in the muon analysis.

CDF uses a cylindrical coordinate system about the beam axis. The \(z\) axis points in the direction of the proton beam along the beam line. The \(y\) axis is chosen to be vertical and the \(z\) axis completes a right-hand coordinate system. More generally particle tracks are identified by pseudorapidity \((\eta = \ln \cot \frac{\theta}{2})\), \(\phi\), and their transverse momentum \((p_t)\). The illustration (Figure 2.1) shows the coordinate system used in this analysis in relation to the detector. Figure 2.2 shows a more detailed view of the central detectors in a side view.

2.1 The VTPC

The Vertex Time Projection chamber was the closest detector to the beam axis for the 1989 run. It consisted of 8 modules. A high voltage grid separated each module into two drift regions of 15.25 cm each with electric fields parallel to the beam line. The chambers were filled with a gas mixture of equal parts argon and
Figure 2.1

An overview of the CDF detector showing the coordinate system used.
Figure 2.2

A cut-away side view of CDF showing the locations of the central detectors used in this analysis. The detector is forward-backward symmetric about the interaction point.
ethane bubbled through isopropanol at 0° centigrade and atmospheric pressure. The chamber was operated with an electric field of $E = 320 \text{ V/cm}$ giving a drift velocity of 46 $\mu\text{m/µs}$. Each of the 8 modules had two proportional chamber endcaps consisting of a wire plane perpendicular to the beam line and pads imbedded in the cathode of each endcap. The endcaps were split into octants with 24 wires and pads in each octant.

The wire position provided radial information on a particle traversing the VTPC while the time of arrival at each wire yielded the particle's position along the beam line. The main purpose of the VTPC was to fix the $z$ location of an interaction along the beam line.[17]

Shown in Figure 2.3 are two modules of the VTPC. The modules were clocked by 11.3° in order to provide limited $\phi$ resolution from the wires if a particle crossed modules. $dE/dz$ and $\phi$ information was available as well from the pad readout but was not used in this analysis.

The VTPC has been replaced since the 88-89 run with a new vertex chamber of similar design (VTX) and a silicon strip detector (SVX). These detectors are in use during the 1992 colliding beam run.

2.2 The Central Tracking Chamber

The CTC is a proportional wire chamber covering $|\eta| < 1.0$ and the full range in $\phi$. There are 84 layers of sense wires that extend from front to back of the cylinder. The wires are grouped in cells of two types, axial and stereo. The axial cells consist of 12 sense wires each and extend straight through the active volume. Axial cells are arranged in super layers at constant radii. There are 9 superlayers of which numbers $0, 2, 4, 6,$ and $8$ consist of axial cells. The CTC has inner and outer radii of 277 and 1380 mm respectively.
An isometric view of two VTPC modules. They are rotated in $\phi$ by $11.3^\circ$ with respect to each other.
Stereo cell sense wires are offset by ±3° with 6 wires per cell. Superlayers 1 and 5 have their wires offset by +3° while superlayers 3 and 7 are offset by −3°. Stereo superlayers provide r-z position information on charged tracks. r − φ information is gathered from both stereo and axial superlayers. Figure 2.4 shows the end view of the CTC emphasizing the positions of the stereo and axial superlayers. The cells are tilted by 45° with respect to the radial coordinate. This compensates for the crossed electric and magnetic fields in the gas volume and gives the drift electrons trajectories perpendicular to the radial coordinate. The electric field is held at 1350 V/cm.

The entire active volume of the CTC is in a 1.4 T solenoidal magnetic field enabling the three dimensional momentum of the tracks to be measured. The transverse momentum resolution of unconstrained tracks is \( \delta P_T / P_T^2 \simeq 0.002 \text{GeV}^{-1} \). [18] (Also see [19], page 26, for a detailed overview of the CTC.)

2.3 The Central Muon Chambers

Each central calorimeter wedge contains a stack of single wire drift chambers behind the layers of steel and scintillator. These drift chambers make up the muon detection system. Each drift cell has a wire extending the length of the cell (2260 mm) which returns through another cell thus permitting readout on one side. [20]

The muon chambers, whose inner faces are located 3470 mm from the beam pipe, consist of 3 towers each consisting of 4 layers of drift cells occupying a single calorimeter wedge. This is illustrated in Figures 2.5 and 2.6.

Within a tower, the sense wires lie on radial lines and are offset by 2 mm from each other in pairs. Thus the ambiguity as to which side of the sense wire the muon passed is resolved by recording which wire was hit first. The angle of the track in the r − φ projection can be found by taking the differences in time between wires
The resolution of the track measurement in the $r - \phi$ plane is 250 microns from cosmic rays.

Because of the rectangular cross section of the muon cells, it is necessary to run the electric field high enough to insure that the drift velocity is saturated all the way from the chamber edge to the sense wire. This requires +3150 V on the sense wire and -2500 V on the C and I-beams. The top and bottom of the chamber is held at ground. This yields a minimum electric field of 100 V/mm corresponding to a drift velocity of 45 $\mu$m/ns.[21]

The muon chamber coordinate system is somewhat different from the rest of CDF (Figure 2.6). A rectangular coordinate system is used with the $z$ axis parallel to the beam line with $z = 0$ at the center of the detector. The $y$ axis points radially outward with $y = 0$ at the inner radius of the muon cell. The $x$ axis is then chosen to form a right-hand coordinate system with $z = 0$ at the center of the muon chamber.

The position of a track along $z$ is determined by charge division. The wire used in each cell is 50 $\mu$m in diameter and made of stainless steel with a resistance of 0.4 $\Omega$/mm. Each sense wire is read out at both ends by ADC’s and the relative pulse heights determine the position of the track along the wire by the formula $R = \frac{Q_L - Q_R}{Q_L + Q_R}$ where $R$ is a dimensionless coordinate. Conversion of $R$ to actual position requires the calibration of the wire and amplifier impedance. For this purpose, $^{56}$Fe sources are located at precise locations along the wire for fiducial $z$ measurements. The resolution of this measurement is 1.2 mm from cosmic ray tracks.

2.4 The muon trigger

It is possible to get a momentum measurement of the muon candidate using the muon chambers alone. Because a track curves in the CTC it approaches the muon chambers at an angle relative to local muon coordinates. Thus, the difference in
arrival time between two wires on the same radial line is related to the momentum of the track. This is illustrated in Figure 2.4. The difference in drift times $\Delta t$ is given by $\Delta t = H \alpha / v$ where $H = 55.0$ mm is the separation between wire layers, $v$ is the drift velocity of ionization electrons in the chamber, and $\alpha$ is the angle of the track (also called a stub) with respect to a radial line. For a magnetic field strength of 1.5 Tesla, and drift velocity of .0045 cm/ns, the transverse momentum is given by,

$$ p_t = \frac{164}{\Delta t} \text{ GeV/c} \quad (2.1) $$

The momentum resolution is

$$ \frac{\delta p_t}{p_t} \approx 60\% \quad (p_t < 100 \text{ GeV/c}) $$

independent of track momentum. Using this property allowed momentum limits to be placed on tracks reducing the trigger rate.

The inclusive muon trigger consisted of three levels of selection. The level one trigger used the fact that the time difference between wires is inversely proportional to the transverse momentum.[22] Wire hit information was sent to a hardware trigger module where a trigger was generated if the time difference between two wires was less than some predetermined value.

$$ \text{MIN}(|t_4 - t_2|, |t_3 - t_1|) < t_{\text{max}} \quad (2.2) $$

Two different thresholds were used in 1988-89, one at 5 GeV/c ($t_{\text{max}} = 30\text{ns}$) and one at 3 GeV/c ($t_{\text{max}} = 70\text{ns}$).[23] Both thresholds reached a plateau efficiency of 92% before $p_t = 12.0$ GeV/c.

The central muon level 2 trigger required a track be found by the Central Fast Tracker (CFT), extrapolated to the muon chambers, and match within $\pm 15^\circ$ in $\phi$ with a muon stub (or track). Since at the trigger level, a muon stub was defined by a
level one trigger, a level two trigger could not occur without a level one having first been satisfied. The level 2 trigger was not operational prior to run 17265; therefore, only runs greater than or equal to 17265 were used in the analysis. The level 2 trigger had a threshold of 9.2 GeV/c and was flat with a 97% efficiency by $p_t = 12.0$ GeV/c.[24]

The level 3 trigger was a FORTRAN program running online during data taking. CTC tracking was done in the $r$-$\phi$ plane. A level 3 trigger occurred when the $z$ intercept of the extrapolated CTC track matched the intercept of the muon stub within $\pm 10$ cm in the muon chamber coordinate system. There was no explicit requirement of a level 1 and level 2 trigger; however, level 3, by requiring a muon stub and by requiring the track be above the level 2 threshold, implicitly demanded that both level 1 and level 2 were satisfied.[25] The level 3 trigger threshold was 11 GeV/c and it was fully efficient by 13 GeV/c as is shown in Figure 2.8. The combined efficiency of the levels 1, 2, and 3 triggers was $\epsilon_{\text{trig}} = .90 \pm .02$ according to [26] for $p_t > 13$ GeV/c and for the muon cuts defined in the next chapter. (In other words, this is the trigger efficiency for muons passing in the fiducial volume of the chambers.)
Figure 2.4

An end view of the Central Tracking Chamber showing the location of the slots in the aluminum endplates. The slots indicate the location of stereo and axial cells on the endplate.
Figure 2.5

End and side view of the Central Muon Chamber showing its location relative to the hadron colorimeter.
Figure 2.6

An end view of a muon chamber tower showing the relative sense wire positions. The sense wires are located along radial lines. The pairs are separated by 2 mm. Note the definition of the muon coordinate system with the z axis out of the page. There are three of these modules per calorimeter wedge.
The relation between the track curvature, transverse momentum, and track angle at the muon chamber.

Figure 2.7

The relation between the track curvature, transverse momentum, and track angle at the muon chamber.
Figure 2.8

The efficiency of the Level 3 muon trigger for the 11 GeV/c threshold. The full curve is a smooth fit to the histogram.
3. THE MUON DATA SAMPLE

The inclusive muon data sample corresponds to an integrated luminosity of $3.79 \pm 0.26 \text{ pb}^{-1}$ from the 1988-89 run. This chapter describes the cuts applied to the data in order to increase the signal to background ratio and explains how the cut efficiencies for muons are measured.

3.1 The Applied Cuts

Several cuts are used to aid in enhancing the signal from prompt muons. These cuts are summarized in Table 3.1. First, as explained on page 16, only runs greater than or equal to 17265 are used in the analysis. Second there are vertex matching cuts. The muon track is required to cross the beam line within 10 cm in $z$ of the actual vertex. This is a very loose cut intended principally to reduce the number of cosmic rays.

A cut on impact parameter ($|D_0|$) is also required. The muon candidate must have an impact parameter within .15 cm of the vertex in the $r$-$\phi$ view. This cut is mainly effective against decays-in-flight, particularly kaons, as well as cosmic rays. The effect of this and other cuts on decays-in-flight will be detailed in a later section. It is desirable to try to keep this cut loose in order to be assured one is not cutting away real prompt muons. This was checked in a Monte Carlo of $b$ quark decays and no measurable loss beyond that due to resolution was seen with $|D_0| < .15 \text{ cm}$.

The muon ADC's will occasionally overflow when a hadron showers and charged particles enter the muon chamber. This is registered as zero ADC counts. Since
these events are all background and since they do not contain any \( z \) information, a cut requiring at least two ADC counts is imposed.

3.1.1 The Track Matching Cuts

The level 3 muon trigger had only a very loose requirement on matching the CTC track to the muon stub. This was only marginally tightened in the creation of the final sample by requiring the difference of the intercepts between the CTC track and the muon stub be less than 10 cm (in the muon coordinate system).

Among the data recorded in a muon event are the difference in slope (\( \delta S \) in radians) and intercept (\( \delta I \) in centimeters) between the muon stub and the extrapolated CTC track in the muon coordinate system (see Figure 2.6). Because of multiple scattering, one would not expect the two tracks to match exactly. However, one can calculate the expected spread of these variables based on multiple scattering and track resolution.[27]

The track measurement in the \( r-\phi \) plane is more accurate than the \( r-z \) measurement for both the CTC and the muon chambers. A muon traversing the material in the calorimeter will scatter electromagnetically off nuclei according to the formula given in the Particle Data Group booklet.[28] The width of the \( \phi \) distribution is given by:

\[
\sigma_\phi = \frac{13.6 \text{MeV/c}}{\beta p} Z_{\text{inc}} \sqrt{L/L_R [1 + 0.038 \ln (L/L_R)]} \text{ (radians)} \tag{3.1}
\]

Where \( \beta, p, \) and \( Z_{\text{inc}} \) are the velocity, momentum, and charge of the particles. \( L/L_R \) is the number of radiation lengths traversed by the muon candidate.

Because of the fact that the multiple scattering angle depends on the distance traveled through the calorimeter, \( \delta S \) and \( \delta I \) are correlated where the correlation coefficient \( \rho \approx 0.87 \). When one takes into account the energy loss in the calorimeter
the expected $\sigma$ of the slope and intercepts can be parameterized to the 1% level by
the expressions below. $S_x$ represents the slope in the $x$-$y$ plane in muon chamber
coordinates whereas $I_z$ represents the intercept in the $y$-$z$ plane. $\rho = \gamma_u/(\sigma_{Sx}\sigma_{Iz})$
relates $\gamma_u$ to the correlation between the slope and intercept.[27]

$$
\sigma_{Sx} = \frac{.131}{p_t} \sqrt{\frac{.27 + .73/\sin \theta}{1 - 1.43/p_t}}
$$

$$
\sigma_{Iz} = \frac{13.8 \text{ cm}}{p_t} \sqrt{\frac{.59 + .41/\sin \theta}{1 - 0.71/p_t}}
$$

$$
\gamma_u = \frac{1.54 \text{ cm}}{p_t^2} \left[ .46 + .54/\sin \theta \right]
$$

$$
\sigma_{Iz} = \frac{13.8 \text{ cm}}{p_t} \sqrt{\frac{.59 + .41/\sin \theta}{1 - 0.71/p_t}} \frac{1}{\sin \theta}
$$

(3.2)

The correlation can be seen when a scatter plot of $\Delta S/\sigma_{Sx}$ versus $\Delta I/\sigma_{Iz}$ is
made. Figure 3.1 shows such a plot using the $\sigma$ as calculated in the equations (3.2).

Given the expressions in Equation (3.2) for the spread in slope and intercept in
the $x$-$y$ plane one can calculate a $\chi^2$ for each event.

$$
\chi_u^2 = \left( \begin{array}{cc} \delta I & \delta S \end{array} \right) \left( \begin{array}{cc} \sigma_{Iz}^2 & \gamma_u \\ \gamma_u & \sigma_{Sx}^2 \end{array} \right)^{-1} \left( \begin{array}{c} \delta I \\ \delta S \end{array} \right)
$$

(3.3)

The $x$-$y$ plane matching cut imposed requires $\chi^2 < 10.0$ while only the intercept is
matched in the $y$-$z$ plane where $|\Delta I_z/\sigma_{Iz}| < 3.0$ is required.

After applying the cuts the $p_t$ distribution is shown in the dashed histogram
in Figure 3.2. One can clearly see the hump in the distribution from W and Z
decay muons. There are 17,672 muon candidates left in the data. Table 3.1 shows
a summary of all the cuts used on the inclusive muon data.
Figure 3.1

A scatter plot of the difference in slope versus the difference in intercept between the extrapolated CTC track and the muon stub. Each of the quantities is divided by the expected $\sigma$ from multiple scattering in the calorimeter.
Figure 3.2

The $p_t$ distribution of the inclusive muon candidates after cuts. The dashed plot is prior to the removal of W and Z bosons.
Table 3.1

The offline cuts applied to the inclusive muon data sample.

<table>
<thead>
<tr>
<th>Use Runs ≥ 17265</th>
</tr>
</thead>
<tbody>
<tr>
<td>Require level 3 Muon trigger</td>
</tr>
<tr>
<td>$p_t &gt; 12.0$ GeV/c</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>Impact parameter &lt; .15 cm</td>
</tr>
<tr>
<td>Hadron Energy &lt; 5.0 GeV</td>
</tr>
<tr>
<td>$# ADC counts ≥ 2$</td>
</tr>
<tr>
<td>$\chi^2 &lt; 10.0$ (of $r - \phi$ match)</td>
</tr>
<tr>
<td>$\left</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>$25 &lt;</td>
</tr>
</tbody>
</table>

$Z \rightarrow \mu^+ \mu^-$ decays are removed when another minimum ionizing central track exists which forms an invariant mass greater than 65 GeV/c² with the muon candidate. $W$ bosons are removed when there is at least 20 GeV of missing energy in the plane transverse to the $z$ axis ($E_t$) and a transverse mass greater than 20 GeV ($M_t = \sqrt{2p_t E_t (1 - \cos(\phi_\mu - \phi_e))}$). These cuts remove about 140 $Z$ bosons and 1280 $W$ bosons from the sample. There is the possibility with the $Z$ boson that one of the muons does not end up in the central calorimeter. When this happens the event...
appears to be a W and is still removed by the cuts.

3.2 The Cut Efficiencies

Determination of the cut efficiencies is aided by three independent sources of muons each of which have advantages and disadvantages. They are $J/\Psi \rightarrow \mu^+\mu^-$ decays, W and Z bosons in the data sample, and cosmic ray runs.

3.2.1 W and Z Bosons

W and Z bosons have the advantage of riding along with the inclusive muon sample; therefore, they have been through all the same code and corrections as applied to the data. However, after cuts there are only 33 Z bosons remaining. Also, nearly all of the muons from W and Z decays have $p_t > 20$ GeV/c whereas the data consists mainly of events near 12 GeV/c. Finally, the decay $W \rightarrow \mu \nu$ tends to produce a very clean event with few tracks other than the muon in the central region. Consequently, the impact parameter is not always well measured and that cut efficiency cannot be determined by W bosons.

3.2.2 Cosmic Rays

Cosmic ray runs were done prior to and after the 1988-89 run at CDF. [29] One can use this sample of guaranteed muons to help determine cut efficiencies. Again, though, this sample is not ideal.

Cosmic rays tend to have a falling spectrum in transverse momentum; however, they will give a flat vertex and impact parameter distribution. Additionally it is not clear that the incoming leg of the cosmic ray will be found with the same efficiency as the outgoing one. This is because the incoming leg loses energy in the calorimeter before reaching the CTC rather than after. In order to use the cosmic ray sample
the vertex was required to be within ±60 cm of the center of the detector along the
beam line and the r-φ position of the track ‘vertex’ be within 1.5 cm of the origin.
We also only use the outgoing leg of the cosmic ray in efficiency studies.

3.2.3 $J/\Psi$ Decays to Muon Pairs

$J/\Psi$ decays are another sample of good muons. Requiring that the $\mu^+\mu^-$ invariant mass be within ±100 MeV/c$^2$ of the $J/\Psi$ mass leaves us with 650 $J/\Psi$'s after
cuts where the muons have $p_t > 4.0$ GeV/c. The only real disadvantage to using
$J/\Psi$ decays is the fact that only low momentum muons are observed. Also, in this
sample, a matching cut in the r-z plane was already applied in order to clean up
the sample.

In summary Table 3.2 lists the offline cuts applied to the data sample and the
efficiencies and the errors associated with them. In those places where multiple
measurements existed from different sources, the weighted average and error was
calculated and is presented in the table. In all of the cases where methods over-
lapped, the efficiencies were consistent within the statistical errors except for the
cut $\chi^2 < 10.0$. We deal with a discrepancy in the results from W muons and cosmic
rays by adding an additional 1% systematic error in quadrature to the efficiency.

3.2.4 The Minimum Ionizing Cut Efficiency

After applying all the previous cuts the energy in the calorimeter tower of the
muon candidate is shown in Figure 4.20 on page 78. Minimum ionizing particles
generally register about 2 GeV in the hadron calorimeter. The cut requiring that
the muon candidate deposit less than 5 GeV of energy in the calorimeter is the only
cut that exhibits a $p_t$ dependence. Also, one cannot derive the efficiency of this cut
using any of the independent data samples previously detailed.
Table 3.2

The weighted average of cut efficiencies where the errors are statistical.

<table>
<thead>
<tr>
<th>Offline Cut</th>
<th>Cut efficiency ($\epsilon_{\text{off}}$)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2 &lt; 10.0$</td>
<td>.883 ± .024 .903 ± .022 .884 ± .009</td>
<td>$W, \cos, \Psi$</td>
</tr>
<tr>
<td>$</td>
<td>\frac{\delta I_a}{\sigma_{I_a}}</td>
<td>&lt; 3.0$</td>
</tr>
<tr>
<td># ADC $\geq$ 2.0</td>
<td>1.0$^{+0.0}_{-0.001}$ .99 ± .002 .993 ± .002</td>
<td>$W, \cos, \Psi$</td>
</tr>
<tr>
<td>HAD $&lt;$ 5.0 GeV, bot.</td>
<td>.910 ± .003 .856 ± .006 .764 ± .012</td>
<td>M.C.</td>
</tr>
<tr>
<td>HAD $&lt;$ 5.0 GeV, charm</td>
<td>.705 ± .005 .549 ± .014 .412 ± .020</td>
<td>M.C.</td>
</tr>
<tr>
<td>$</td>
<td>z_{\text{vert}} - z_{\text{tr}}</td>
<td>&lt; 10.0$ cm</td>
</tr>
<tr>
<td>$</td>
<td>D_0</td>
<td>&lt; .15$ cm</td>
</tr>
<tr>
<td>Total $\epsilon$</td>
<td>.746 ± .028 .728 ± .022 .627 ± .015</td>
<td></td>
</tr>
</tbody>
</table>

An additional ±.01 systematic error is added.
W's, Z's, and cosmic rays are all unsuitable because they tend to be extremely clean events with only one or two tracks in the central region. This means one can be assured only the muon passed through the calorimeter tower used to determine the energy. Even with $J/\Psi$ decays it is not clear the event topology properly represents typical muons from b quarks. Additionally, the average energy of the muons is about 4 GeV which is significantly smaller than in the data. For these reasons it was decided to use a Monte Carlo program and detector simulation to determine the efficiency of the minimum ionizing cut.

The Monte Carlo generator ISAJET was used to produce a large number of $b - \bar{b}$ quark events with $p_t > 20$ GeV/c. These events were then run through the detector simulation QFL which has the latest information on calorimeter performance based on test beam runs in 1991.[30] Figure 3.3 shows the efficiency of the minimum ionizing cut for these Monte Carlo events. The reason a transverse momentum dependence exists is due to the fact that high momentum muons come from high momentum b quarks. As the momentum of the quark increases the jet associated with its own fragmentation and also with the other quarks produced in its decay become more collimated. The muon is then closer to the associated jet and the probability that other particles shower in the same tower the muon enters increases. This means it is more likely the energy in the tower of the muon candidate will exceed 5.0 GeV and fail the cut.

A similar sized sample of $c\bar{c}$ pairs were simulated with ISAJET+QFL. The efficiency of the minimum ionizing cut for muons from charm quarks is shown in Figure 3.3 as well. The lower efficiency for charm muons is due to less isolation from other particles in the event than for bottom muons. The actual numbers are presented along with the errors in Table 3.2 for both bottom and charm quarks whereas the total efficiency is calculated using only the b quark efficiency.
The $p_t$ dependence of the efficiency of requiring less than 5 GeV of energy in the tower of the muon candidate. $b - \bar{b}$ events were used from Isajet+QFL Monte Carlo.
3.3 Other Cuts

The last two entries in Table 3.1 are applied to ensure that the muon candidate is within the fiducial volume of the detector. The position of the event vertex in $z$ ($z_{\text{vert}}$) must be within $\pm 60$ cm of the center of the detector. Additionally, the $z$ position of the muon track at the inner surface of the muon chambers ($|Z_0|$) must be at least $\pm 10$ cm from the ends of the muon chamber in order to avoid situations where a steeply angled track in the $r$-$z$ plane only encounters 2 or 3 of the muon layers in a tower.
4. BACKGROUNDS IN INCLUSIVE MUONS

Various background processes contribute to the overall number of muon candidates in the data, even after the cuts have been applied. The main backgrounds come from three sources: decays of pions or kaons in flight, hadrons that travel through the calorimeter without showering, and hadrons that shower toward the end of the calorimeter but produce a muon trigger nonetheless. Muons from the decay of prompt $c$ or $\bar{c}$ quarks are also a background if one wishes to measure a $b$ quark cross section; this is discussed in chapter 5. Other potential sources of background exist such as muons from $W$ or $Z$ boson decays or cosmic rays, but it will be shown that these sources are small. This chapter will explain just how the backgrounds are estimated and what percentages they contribute to the overall sample.

4.1 Estimating the Decay-In-Flight Background

One of the backgrounds in the muon sample from the 1988-89 run is the decay of kaons or pions to muons which then register in the central muon detector. It is important to understand this background in order to measure the bottom quark cross section using the inclusive muon data sample. This section describes the Monte Carlo method used to determine the decay in flight spectrum and presents the results.

The probability that a relativistic particle of lifetime $\tau$ will decay after a time $t$ in the lab frame of reference is $P(t) = 1 - e^{-t/\tau}$. One can put this expression in a more relevant form for CDF by transforming into variables of particle mass $M$ and
transverse momentum $p_t$.

Calling $r$ the path length the particle traverses, with a velocity $v$ and momentum $p$ in the lab frame of reference, $t = r/v$ while:

$$P(r) = 1 - \exp\left(-\frac{r}{\gamma v}\right)$$

But it is also true that $p/M = \gamma v$. So making the substitution into 4.1 yields:

$$P(r) = 1 - \exp\left(-\frac{rM}{pt}\right)$$

Since $p_t = p\sin \theta$ and $R = r\sin \theta$, the theta dependence cancels and one is left with an expression in terms of the transverse momentum, the particle mass, and the arc-length ($R$) projected in the x-y plane ($\theta$ is referenced from the beam line).

$$P(p_t) = 1 - \exp\left(-\frac{RMc}{p_t ct}\right)$$

The only particles one needs to include in a decay-in-flight calculation are kaons and pions. The reason this is true lies in the fact that all other charged particles are either produced rarely, are prompt, or do not decay into muons with a significant probability.

The momentum spectrum of muons from pion or kaon decay is flat. This is easily demonstrated with a Monte Carlo program, the results of which are shown in Figure 4.1. Muons from K decays have an allowed momentum range of $0.045p_K < p_\mu < p_K$ while for pions it is $0.57p_\pi < p_\mu < p_\pi$. This is also true to a good approximation for transverse momentum for relativistic, central mesons.

Using this information one can calculate the probability distribution for a meson to decay into a muon of transverse momentum $p_{t\mu}$ in $dp_{t\mu}$ before reaching radius $R$. [31]

$$\frac{dP(p_{t\mu})}{dp_{t\mu}} = \int_{p_{t\mu}}^{3.742p_{t\mu}} dp_{t\pi}(1 - \exp\left(-\frac{RMc}{p_{t\pi} ct}\right)) \frac{1}{0.426p_{t\pi}} \text{ (pions)}$$
The momentum distribution of muons that decayed from 16 GeV/c pions and kaons as modeled in a Monte Carlo simulation.

Figure 4.1

\[ dP(p_{t\mu}) = \int_{p_{t\mu}}^{1.2p_{t\mu}} dp_{tK} \left( 1 - \exp \left( - \frac{RMc}{p_{tK}c} \right) \right) \frac{1}{0.954p_{tK}} \times (BR) \text{ (kaons)} \]  

(4.4)

4.1.1 Estimating \( \frac{d\sigma_{\text{decay}}}{dp_t} \)

Since we can calculate the decay probabilities we can also estimate the cross section for observing decay-in-flight muons in CDF. This section will present such an estimate. Later sections will explain the problems with making these kinds of calculations and present a more reliable result using a Monte Carlo.
For a given meson momentum, we must multiply the inclusive charged particle cross section for that momentum by the probability of its decay. A correction for the ratio of kaons to pions and a correction for the fraction of charged particles that are pions should be included as well. This yields a cross section for decay muons as a function of transverse momentum.

\[
\frac{d\sigma_{\text{DIF}}}{dp_{t\mu}} = f \int_{p_{t\mu}}^{0.742p_{t\mu}} \frac{d\sigma_{\text{MBS}}}{dp_{t\pi}} (1 - \exp(-\frac{\alpha}{p_{t\pi}})) \frac{1}{.426p_{t\pi}} dp_{t\pi} + \frac{f}{\pi} \frac{K}{BR} \int_{p_{t\mu}}^{2.9p_{t\mu}} \frac{d\sigma_{\text{MBS}}}{dp_{tK}} (1 - \exp(-\frac{\beta}{p_{tK}})) \frac{1}{.954p_{tK}} dp_{tK}
\]

(4.5)

where \( \alpha = 1.79 \times 10^{-4}R \) and \( \beta = 1.33 \times 10^{-3}R \) represent the factors previously mentioned for the decay probabilities, BR is the \( K \rightarrow \mu \nu \) branching ratio, while \( f \) is the fraction of charged particles which are pions and \( \frac{K}{\pi} \) is the ratio of charged kaons to pions.

CDF has measured the inclusive single particle \( p_t \) spectrum out to 9 GeV/c and provided a functional form that reproduces the data \([32, 33]\). To extend the spectrum to higher momentum, Claudio Campagnari used data from minimum bias, stiff track, and Jet.20 triggers.\([34]\)

The stiff track data from a trigger requiring one track of \( p_t > 5.0 \) GeV/c was compared to the minimum bias spectrum in the region from 6–10 GeV. The cross section of the minimum bias data between 6 and 9 GeV/c is \( \sigma(6 \leftrightarrow 9)_{\text{MBS}} = 25.9 \pm 2.5 \) \( \mu b \) for \( \eta < .6 \) while for the CFT data \( \sigma(6 \leftrightarrow 9) = 23.4 \pm 0.5 \mu b.\([34]\) Similarly in the range from 7 to 12 GeV/c, \( \sigma(7 \leftrightarrow 12)_{\text{MBS}} = 12.1 \pm 1.7 \mu b \) and \( \sigma(7 \leftrightarrow 12)_{\text{CFT}} = 10.4 \pm .5 \mu b. \) Using the ratios of these numbers the minimum bias values were adjusted down to match the stiff track cross section to generate the spectrum below 6 GeV. The CFT value was used because of the limited statistics of the minimum bias data and because the luminosity of the CFT data was better understood. For example, corrections could be made for Event Builder losses in the
Figure 4.2

The total charged track spectrum used in the decay in flight calculation for $|y| \leq 1.0$. The errors shown are statistical. The solid line is a parameterization from the 1987 run fitted to data below 10 GeV/c.
The points show the result of calculating the decay in flight cross section using the minimum bias spectrum and equation 1. The triangles are for $K/\pi = .33$ while the crosses for $K/\pi = .10$. 

Figure 4.3
The Jet.20 data was compared to stiff track data from a trigger requiring a track of $p_t > 9.2$ GeV/c in the range 16–28 GeV. Since the $p_t$ dependences of the samples were found to differ, a function of the form

$$\frac{d\sigma}{dp_T}_{\text{stiff}} = \frac{d\sigma}{dp_T}_{\text{Jet.20}} + \frac{\alpha}{p_T}$$

(4.6)

was used to adjust the Jet.20 data. The best fit occurred for $\alpha = 10.14$ and is limited to $\approx 20\%$ accuracy above 20 GeV/c due to limited statistics in the stiff track data. The Jet.20 data was then used to extend the spectrum past 20 GeV/c where each bin was modified by the function above.

The reason for using the Jet.20 data is to smooth the spectrum of the limited statistics CFT data above 20 GeV/c. By using the Jet.20 data in this manner, the Jet.20 data passes through the CFT data and acquires the CFT normalization. The adjusted Jet.20 data is used above 20 GeV/c. The result of combining all the techniques above is shown in Figure 4.2 with statistical errors only. Also plotted is the functional form of the cross section given in [32] for $p_t$ less than 10 GeV/c (and extended well past its useful range).

It is a relatively simple matter to numerically integrate Equation 4.5 given the single particle spectrum and making some naive assumptions about particle types. Since most of the charged hadrons that reach the calorimeter will shower before they decay, taking $R$ to be 200cm (the distance to the central hadron calorimeter) is a good first order guess. We used $K = .33$ and assumed $K \sim \frac{2 + \bar{f}}{3}$ in order to fix $f$. One then obtains the result shown in Figure 4.3. Summing the contributions greater than 12 GeV yields $\sigma_{>12\text{GeV}} = 4.09$ nb. For the inclusive muon data sample of $\approx 3.7$ pb$^{-1}$ luminosity, this is about 15,000 decays. Also shown is the estimate obtained when $K = .10$ is assumed which is 1.71 nb. The cross section obtained is
about 2.4 times less than when we had a larger fraction of kaons indicating a strong dependence on the $K/\pi$ ratio.

Several items have not been correctly accounted for in the somewhat oversimplified equation 4.5.

- The muon chamber acceptance and the efficiency of the muon trigger for decay muons was not included.

- A decay in the CTC may or may not reconstruct within the allowed physical $p_t$ range of the muon because of the kink in the track.

- The kaon or pion may enter the calorimeter and then decay before showering. The calculation assumes all hadrons entering the calorimeter will neither punch-through nor decay to muons.

- Applying offline track matching cuts and impact parameter cuts will preferentially select prompt muons over decay muons. The efficiency of these cuts must be folded in as well.

- Mesons that shower can produce other mesons that can decay.

Some of the above effects increase and some decrease the final result (though admittedly most of the large effects cause a decrease). In order to obtain a more precise estimate we have been employing a Monte Carlo method of determining the decay in flight spectrum.

4.2 The Monte Carlo Method

Because of the difficulties in making a calculation to accommodate the previously mentioned effects, a Monte Carlo method was developed for the 1987 run by Dave Smith to help deal with them [21]. What follows is a review of the method and the results of the detector simulation on kaon and pion decays in flight.
4.2.1 Simulating the Decays

Prior to simulation, one first runs a program that generates a pion (kaon) at a given transverse momentum. Then, a subroutine forces the pion (kaon) to decay into a muon before the muon chamber. This routine takes into account dE/dx of the meson in the calorimeter prior to decay and produces the probability of decay and decay path-length as outputs. This is repeated for a large number of pions (kaons).

These single tracks are then passed through the detector simulation in order to model the detector and trigger effects and to generate CMUO\(^*\) data banks. The simulation steps the kaon or pion through the detector, comparing its current position to the path-length. When its current position exceeds the path-length, the particle is forced to decay unless it has already showered in the calorimeter. Decays of shower products have been ignored in this study.

4.2.1.1 Effects of Tracking and Cuts

Five thousand single tracks were generated at each integer value of \(p_t\) from 6 to 31 GeV/c with \(|\eta|<.8\) for both kaons and pions for a total of 260K tracks. Even though kaons have a higher probability of decaying in the active detector volume, fewer kaons reconstruct as CMUO objects.

This is illustrated by Figure 4.4 which shows the reconstructed transverse momentum of all the CMUO's from kaons and pions. Only 1718 muons from pions and 861 muons from kaons with an original energy of 15 GeV/c are reconstructed at all. The wide distributions of both types of particles in Figure 4.4 is due to two effects. One is simply the momentum spread due to a decay which occurs before the meson enters the CTC. The other occurs when the parent decays within the CTC.

\*A CMUO occurs when a track in the muon chamber is found to correspond to a track in the central tracking chamber within 15 degrees.
Comparison of how kaon and pion decays reconstruct in the detector. The solid line represents decay muons from 15 GeV/c pions while the dashed line represents decay muons from kaons of the same parent momentum.
Table 4.1

The table shows the offline cuts applied to the Monte Carlo events after simulation.

<table>
<thead>
<tr>
<th>Offline cuts</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>z_{\text{track}} - z_{\text{vertex}}</td>
</tr>
<tr>
<td>impact parameter</td>
<td>(&lt; .15 \text{ cm} )</td>
</tr>
<tr>
<td># of ADC hits used in stub</td>
<td>(\geq 2.0 )</td>
</tr>
<tr>
<td>(\chi^2_z)</td>
<td>(&lt; 10.0 )</td>
</tr>
<tr>
<td>(</td>
<td>\delta I_z/\sigma I_z</td>
</tr>
</tbody>
</table>

The tracking code can then actually straighten the track on occasion producing the unphysical points above 15 GeV/c. The wide distribution has a sharp peak from well measured tracks from particles that decayed after the CTC.

The offline cuts are then applied to the resulting central muon tracks. The cuts that have a significant effect on the decay mesons are the impact parameter \((D_0)\) and track-stub matching cuts. The cuts are summarized in Table 4.1. All of these cuts are more extensively described in chapter 3.

Since there is a kink in the track, the chance of a decay muon having a measurable impact parameter is quite high. Figures 4.5 and 4.6 show the effect of the impact parameter and stub matching cuts for pions and kaons respectively. These figures are plots from the simulation showing the radius at which 14–19 GeV/c parent mesons decayed into muons producing CMUO data banks. The solid histogram in Figures 4.5 and 4.6 has no offline cuts applied. The short-dashed histogram in both
The radius from the beam line at which the parent pion decayed. Parent particles from 14 to 19 GeV are included. The cuts applied to the histograms are described in the text.

Figures 4.5 and 4.6 indicates the radius of decay after the impact parameter cut $|D_0| < 0.15$ cm has been applied. As expected for any muon cut, we observe that this cut is not effective for mesons which decay close to the interaction point.

Using the expected multiple scattering of a minimum ionizing particle in the calorimeters, an expected variance ($\sigma$) of events in slope or intercept is calculated.[27] Using this, a $\chi^2$ value is assigned to the $x$-$y$ track which is a measure of how well the stub matches the track in that plane.

The long-dashed histogram in Figures 4.5 and 4.6 shows the effect of cutting on $\chi^2$, impact parameter, and on the match of the intercepts ($\delta I_x/\sigma_{I_x}$) in the $y$-$z$ plane.
Decay radius for parent kaons. Note the increased effectiveness of the cuts due to the larger transverse 'kick' given the muon.

weighted by the variance as calculated in reference [27]. The track-stub matching cuts are particularly effective at eliminating kaons that decay toward the end of the CTC and in the first layers of the calorimeters (Figure 4.6).

4.2.2 Obtaining $\frac{d\sigma_{\text{decay}}}{dp_t}$ from Monte Carlo

After collecting the sample of decay muons, one must calculate the decay in flight cross section for a given parent momentum distribution. Since there was a muon trigger for the 1988-89 run and since large numbers of kaons and pions are produced in normal QCD events, the parent momentum distribution would be the
total charged particle cross section \( (d\sigma_{\text{MBS}}/dp_t \text{ as shown in figure 4.2}) \). When the quantities below are defined:

\( j \) - The momentum index of the parent particle as generated by FAKEV. \( j = 6, 7, ..., 31 \) for this analysis.

\( p_{ti} \) - The original transverse momentum of the meson. This is always of integer value equal to \( j \).

\( i \) - The momentum index of the reconstructed muon from CDFSIM in 1 GeV/c bins.

\( p_{ti} \) - The muon transverse momentum in bin \( i \) as reconstructed in CDFSIM and 5.1 tracking.

\( d_j^{\text{K}(\pi)} \) - The probability that a kaon (pion) of transverse momentum \( p_{ti} \) decays before reaching the muon chamber.

\( N_j^\pi(p_{ti}^\pi) \) - The number of pions with parent transverse momentum \( p_{ti} \) which reconstruct with transverse momentum \( p_{ti} \). (i.e. lie in the \( i^{\text{th}} \) bin)

\( \frac{K}{\pi} \) - The ratio of kaons to pions in the total charged particle spectrum.

\( F \) - The fraction of charged particles which are pions in minimum bias events.

\( \text{BR} \) - The branching ratio of kaons to muons.

\( \epsilon_T \) - The efficiency of the level 1 and level 2 muon triggers. Level 3 is handled inside the simulation.

\( \epsilon_{\text{off}} \) - The efficiency of the offline cuts to muons since single tracks in CDFSIM are ideal cases.

\( \eta_{\text{MC}} \) and \( \eta_{\text{MBS}} \) - The pseudorapidity ranges of the Monte Carlo generated events and the minimum bias cross section respectively.

Then the full formula for calculating the decay in flight cross section is given by [21]:

\[
\left. \frac{d\sigma_{\text{DIFF}}}{dp_t} \right|_{p_t=p_{ti}} = \frac{\epsilon_T \epsilon_{\text{off}}}{5000} \sum_j \left[ \text{BR} \times d_j^{\text{K}} N_j^{\text{K}}(p_{ti}^\text{K}) \frac{K}{\pi} + d_j^{\pi} N_j^{\pi}(p_{ti}^\pi) \right] \\
\times F \times \left( \frac{\eta_{\text{MC}}}{\eta_{\text{MBS}}} \right) \left. \frac{d\sigma_{\text{MBS}}}{dp_t} \right|_{p_t=p_{ti}}
\]

(4.7)
$\epsilon_T$ has been measured to be $90 \pm 2\%$ by Alain Gauthier [24]. The author has measured $\epsilon_{\text{eff}} = .722 \pm .032$ using muons from W and Z boson decays, cosmic rays, and $J/\Psi$'s. The factor $\eta_{MC}/\eta_{MBS}$ is needed to account for the fact that the particles generated by FAKEV were in the range $|\eta| < .8$ in pseudorapidity while the inclusive charged particle cross section from [32] is valid for $|\eta| < 1.0$. The $K/\pi$ ratio is less well known. The quantity is an increasing function at transverse momenta less than 1.4 GeV but seems to become flat at higher momentum. The ratio is bounded on the low side by $.25$ by the C0 [35] experiment and on the high side by estimates using fixed target data (see Appendix A). The range was varied from $.1 < K/\pi < .33$ and yielded only a 3.5% variation in the final result. This is an overestimate so we use $\pm 3\%$ as the systematic error for variations in $K/\pi$. Figures 4.7 and 4.8 demonstrate why large variations in $K/\pi$ do not effect the decay-in-flight cross section significantly. The number of events after cuts from kaons and pions are similar leaving the total cross section insensitive to the relative fraction of each particle. A value of $K/\pi = .28$ was chosen for the final muon cross section. Additionally, it is assumed $(p + \bar{p})/\pi$ is 4% (see Appendix A).

### 4.2.2.1 Error estimates

Listed in Table 4.2 are what are believed to be the largest of the statistical and systematic errors. The $K/\pi$ ratio was varied and it has been shown that the calculation is insensitive to this quantity. Related to this is the value chosen for $(p + \bar{p})/\pi$. Using the results from reference [36] and Appendix A, one obtains $(p + \bar{p})/\pi = .04 \pm .01$. This error translates to a .7% systematic variation in the final decay cross section.

The absorption lengths used in CDFSIM do not differ between particles of opposite sign [37]. This causes a systematic error particularly in the case of the positively
Figure 4.7

The simulated decay in flight cross section after all cuts (except the minimum ionizing cut) and corrections have been applied.
The simulated decay in flight cross section after all cuts and corrections have been applied for $K/\pi = .1$. 

Figure 4.8
Table 4.2

The table shows the sources of error and their contribution to the uncertainty in the decay-in-flight cross section for all events with $p_t > 12$ GeV/c.

<table>
<thead>
<tr>
<th>Error Estimates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/\pi$ ratio</td>
<td>±3% (sys)</td>
</tr>
<tr>
<td>Min-bias spectrum</td>
<td>±3% ± 7% (stat+sys)</td>
</tr>
<tr>
<td>Trigger eff.</td>
<td>±2% (stat)</td>
</tr>
<tr>
<td>offline cuts</td>
<td>±2.2 ± 1.0% (stat+sys)</td>
</tr>
<tr>
<td>$\lambda_{ABS}$ in CDFSIM</td>
<td>±2% (sys)</td>
</tr>
<tr>
<td>CDFSIM tracking</td>
<td>±4% (sys)</td>
</tr>
<tr>
<td>$(p + \bar{p})/\pi$</td>
<td>±.7% (sys)</td>
</tr>
</tbody>
</table>
charged kaon. In order to account for this, the simulation was modified to correct the absorption lengths based on information in reference [28]. The quoted error is estimated from the differences in the numbers of reconstructed events between the two simulations. Several bins of kaons were tested from 6 to 20 GeV/c with this simulation. The numbers of kaons surviving the cuts increased by about 5% in each bin tested. This has a two percent effect on the overall decay-in-flight cross section and the quoted number is corrected for this factor. The 2% correction was made in the final cross section and added in quadrature as a systematic error.

Additionally, tracking in CDFSIM is always better than in reality [38] and a correction with its associated errors must be included. Another modified version was run where the hit efficiency in the CTC for real tracks was imposed on the wires. This was tuned so that the shape of the reconstructed hit efficiency was the same as in the inclusive muon data sample after the cuts. This caused a 4% drop in the number of tracks reconstructed as muons after the cuts were applied. (The definition of hit efficiency used here is the same as in [38]. For a given sense wire, the number of hits in the CTC used after reconstruction is divided by the total number possible.)

The contribution from meson decays after showering is assumed to be negligibly small given the effectiveness of the track matching cuts at screening out decays after the CTC.

4.2.2.2 Minimum ionizing cut efficiency

Because of the fact that the simulation used only single particles, it does not properly simulate the effect of the minimum ionizing cut.* Jet structure can result

---

*The minimum ionizing cut requires the energy deposited in the hadron calorimeter be less than 5.0 GeV in the tower entered by the muon candidate.
in more than one particle entering the tower containing the muon candidate. Additionally, it is also probable that a shower in a neighboring tower can cross tower boundaries. Both of these effects can cause additional energy deposition in the candidate's tower and potentially effect the cut efficiency.

The cut efficiency will be highly dependent on the type of events as well. If a typical decay event occurs within a high energy jet, one would expect a completely different dependence of the minimum ionizing cut on transverse momentum than if a typical decay occurs with only one really stiff track. For this reason it is very important to try to accurately model the event topology as best one can in simulating the effect of the minimum ionizing cut.

The level 3 muon trigger requires a stiff track in the CTC in addition to a loosely matching stub in the muon chambers. The data taken with the CFT trigger only requires a stiff track in the CTC, so one would expect this sample would provide the correct event structure for decay hadrons. There are only 4 CMUO's in this sample, however, passing all the cuts.

To overcome the problem of the size of the CFT data sample it was used as the basis of a Monte Carlo sample. All tracks with $p_t > 12$ GeV/c were labeled muons and the rest were labeled pions for each event. The momentum and position of each track was retained. These generated tracks were then run through a detector simulation (QFL) and the cuts applied. Figure 4.9 shows the results indicating the cut efficiency is around 90% for the range of interest and is flat in $p_t$.

4.2.3 Results for Decays-in-Flight

When the effect of the minimum ionizing cut is included, the results are shown in Table 4.3 with statistical and systematic errors. Using a value of integrated luminosity for the MUO4 stream of 3.79 pb$^{-1}$ [39] one can obtain the number of
Figure 4.9

A plot showing the efficiency of the minimum ionizing cut for Decays-in-flight.
Table 4.3

The decay-in-flight cross sections at the muon chambers after the cuts have been applied. $K/\pi = .28$ and $p + \bar{p}/\pi = 4\%$ are assumed. The 7% systematic from the luminosity is not included here as it is correlated with the luminosity systematic in the data sample.

<table>
<thead>
<tr>
<th>Decay-in-flight Cross section</th>
<th>$\delta\sigma_{DIF}$ per 5 GeV/c bins</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 - 17</td>
<td>$0.881 \pm 0.051 \pm 0.048$ nb</td>
</tr>
<tr>
<td>17 - 22</td>
<td>$0.130 \pm 0.029 \pm 0.007$ nb</td>
</tr>
<tr>
<td>22-27 GeV/c</td>
<td>$0.034 \pm 0.018 \pm 0.002$ nb</td>
</tr>
</tbody>
</table>

decay muons expected in each bin of interest.

4.2.3.1 Checking the impact parameter

There is always the fear when using a purely Monte Carlo method in an analysis that the model is wrong in some subtle but fundamental way and the data are not being well simulated. Consequently, it is beneficial to attempt some reasonable checks on Monte Carlo methods if possible.

One possible problem with this calculation involves the fact that only single tracks were used in each event. This was necessary to collect a statistically significant sample of decay particles for the calculation, but it leaves open the question of how the impact parameter was calculated. In the Monte Carlo, the event vertex was placed and smeared externally based on real data events. The tracking code
measured an impact parameter from this "placed" vertex. Since this in no way actually simulates how an event vertex is found from the data, there was concern that this could make the impact parameter cut unrealistic in the simulation.

In order to determine if, in fact, the simulation was valid, a second Monte Carlo was performed. In this second simulation each event from the CFT trigger was stripped of all charged particles except the four of highest momentum. Any particle with $p_t > 9.5$ GeV/c was labeled a kaon(pion) and forced to decay before entering the muon chambers as described earlier in section 4.2.1. The transverse momentum of the particles was retained but all other quantities randomized. In this way one can rerun the simulation using the same starting data but changing random number seeds. Including the other tracks provides the Monte Carlo with a realistic event vertex. The vertex finding code was then employed normally, and no ad hoc vertex was necessary.

The results agree to within the errors and are shown in Figure 4.10. The dashed values are the results of the Monte Carlo method outlined previously while the solid curve is from the simulation that allowed multiple tracks.

4.3 Hadronic Backgrounds

Because hadrons are so copiously produced in QCD processes, a significant portion of the background is due to charged hadrons faking a muon. There are two different types of processes in the calorimeter which might cause the spurious identification of a hadron as a muon. One is non-interacting punch-through (NIP) where the charged hadron simply passes through the steel in the central calorimeter without showering, thus depositing only the energy associated with a minimum ionizing particle. The other occurs when a hadron showers late and one of the shower particles passes the matching cuts between the CTC track and the muon
The decay-in-flight cross section after cuts and with no minimum ionizing cut included. The dashed values with their associated errors are from the single track Monte Carlo while the solid line is from the multi-track simulation. The errors on the solid curve (not shown) are similar in size to the dashed values.
stub. The punch-through of minimum ionizing hadrons is a particularly insidious background and will be treated first followed by an estimate of the leakage background from showering hadrons. As will be shown, the method used to estimate the non-interacting punch-through also includes the leakage background. A consistency check is made using the measured charge asymmetry of the muon candidates.

4.3.1 A Fit Method to Measure Punch-through Background

Hadrons that fail to shower in the steel of the calorimeter are indistinguishable from muons on an event by event basis. Hence, a pion or kaon could trigger data acquisition as a muon. While it is impossible to apply a cut that will preferentially remove minimum ionizing hadrons, it is possible to measure the percentage of background they represent and correct for that percentage in the final cross section statistically.

The probability that a hadron will pass through $\lambda$ absorption lengths of material without interacting inelastically is:

$$ P = e^{-x\rho/\Lambda_{abs}} = e^{-\lambda} $$

$x$ and $\rho$ are the length and density of the material. The value of the exponent is the number of mean free paths in the detector with regard to inelastic nuclear interactions. The absorption length ($\Lambda_{abs}$) is related to the material and the absorption cross section by

$$ \Lambda_{abs} = \frac{N_A\sigma_{abs}}{A} \quad (4.8) $$

where $A$ is the molecular weight of the material, $N_A$ is Avogadro’s number, and $\sigma_{abs}$ is the absorption cross section of the hadron on the material.

$\sigma_{abs}$ is available from the literature. However, since $\frac{EP}{F} = \lambda(\delta\sigma_{abs}/\sigma_{abs})$, a small error in the absorption cross section translates into a large error in one’s knowledge
of the punch through probability. The absorption cross sections for kaons, pions, and protons were measured in the relevant momentum range by Denisov et. al. [40, 41] on the types of materials used in the construction of the CDF calorimeters. The Particle Data Group provides a parameterization formula for the absorption cross section of hadrons on proton targets as a function of particle momentum. [28]

Normalizing the parameterizations from the Particle Data Group to the absorption cross sections from Denisov allows one to plot the absorption lengths for a given hadron at normal incidence to the calorimeter as a function of hadron energy. This is shown in Figure 4.11. The main contribution to NIP is therefore going to come from positive kaons, with negative kaons and pions making up the rest. The proton and antiproton fractions of the punch-through will be negligible as they more strongly interact with the steel in the calorimeter.

Another point that should be stressed about Figure 4.11 is the difference between the positive and negative kaon absorption lengths. The reason for this is the strange quark in the $K^-$ can form $\Lambda$ or $\Sigma$ baryons in collisions. The $K^+$, containing an $\bar{s}$ quark cannot form these baryons with normal matter. Since those processes are not available to the $K^+$, the absorption cross section is smaller.

4.3.1.1 The Fit Method on Noninteracting Punch-through

As a hadron's angle of incidence in the calorimeter changes from normal, the hadron passes through more steel. Thus $x$ in Equation 4.8 increases and the amount of material traversed by the particle goes as $1/\sin \theta$ where $\theta$ is the polar angle. Specifically, the probability that a hadron encountering $\lambda$ absorption lengths at normal incidence will pass through the calorimeter without showering is:

$$P = \exp \left( -\frac{\lambda}{\sin \theta} \right). \quad (4.9)$$
Absorption Lengths At Normal Incidence

Figure 4.11

Thickness of the CDF calorimeter in absorption lengths for particles at normal incidence.
However, since a muon only minimally ionizes in the calorimeter, it will pass through relatively unhindered. If one compares samples of hadrons and muons produced with flat distributions in pseudorapidity, the number of noninteracting hadrons emerging from the calorimeter would fall rapidly in $|\eta|^*$ (because of 4.9) while muon distribution would remain flat. It is this fundamental difference in the expected shapes of the two distributions that provides a handle on the minimum ionizing hadronic background.

If the muon candidates were all produced at the center of the detector and if the muon chambers had no cracks in $|\eta|$ it would be a simple matter to fit the expected shape using terms from (4.9) in addition to a nearly flat distribution for muons to the data and estimate the relative numbers of punch-through versus prompt muons. However, a couple of factors put an additional shape on the particle distributions in $|\eta|$.

First, there are purely geometrical factors. The muon chambers are of finite size and only cover approximately $-.6 < \eta < .6$ (see Figure 2.5 on page 19). They also have a gap at $90^\circ$ to the beam line of about 15 cm. Additionally, particle interactions do not in general take place at the center of the detector; rather, interactions are gaussianly distributed about the center of the detector with a variance of 29 cm. These facts mean that it is possible for a given muon candidate to have $|\eta| > .6$ and still intercept the muon chamber. By using a Monte Carlo simulation of the detector it is possible to get the geometrical acceptance of the muon chambers as a function of $\eta$. The data can then be corrected for the fiducial effects.

Figure 4.12 shows the geometrical acceptance of simulated muons with the statistical errors as well as the stub finding efficiency of the muon code. 200,000 single muon tracks were generated in the detector simulation CDFSIM with flat $\eta$ and $\phi$

* $\eta = \ln \cot \theta/2$
This plot shows the muon chamber efficiency as a function of $|\eta|$ for muons which were produced flat in pseudorapidity and passed through a detector simulation. A cut required the candidate to pass at least 10 cm from the ends of the muon chamber to remove edge effects.
distributions. The plot shows that the acceptance as a function of $\eta$ has a plateau at 77% and rapidly falls near the edges of the muon chambers. The $\phi$ cracks alone between the muon chambers reduce the acceptance to 84% \cite{42}; however, other detector cuts such as requiring that the event vertex lie within $\pm 60$ cm of the detector center are also modeled. The absolute value of the acceptance has no effect on the results of an $|\eta|$ dependent fit, which depend only on the shape of Figure 4.12, but these other effects were included so the figure would yield information on the overall detector acceptance as well. A histogram showing the data prior to correcting for these effects is shown in Figure 4.13.

The second factor that had to be considered was the production $\eta$ distribution for hadrons and muons. For hadrons and decays-in-flight the expected flatness in $\eta$ was checked using the data from the Central Fast Track trigger. The CFT trigger simply requires that a stiff track with $p_t > 9.2$ GeV/c be in the CTC. The CFT trigger is an unbiased trigger for the high-rate QCD processes which produce charged hadrons. Figure 4.14 shows the $\eta$ distribution for all charged tracks that satisfied the trigger above a transverse momentum of 12 GeV/c. Using a least squares fitting program \cite{43}, the plot was fit to an arbitrary second order polynomial of the form $P1(1 + P2 \times \eta + P3 \times \eta^2)$, and the $\chi^2$ reported here is per degree of freedom (there are 23 degrees of freedom). As one can see, the best fit multipliers to the linear and squared portion of the function are consistent with zero with $P2 = .058 \pm .46$ and $P3 = -.011 \pm .48$ confirming the expected flatness in pseudorapidity for charged hadrons.

The expected $\eta$ distribution for prompt muons was determined from a Monte Carlo sample of prompt muons from B meson decays since most of the prompt muons with high transverse momentum will originate from B mesons.\cite{44} To do this, the Isajet Monte Carlo was run using a B meson decay simulation tuned to the results
The inclusive muon data after cuts as a function of $\eta$ prior to correcting for acceptance shaping effects.
The $\eta$ distribution of tracks satisfying a stiff-track trigger. The trigger requires a track in the CTC with a transverse momentum of at least 7 GeV/c. Plotted tracks have $p_t > 12$ GeV/c.

from the CLEO B experiment.[45] The results are shown in Figure 4.15. In this case there is a slope to the $\eta$ distribution which is well modeled by a straight line of slope $= -0.17 \pm 0.02$. Apart from geometrical acceptance, one expects this distribution to remain virtually unchanged in detector simulation as there is little chance of a 12 GeV muon stopping in the calorimeter. The contribution from decays-in-flight and charm will be discussed later.

The inclusive muon data was examined as a function of $\eta$ and corrected for the geometrical efficiency as given in Figure 4.12. Fitting the data to the linear
The $\eta$ distribution of muons from the Isajet Monte Carlo with a Monte Carlo tuned to the CLEO data to model the B meson decays. No detector simulation has been performed.

distribution from the b-quark Monte Carlo plus the exponentially decreasing form expected for punch through hadrons yields the relative numbers of prompt or decay muons versus the punch through or leakage hadrons. The plot showing the results of this fit is Figure 4.16 where $P1$ and $P2$ represent the relative contributions to the total curve for hadrons and muons respectively. Figure 4.16 contains muon candidates with transverse momenta between 12 and 17 GeV/c. The figures that follow show the same curves for $p_t$ ranges of 17−22 GeV/c and 22−27 GeV/c. The
Fit to the percentage of hadronic background in the inclusive muon data after all the cuts have been applied. $P_2/(P_1 + P_2) = \text{the fraction of muons in the data.}$ The dashed and dotted curves explicitly show the muon and punch-through contributions to this fit.
Figure 4.17

Fit determining the percentage of hadronic background in the inclusive muon data for $17 < p_{\mu} < 22 \text{ GeV/c}$. $P2/(P1 + P2) =$ the fraction of muons in the data.
Figure 4.18

Fit results determining the percentage of hadronic background in the inclusive muon data for $22 < p_{t\mu} < 27$ GeV/c. This $p_t$ range is consistent with having no muons.
function used to represent the hadron fraction is given by equation 4.10.

\[
\left(\frac{K/\pi}{1 + K/\pi + \frac{p^+/\pi}{p^-/\pi}}\right) \\
\times \left(\exp\left(-\frac{\lambda_{K^+}}{\sin \theta}\right) + \exp\left(-\frac{\lambda_{K^-}}{\sin \theta}\right)\right) \\
+ \left[\frac{1}{1 + K/\pi + \frac{p^+/\pi}{p^-/\pi}}\right] \\
\times \left(\exp\left(-\frac{\lambda_{\pi^+}}{\sin \theta}\right) + \exp\left(-\frac{\lambda_{\pi^-}}{\sin \theta}\right)\right) \\
+ \left[\frac{(p + \bar{p})/\pi}{1 + K/\pi + \frac{p^+/\pi}{p^-/\pi}}\right] \\
\times \left(\exp\left(-\frac{\lambda_{p^+}}{\sin \theta}\right) + \exp\left(-\frac{\lambda_{p^-}}{\sin \theta}\right)\right)
\] (4.10)

Where \(\lambda_{\pi}, \lambda_{K^-}, \ldots\) are the absorption lengths for each possible punch-through particle at normal incidence. The function used for the muons is given by the simple straight line.

\[1 - 0.173 \times \eta\] (4.11)

Both of these equations are multiplied by a normalizing constant in such a way that \(P1\) and \(P2\) can be directly compared. For the case where the muon candidates have transverse momentum greater than 12 GeV/c, this fitting procedure indicates that 52.3% of the sample consists of prompt or decay muons. The rest of the data is due to hadron punch-through and leakage background. Table 4.4 summarizes the results and statistical errors for the data broken into 5 GeV/c bins in transverse momentum. Note that this muon fraction includes decays-in-flight.

### 4.3.1.2 Fit method systematic errors

In order to try to estimate the systematic errors of the fitting procedure the slope of the straight line and the \(K/\pi\) ratio were varied from their nominal values. This was done on a subset consisting of 3/4 the total data where the median values for
Table 4.4

The table shows the percentage of muons as determined by fitting the expected distributions of muons and hadrons as functions of pseudorapidity.

<table>
<thead>
<tr>
<th>$p_t$ bin</th>
<th>muon fraction ($f_\mu$)</th>
<th>fit error (stat)</th>
<th>Total # of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-17 GeV/c</td>
<td>.55</td>
<td>±.01</td>
<td>14,667</td>
</tr>
<tr>
<td>17-22 GeV/c</td>
<td>.47</td>
<td>±.04</td>
<td>2,418</td>
</tr>
<tr>
<td>22-27 GeV/c</td>
<td>.07</td>
<td>±.10</td>
<td>554</td>
</tr>
</tbody>
</table>

these quantities were chosen as, slope = −.17 and $K/\pi = .23$. The sample included all muon candidates passing the cuts with $p_t > 12$ GeV/c. The value of slope was determined by a $b\bar{b}$ Monte Carlo. The $K/\pi$ ratio was chosen close to the value measured by the C0 experiment.

The slope of the muon function was then varied by ±12% while $K/\pi$ was held fixed. Variations of 12% were chosen because this corresponds to the difference in the slope of the fitted line when the $\eta$ range was extended to 1.0 from .8 in the Monte Carlo.

Similarly, $K/\pi$ was allowed to vary from .1 to .33 while the slope was held fixed. $K/\pi = .28$ is about the ratio one would expect using results from fixed target experiments and the C0 experiment at the Tevatron (see Appendix A).

In both these cases, the variation in the final fit fraction was ±.01. Table 4.5 shows the effect on the fraction of muons when these variations are made.

Another systematic involves the kaon absorption cross section. The variation
Table 4.5

The variation in the fraction of muons as a function of the $K/\pi$ ratio and the slope of the line used to represent the muon distribution in $|\eta|$. These numbers were taken with $3/4$ of the data. The $\chi^2$ of the fit is per degree of freedom and $(p+\bar{p})/\pi = 4\%$.

<table>
<thead>
<tr>
<th>Muon fraction for various fit parameters</th>
<th>$K/\pi = .23$</th>
<th>$K/\pi$</th>
<th>$\chi^2$</th>
<th>Muon fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>$\chi^2$</td>
<td>Muon fraction</td>
<td>$\chi^2$</td>
<td>Muon fraction</td>
</tr>
<tr>
<td>-.15</td>
<td>.77</td>
<td>.505</td>
<td>.1</td>
<td>.799</td>
</tr>
<tr>
<td>-.17</td>
<td>.781</td>
<td>.512</td>
<td>.23</td>
<td>.781</td>
</tr>
<tr>
<td>-.20</td>
<td>.79</td>
<td>.522</td>
<td>.33</td>
<td>.77</td>
</tr>
</tbody>
</table>

in this cross section is about $\pm 3\%$ according to reference [40]. Imposing such a variation on the hadron function causes about $0.006$ variation in the muon fraction. Increasing this variation to $\pm 5\%$ increases the uncertainty to $\pm 0.01$. This is shown in Table 4.6 along with the $\chi^2$ per degree of freedom for the fits. A $5\%$ variation is used to account for uncertainties in the amount of material between the CTC and the muon chambers.

A systematic error was applied to account for the fact that the $\eta$ distribution for decay muons is flat but the fit was performed with a sloping function. The fit was redone with a third flat term in $\eta$ which, combined with the curve from the Monte Carlo, gave a muon fraction of $53\%$. Combining all these errors in quadrature fixes the systematic error due to the fit method at $\pm 0.028$.

Finally it was not clear that the fit would remain stable for the small muon
The absorption cross section of positive kaons was varied by up to ±5%. The data used includes muon candidates with $12 < p_t < 17 \text{ GeV/c}$.

Table 4.6

<table>
<thead>
<tr>
<th>Variation in $f_\mu$ with Kaon absorption cross section</th>
<th>$\Delta \sigma_{ABS}$</th>
<th>$\chi^2$ per DOF</th>
<th>% muons+DIF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+5%</td>
<td>.692</td>
<td>55.9%</td>
</tr>
<tr>
<td></td>
<td>+3%</td>
<td>.691</td>
<td>55.5%</td>
</tr>
<tr>
<td></td>
<td>nominal</td>
<td>.689</td>
<td>54.9%</td>
</tr>
<tr>
<td></td>
<td>-3%</td>
<td>.687</td>
<td>54.2%</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>.687</td>
<td>53.7%</td>
</tr>
</tbody>
</table>

Slope = −.17 and $K/\pi = .28$
fractions and low statistics in the last bin. A program was written which generated events randomly based on the two distributions (Equations (4.10) and (4.11)). One enters the number of hadrons and muons and the program generates a histogram using the assumed distributions. This program was run repeatedly and the results of the fit program recorded for distributions with 600 hadrons and no muons in order to simulate the statistics in the last bin. The variation of the fit to low statistics and small muon fractions was $\delta f_\mu = \pm 0.1$. The statistical error on the data was also $\pm 0.1$ consequently, no additional uncertainty was indicated for the 22–27 GeV/c bin.

4.3.1.3 Leakage background

Leakage background occurs when a kaon or pion showers late in the calorimeter and a secondary particle gives an acceptable stub in the muon chamber. The leakage background is the main contributor to the long tail seen in Figure 4.20 (the distribution of hadronic energy for the muon candidates). Because there are large fluctuation in the calorimeter energy at energy scales near 12 GeV, some of the leakage events have energies which actually sit under the minimum-ionizing peak in a histogram of the calorimeter energy for muon candidates. In addition, hadronic energy may not be fully contained for late interacting particles.

If one chooses not to believe these arguments, however, one can attempt to subtract off the leakage background and re-perform the fit done in the previous section. Using the hadronic energy distribution from the Isajet+CLEO b quark Monte Carlo and a detector simulation (in this case QFL), one obtains the hadron energy histogram for prompt muons. This is shown in Figure 4.19 for muons from b quarks. The actual Landau tail is quite small in size on this scale. [46] The tail evidenced in Figure 4.19 is due to other particles either sharing their energy with the same calorimeter tower as the muon or other particles actually showering in the
same tower.

Using Figure 4.19 in conjunction with a parabolic function it is possible to determine the leakage background using a fit to the data. The results are shown in Figure 4.20. \( F(x) \) (Equation 4.12) is merely the function used to parameterize Figure 4.19 and is essentially a step function multiplied by two falling exponentials. The parameter \( \sigma \) in \( F(x) \) was chosen for the best fit with Figure 4.19 giving a \( \chi^2 \) per degree of freedom of about 1.7.

\[
F(x) = (e^{-1.5x} + 0.015e^{-3.3x}) * \frac{1}{\sqrt{\pi \sigma}} \int \exp \left( \frac{(y - 1.6)^2}{2\sigma^2} \right) dy
\]  

(4.12)

The parabola multipliers are given by \( P1 \) and \( P2 \) and represent the leakage contribution to the overall data plot. Using this fit, an 8% leakage background is measured for hadronic energies < 5.0 GeV.

Based on the Monte Carlo of Figure 4.19, there are few minimum ionizing particles above 6 GeV in the data. One can therefore use the \( \eta \) distribution of those events that pass all the cuts but have deposited more than 6 GeV in the calorimeter to represent the shape of the leakage distribution in the final sample. When the shape of the leakage background is subtracted from the pseudorapidity distribution of the data at the 8% level and the fraction of muons in the final sample recalculated; one obtains 53.1% muons as opposed to 52.3% mentioned earlier (page 71). This discrepancy is quite small and confirms the previous arguments that the leakage background is properly modeled in the fit. This difference was included in the systematic errors from the previous section.

### 4.3.2 Charge Asymmetry as a Consistency Check

Figure 4.11 on page 61 indicates there should be an asymmetry in the total charge of the muon candidates. Since, as has been claimed earlier, almost half the
The plot shows the energy deposition of muons from the Isajet Monte Carlo with an additional decay procedure tuned to the CLEO data. The events were passed through a detector simulation (QFL).
The plot shows the energy deposition of muons from the data in the tower of the muon candidate. The cut placed on calorimeter energy is 5 GeV. The fit uses a parameterization of the previous figure to represent minimum ionizing particles and a parabolic function to represent leakage.

\[ P_1 - P_2 \times (x - 7.5)^2 + P_3 \times F(x) \]

\[ F(x) = (e^{-1.3x} + 0.015 e^{-2.3x}) \times (1/\sqrt{\pi}) \times \exp(- (y-1.6)^2/2 \sigma^2) \times dy \]
final sample is hadronic background, some of those hadrons will be kaons. Because the $K^+$ traverses fewer absorption lengths in the calorimeter, it is more likely a positive kaon will pass through the calorimeter than a $K^-$. 

In fact, there is a charge asymmetry in the muon data. The asymmetry is measured as:

$$A = \frac{N^+ - N^-}{N^+ + N^-} = 0.079 \pm 0.007$$ \hspace{1cm} (4.13)

where the error quoted is statistical.

It is possible to write an expression for the charge asymmetry as a function of: the $K/\pi$ ratio ($r$), the total number of muon candidates ($N_{\text{tot}}$), the total number of real muons ($N_\mu$), the ratio of the probability a $K^+$ punches through to the probability a $\pi^+$ punches through ($\xi_+$), and the same ratio but for $K^-$ and $\pi^-$ ($\xi_-$).

Because of the fact that the proton fraction is small (see Appendix A) and also since there are over six absorption lengths for protons at normal incidence from Figure 4.11, it is reasonable to assume that the proton contribution to the charge asymmetry is negligible. This reduces the expression for $N^+$ to:

$$N^+ = \frac{1}{2}N_\mu + \frac{1}{2}N_\pi + \frac{r}{2}N_\pi \frac{P_{K^+}}{P_{\pi^+}}$$ \hspace{1cm} (4.14)

where $P_{\pi^+}$ and $P_{K^+}$ are the punch-through probabilities given by equation (4.9). $N_{\pi(\mu)}$ is the total number of pions(muons) in the final sample. The expression for $N^-$ is exactly the same except all superscript +'s are exchanged for '-'s.

Placing the expressions for $N^+$ and $N^-$ into (4.13) and substituting $\xi = P_K/P_\pi$ one obtains:

$$A = \frac{\frac{r}{2}N_\pi(\xi_+ - \xi_-)}{N_\mu + N_\pi + N_\pi \frac{1}{2}(\xi_+ + \xi_-)}$$ \hspace{1cm} (4.15)

Since $N_{\text{tot}}$ equals the denominator of the above expression one can remove $N_\pi$ and be left with the charge asymmetry as a function of the variables stated. Some algebra leads to an expression for the muon fraction as a function of the charge
asymmetry \((A)\), the \(K/\pi\) ratio \((r)\), and the relative punch-through probabilities of kaons and pions \((\xi)\).

\[
f_\mu = \frac{N_\mu}{N_{\text{tot}}} = 1 - A \frac{1 + \frac{\xi}{2} (\xi_+ + \xi_-)}{\xi (\xi_+ - \xi_-)} \quad (4.16)
\]

The muon fraction in the above expression is strongly dependent on the \(K/\pi\) ratio given a measured asymmetry. Using the average energy and pseudorapidity of the particles in the data, \(\xi_+ = 3.372\) and \(\xi_- = 1.315\), and then using the measured value of \(A\) one obtains the curve shown in Figure 4.21. One can put reasonable bounds on the ratio of kaons to pions by using data from the C0 experiment [35] and from the fixed target results of E605 [36]. The C0 data only goes out to a transverse momentum of 1.4 GeV/c and provides a lower bound on \(K/\pi\) of .25 (see Appendix A).

Unfortunately, this is not the only uncertainty in the muon fraction using the charge asymmetry. The muon fraction depends on the kaon punch-through probability as well. Using the errors cited in Denisov [40], and varying the kaon absorption cross section by \(\pm 3\%\), one gets values of the muon fraction ranging from 58 to 42\%.

At the two extremes of \(K/\pi\), \(N_\mu/N_{\text{tot}} = .513\) for \(K/\pi = .25\) and \(N_\mu/N_{\text{tot}} = .581\) for \(K/\pi = .33\). Using these values as a systematic error and adding the statistical error on the charge asymmetry measurement in quadrature yields

\[
f_\mu = N_\mu/N_{\text{tot}} = .546 \pm .053
\]

from the charge asymmetry alone. One must keep in mind the additional uncertainty of \(+.04 - .12\) from measurement error in the kaon absorption cross section. This agrees well with the measurement made using the fit of \(f_\mu = .523 \pm .01 \pm .018\) from section 4.3.1.1.

The problem with using the charge asymmetry throughout the analysis is purely one of statistics. Already using all events with \(p_t > 12.0\) GeV/c the statistical
The fraction of muons present in a sample as a function of the $K/\pi$ ratio. The charge asymmetry is also an input and was fixed at its measured value of .079.
error dominates the measurement. An attempt to use smaller \( p_t \) bins would further reduce the effectiveness of this method. The fit method of section 4.3.1.1 makes more efficient use of the statistics.

4.4 Other Backgrounds

In chapter 3 it is shown that W and Z bosons present little trouble as a background. This is because they have unique signatures and are thus easily distinguished from other muon events. However, there is a residual component of muonic W boson decays that will not be subtracted. By running a Monte Carlo of W decays to muons the number missed by the subtraction method can be estimated. This amounts to 29 events in the first 5 GeV/c bin, 22 events in the second, and 7 events in the third bin.

It is possible for a W or Z boson to decay into a \( \tau \) lepton, which then decays into a muon with a branching ratio of 17.5%.[28] To first order we can estimate the number of muons from \( W \rightarrow \tau \rightarrow \mu \) and \( Z \rightarrow \tau \rightarrow \mu \) from the numbers found in the data. Given 1300 \( W \rightarrow \mu \) events one would expect about 230 muons from \( W \rightarrow \tau \rightarrow \mu \). Based on Monte Carlo studies of this decay, the method used for subtracting W bosons from the data is 95% efficient at removing this decay mode leaving 12 muons. Given the combined offline cut efficiencies we expect fewer than 10 such muons and they would all be in the 12–17 GeV/c muon bin, so this is a negligible effect.

Given about 140 Z boson decays found in the data one would expect about 45 muons from \( Z \rightarrow \tau^+\tau^- \rightarrow \mu^\pm \) using the branching ratio alone. In this case the method used to distinguish Z's is not as effective as there is little missing energy and most of the time the other tauon will decay hadronically. However, the \( p_t \) spectrum of \( Z \rightarrow \tau^+\tau^- \rightarrow \mu^\pm \) is flat from 1 to 45 GeV/c. This translates into only 4 events
in each of the 5 GeV/c bins prior to corrections for cut efficiencies.

Another possible source of background are cosmic rays. Several factors work to make this background negligible as well. The offline cuts require the muon candidate to be associated with a good vertex. The z position of a cosmic ray would have to be within 10 cm of that vertex along the beam line. The impact parameter of a cosmic ray would have to be within .15 cm of the vertex as well. Additionally, because cosmic ray events tend to be out-of-time with respect to the collision, they often do not form three dimensional tracks in the CTC, which is a requirement in this analysis. All of these cuts will screen cosmic rays.

A cosmic ray filter was developed at CDF which uses cuts similar to the ones in this analysis.[47] The filter places a cut of 5 cm on \( |z_{tr} - z_{vert}| \), requires a 3-D track, and requires \( |D_o| < .5 \) cm. Less than .37% of the test sample (W and Z muons) remained as cosmic rays after the filter was used. Since the cuts used here are tighter than the filter, it is safe to conclude that fewer than 65 events are cosmic rays after cuts. This too is therefore negligible. Based on the cosmic ray spectrum from a cosmic ray trigger sample and including efficiencies, 45 events are expected in the first bin and 4 in the second.

Finally there are Drell-Yan muons. For Drell-Yan events with invariant masses greater than 65 GeV/c\(^2\), the method used to remove W and Z bosons will effectively remove this background. Since \( p_t > 12.0 \) GeV/c is required for muon candidates, one can safely ignore Drell-Yan muons with invariant masses below about 25 GeV/c\(^2\). CDF has measured the integrated Drell-Yan cross section for electron pairs for invariant masses greater than 30 GeV/c\(^2\) to be approximately 230 pb.[48] Extrapolating back to 25 GeV/c\(^2\) we estimate the total Drell-Yan cross section to be less than 250 pb. This is the total cross section. The muon detector acceptance alone will reduce this by at least a factor of 3. Additionally, most of the integrated
Table 4.7

Additional background sources quantified. The final results are subtracted from the muon candidates. The uncertainties are systematic.

<table>
<thead>
<tr>
<th>Background</th>
<th>12-17 GeV/c</th>
<th>17-22 GeV/c</th>
<th>22-27 GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W \to \mu$</td>
<td>29</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>$W \to \tau \to \mu$</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Z \to \tau\tau \to \mu$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Drell-Yan</td>
<td>45</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>DY $\to \tau \to \mu$</td>
<td>15</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Cosmic rays</td>
<td>45</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>148 ± 37</td>
<td>36 ± 8</td>
<td>11 ± 4</td>
</tr>
</tbody>
</table>

cross section comes from events with an invariant mass greater than 65 GeV/c$^2$ according to [48]. We estimate the remaining Drell-Yan background to be less than 13 pb after cuts or less than 45 events in the first bin and fewer than 5 events in the second.

Table 4.7 summarizes the efforts explained in this section. The totals in the last row were subtracted from their respective $p_t$ bins. The uncertainties were added in quadrature to the systematic error. The uncertainties were estimated on the basis of the accuracy of the calculations used to derive the backgrounds.
5. THE INCLUSIVE MUON CROSS SECTION

The formula needed to tie together all the elements discussed thus far and generate an inclusive muon cross section is given for each $p_t$ bin by:

$$\delta \sigma_\mu = \frac{N_{tot} f_\mu - \sigma_{DIF} \int L \, dt}{\epsilon_{off} \epsilon_{\text{trig}} \text{acc} \int L \, dt}$$  \hspace{1cm} (5.1)$$

The terms in equation (5.1) bear explanation. $N_{tot}$ is the total number of muon candidates after the cuts are applied. $f_\mu$ is the fraction of the sample that contains muons. This quantity was introduced in section 4.4. $\int L \, dt$ is the integrated luminosity of the MUO4 data stream and is given by $\int L \, dt = 3.79 \pm 0.26$ pb$^{-1}$.\[^{39}\] $\sigma_{DIF}$ is the decay-in-flight cross section after cuts as calculated using the Monte Carlo method described in section 4.2.3. $\epsilon_{off}$ is the offline cut efficiency, and $\epsilon_{\text{trig}}$ is the muon trigger efficiency. acc is the acceptance of the muon chambers in $-1 < y < 1$.

Collecting all the terms in the numerator of equation (5.1) one obtains the results shown in Table 5.1. In this table the systematic error includes the error estimated in the decay-in-flight calculation and the systematic error estimated for the fitting procedure added in quadrature. The prompt muons shown have also had cosmic rays, Drell-Yan, and residual $W$ and $Z$ bosons subtracted.

Figure 4.12 on page 63 shows the muon chamber efficiency as a function of $\eta$. It contains all the cuts that effect the detector acceptance as well as the efficiency for finding muon stubs. Three such cuts are made in this analysis. First there is a cut requiring the event vertex lie within $\pm 60$ cm of center of CDF along the $z$ axis. This cut removes about 5% of the total acceptance and is needed to remove tracks that originate at the edge of the CTC.
Table 5.1

Table showing the numbers of decay muons, prompt muons, and total events for each of the transverse momentum bins of interest. The prompt muons have had all backgrounds subtracted.

<table>
<thead>
<tr>
<th>$P_t$ GeV/c</th>
<th>Decay muons</th>
<th>prompt muons ($N_{tot}f_\mu - DIF$)</th>
<th>Total events</th>
</tr>
</thead>
<tbody>
<tr>
<td>12–17</td>
<td>3245 ± 193 ± 182</td>
<td>4674 ± 243 ± 437</td>
<td>14,667</td>
</tr>
<tr>
<td>17–22</td>
<td>492 ± 110 ± 26</td>
<td>609 ± 147 ± 71</td>
<td>2,418</td>
</tr>
<tr>
<td>22–27</td>
<td>129 ± 68 ± 7</td>
<td>&lt; 150 events @ 95% C.L.</td>
<td>554</td>
</tr>
</tbody>
</table>

Errors shown are ±(stat) ± (systematic)

Secondly, in order to get away from the edges of the chambers where the behavior is not well understood, a cut was used requiring an extrapolated track to be at least 10 cm away from the edge of the muon chamber. This cut removes an additional 9% of acceptance.

Finally only tracks with $|\eta| > .066$ were used in the analysis. The reason for this is that when a track points at right angles to the beam line, it is twice as likely to pass through two calorimeter towers as opposed to only one. In calculating the energy deposited by a muon candidate, the muon code sums the energy in all towers the candidate traversed. Consequently, if the muon went through two towers, there is a greater chance another particle in the event landed in one of the two towers making the minimum ionizing cut less efficient. Rather than try to correct for this effect in those two $\eta$ bins, they were not included in the analysis. This effectively removes another 5% from the acceptance.
Table 5.2

The inclusive muon cross section in a rapidity range of ±1.0 in 5 GeV/c bins and the estimated charm fraction.

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$\delta \sigma_\mu$</th>
<th>Est. Charm fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 - 17</td>
<td>$4.49 \pm .31 \pm .54$ nb</td>
<td>.15</td>
</tr>
<tr>
<td>17 - 22</td>
<td>$0.58 \pm .14 \pm .08$ nb</td>
<td>.12</td>
</tr>
<tr>
<td>22-27</td>
<td>$&lt; .17$ nb @ 95% C.L.</td>
<td>.10</td>
</tr>
</tbody>
</table>

Another correction must be made for the fact that the muons were generated flat in $\eta$ but they actually have a slope according to the $b\bar{b}$ Monte Carlo. Since the Monte Carlo is modeled in the fit to extract muons from the NIP background, no correction is necessary out to $|\eta| < .7$. Correcting for this slope increases the acceptance by 2.7%. This is also used as an estimated systematic error in the acceptance. Combining all these effects one obtains $\text{acc} = .421 \pm .002 \pm .011$ for $-1 < y < 1$ for the central muon chambers at CDF.

The efficiency of the offline cuts was discussed in chapter 3 and is shown in Table 3.2. The efficiency of the muon trigger was discussed in detail in chapter 2 and is also given in reference [49]. The value used here is $\epsilon_{\text{trig}} = .90 \pm .02$ from [26]. There is an additional correction of .972 applied to the first bin to correct for the trigger turn on. Placing these quantities in equation (5.1) and adding the systematic and statistical errors separately in quadrature, one obtains the result shown in Table 5.2.
Table 5.3

The inclusive muon cross section in a rapidity range of ±1.0 in 5 GeV/c bins where the muons are from b quark decays.

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$\delta\sigma_{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 - 17</td>
<td>3.90 ± .27 ± .51 nb</td>
</tr>
<tr>
<td>17 - 22</td>
<td>.52 ± .13 ± .08 nb</td>
</tr>
<tr>
<td>22-27</td>
<td>&lt; .17 nb @ 95% C.L.</td>
</tr>
</tbody>
</table>

The cross sections in Table 5.2 include contributions from charm decay. In order to determine the charm fraction we first determine the ratio $(N_c/N_b)$ of c to b quarks that produce 12 GeV/c muons using the Monte Carlo ISAJET. According to ISAJET at the generation level $N_c/N_b = .193 ± .002$ (stat). Muons with $p_t\mu > 12$ GeV/c come primarily from b and c quarks with quark transverse momentum $p_{tq} > 20$ GeV/c.

The systematic error associated with $N_c/N_b$ will be due to any $p_t^b$ dependence of the fragmentation function in ISAJET and our knowledge of the relative b and c quark fragmentation properties.[44] To test the $p_t^b$ dependence of the ratio, another Monte Carlo run requiring $p_{t\mu} > 20$ GeV/c was done and yielded $N_c/N_b = .202±.004$ (stat). Therefore, any $p_t$ dependence of $N_c/N_b$ in ISAJET is slight. The other factor is our knowledge of the parameters in the fragmentation of heavy quarks (see [50] and [51] for experimental results and [52] for an explanation of the fragmentation
function used). We choose $N_c/N_b$ and its error to be $0.2 \pm 0.1$ (sys). This systematic uncertainty was chosen on the basis of further Monte Carlo studies where the average momentum fraction was varied by 5% and by similar tests done by [44].

The fact that the minimum ionizing cut results in higher efficiencies for muons from b quarks than c quarks reduces the charm fraction to about 15%. Using the minimum ionizing cut efficiencies from Table 3.2 the total cross section of muons from bottom quarks is obtained and shown with statistical and estimated systematic errors in Table 5.3.
BIBLIOGRAPHY
BIBLIOGRAPHY


42. Private communication, Vic Scarpine, June 1992.

43. Fitter used is MINUIT in PAW (for Physics Analysis Workstation).


APPENDICES
Appendix A: The $K/\pi$ Ratio

Previously in section 4.3.2 assumptions were made about the ratio of kaons to pions in order to measure the percentage of punch-through in the muon data using the charge asymmetry supplemental analysis. This ratio enters in the main analysis in two places; the decay-in-flight cross section, and in determining the fraction of muons ($f_\mu$) that pass the cuts. It was shown in section 4.2.3 that the decay-in-flight cross section is insensitive to $K/\pi$. It was also shown in Table 4.6 that the muon fraction, $f_\mu$, was insensitive the $K/\pi$ ratio. This appendix describes in greater detail how we arrived at estimates of the ratio of kaons to pions.

During the 1988-89 run another colliding beam experiment was collecting data at the C0 intersection at Fermilab. It used a magnetic spectrometer and time of flight counters to identify particle types out to a transverse momentum of 1.4 GeV/c. [35] Figure A.1 shows the results they obtained for the ratios of $\bar{p}$ to $\pi^-$ and kaons to pions. In the same plot is a comparison to UA1 data indicating that $K/\pi$ does not depend on $\sqrt{s}$. The ratio of $\bar{p}$ to $\pi^-$ appears to increase uniformly in $p_t$ out to near 1.4 GeV/c, while $K/\pi$ increases to about 1 GeV/c where it appears to level off at about .25. Unfortunately, we need to estimate these ratios in the neighborhood of 12 GeV/c and so must make use of other data to guide us in extrapolating.

Another measurement related to the $K/\pi$ ratio was done with the 1987 data by CDF.[53] In that case, the ratio of $K^0_s$ to charged hadrons as a function of $p_t$ was measured out to $p_t \simeq 4$ GeV/c. Figure A.2 shows rather good agreement between that quantity and the average ratio of negative and positive charged kaons to charged pions from $pp$ collisions at ISR. Since the $K^0_s$ cross section should be equal to the $K^+$ or $K^-$ cross section in $p\bar{p}$ collisions, we will assume $K/\pi$ is equal to the average ratio of $K^+/\pi^+$ and $K^-/\pi^-$ which is measured at large transverse momentum in $p$-nucleon collisions in Fermilab experiment E605. Within the stated errors of E605
Figure A.1

Ratios of $K/\pi$ and $\bar{p}/\pi^-$ as a function of $p_t$. The dotted points are from C0. The other points were measured at UA2.
there is no difference in $K^-/\pi^-$ for beryllium or tungsten targets. Based on this we assume that there is no significant difference between $pp$ and $p$-nucleus collisions for $K/\pi$ averaged over charge.

E605 was a fixed target experiment at Fermilab which used a ring imaging Cherenkov detector in order to identify particles out to high transverse momentum. The collisions involved 800 GeV protons impinging on either a beryllium or tungsten target for $\sqrt{s} = 38.8$ GeV. This experiment was able to measure $K^+/\pi^+$, $p/\pi^+$ and
the ratios of their antiparticles out to $p_t = 10.0$ GeV/c. Figure A.3 and A.4 show their results. Also shown is 400 GeV Chicago-Princeton data indicating that there is little dependence of $K/\pi$ on $\sqrt{s}$.[54]

We use the average of $K^+/\pi^+$ and $K^-/\pi^-$ at the highest momenta in E605 of the beryllium collisions as our best estimator of the $K/\pi$ ratio at high $p_t$ in CDF. This yields an average value of $K/\pi = .28 \pm .027$. We chose the uncertainty of $K/\pi$ for the charge asymmetry measurement of the muon fraction to be $K/\pi = .28^{+.05}_{-.03}$.

E605 also measured $p/\pi^+$ and $\bar{p}/\pi^-$. One can see from Figure A.3 and A.4 that the proton fraction decreases dramatically for high transverse momentum. To estimate $(p + \bar{p})/\pi$ for this analysis we averaged over the proton and antiproton fraction in the last bin ($p_t = 9.5$ GeV/c). This yields $(p + \bar{p})/\pi = .04 \pm .0035 \pm .01$ (stat+sys) where we chose a 25% uncertainty as the systematic error.

The variation in $K/\pi$ estimated in this section was solely for the purpose of checking the hadron fraction from the main analysis using the charge asymmetry in the muon sample. It is important to raise the lower limit on $K/\pi$ in a charge asymmetry measurement. This is shown in Figure 4.21 on page 81 where the charge asymmetry is very sensitive to the lower limit. Throughout the muon analysis $K/\pi$ was allowed to vary from .1 to .33. The resulting effect on the final muon cross section is at the 3% level.
Figure A.3

Ratios $K^+/\pi^+$ (squares) and $p/\pi^+$ (triangles) vs $p_t$ for hadrons produced in p-Be (above) and p-W (below) collisions. Open symbols are from Chicago-Princeton data with $s^{1/2} = 27.4$ GeV/c.
Figure A.4

Ratios $K^-/\pi^-$ (squares) and $\bar{p}/\pi^-$ (triangles) vs $p_t$ for hadrons produced in $p$-Be (above) and $p$-W (below) collisions. Open symbols are from Chicago-Princeton data with $s^{1/2} = 27.4$ GeV/c.
Appendix B: Getting a Bottom Quark Cross Section

The main focus to this point has been the measurement of the prompt muon cross section. The muon cross section is the quantity which can be measured unambiguously, and the cross section of muons from b quarks only requires dependence on Monte Carlo models for the relative bottom to charm production, not any absolute values. However, what is more interesting from a theoretical standpoint is the b quark cross section. It is for this reason the method for obtaining the b quark cross section will be explained. The method will also be demonstrated using a lowest-order $b\bar{b}$ Monte Carlo and the preliminary result presented.

The background subtracted muons measured in the last chapter of this thesis come from the decays of primary b and c quarks. Since decays-in-flight have been subtracted, there is no strange contribution to the muon cross section. Top quarks were produced so rarely in the 1988-89 run (if at all) that it is only possible to place a lower limit on the top mass. Consequently, top quark semi-leptonic decays are not a consideration. However charm and bottom have similar lifetimes and similar production cross sections. Therefore, it is expected the muon cross section results from decays of those two types of parent partons.

As explained on page 89 according to the ISAJET $b\bar{b}$ and $c\bar{c}$ Monte Carlo, charm quarks contribute at about 20% the rate of bottom quarks to our data sample. This is due mainly to the fact that the charm quark has a softer fragmentation function in the production of $D$ mesons than $b \rightarrow B$ fragmentation.[52] The only cut that effects muons from charm differently than muons from bottom decays is the hadron energy cut. This cut reduces the percentage of muons from charm to 15% in the total background subtracted sample. Removing the charm muons yields Table 5.3, the total muon cross section in $|y| < 1.0$ for muons from b and $\bar{b}$ quark decays.

The method used for obtaining a b quark cross section from Table 5.3 is outlined
in [55]. A Monte Carlo of b quark production, fragmentation, and decay is run. The threshold for $p_t$ of the b quark ($p_t^{\text{min}}$) is set by plotting the $p_t$ distribution of quarks from each muon bin. One then fixes $p_t^{\text{min}}$ so that 90% of the b quarks have transverse momentum exceeding it. The plots in Figure B.1 show where $p_t^{\text{min}}$ is defined for each of the bins used in this analysis. The b quark minimum transverse momenta for the bins $12 < p_{t\mu} < 17$, $17 < p_{t\mu} < 22$, and $22 < p_{t\mu} < 27$ GeV/c are 21, 29, and 37 GeV/c respectively.

Keeping track of the ratio of the b quark cross section to the muon cross section in each $p_t$ bin from the Monte Carlo allows one to estimate the b quark cross section for those bins. Equation (B.1) shows the formula applied to each bin. The factor of $1/2$ is because both b and $\bar{b}$ quarks are in the data and simulation, but the theoretical curve on our result is only for b quarks.

$$
\sigma_{p_t^{>p_t^{\text{min}}}} = \frac{1}{2} \sigma_{\mu\text{data}} \frac{\sigma_{MC}(p_t^{b} > p_t^{\text{min}})}{\sigma_{MC}(p_{t\mu\text{ bin}})}
$$

Equation (B.1) was determined by running the ISAJET+CLEO Monte Carlo with $b\bar{b}$ events with the minimum energies mentioned previously. The results, along with the theoretical curve based on structure functions from Diemoz[56] are shown in Figure B.2 and in Table B.1. The theoretical curve is from Nason, Dawson, and Ellis with leading and next-to-leading order QCD calculations.[10] The dashed curves show the effect of estimated theoretical uncertainties. Also shown are the results of independent CDF analyses on the b cross section from $B \to J/\Psi K$, $B \to D^0 e$, and inclusive electron decays of the $B$ meson.

There are additional systematic errors associated with the Monte Carlo. These have been studied by Fumi Ukegawa and Barry Wicklund [44] for the b cross section from its decay to electrons. The error associated with our knowledge of the $B \to \mu+$
The three plots show the momentum distribution of b quarks that produced muons in the bins $12 < p_t < 17$ GeV/c, $17 < p_t < 22$ GeV/c, and $22 < p_t < 27$ GeV/c respectively. The dotted lines show the b quark $p_t$ which is exceeded by 90% of the momenta in that bin.
Inclusive cross section for the production of b quarks with $p_t > p_{T_{\text{min}}}^{\text{min}}$. The data points are shown along with the theoretical plot from Nason, Dawson, and Ellis. The data points from inclusive muons include the muon cross section statistical and systematic errors added in quadrature.
Table B.1

The b quark cross section for b quarks with $p_t^b > p_t^{\text{min}}$.

<table>
<thead>
<tr>
<th>$p_t^{\text{min}}$ GeV/c</th>
<th>$\sigma_b(p_t^b &gt; p_t^{\text{min}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>306 ± 24 ± 75 nb</td>
</tr>
<tr>
<td>29</td>
<td>61 ± 15 ± 16 nb</td>
</tr>
<tr>
<td>37</td>
<td>$\sigma_b &lt; 24$ nb @ 95% C.L.</td>
</tr>
</tbody>
</table>

$X$ branching ratio is about 10%.\[45] The uncertainty in the fragmentation parameter $< z >$ was estimated to be 15%. Finally there is an uncertainty associated with the model of the $B$ meson decay. This occurs because the spectator process where the b quark is considered to be unaffected by the other quark in the meson is not a perfect description. This uncertainty is estimated at 10%. These systematics are included in quadrature with the previous quantities in Figure B.2.
VITA
VITA

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