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EVOLUTION OF DOMAIN WALLS

in the Early Universe *

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Abstract

The evolution of domain walls in the early universe is studied via two-dimensional computer simulation. The walls are initially configured on a triangular lattice and then released from the lattice, their evolution driven by wall curvature and by the universal expansion. The walls attain an average velocity of about $0.3c$ and their surface area per volume (as measured in comoving coordinates) goes down with a slope of -1 with respect to conformal time, regardless of whether the universe is matter or radiation dominated. The additional influence of vacuum pressure causes the energy density to fall away from this slope and steepen, thus allowing a situation in which domain walls can constitute a significant portion of the energy density of the universe without provoking an unacceptably large perturbation upon the microwave background.

I. Introduction

Phase transitions in the early universe, brought about by the breaking of symmetries in physical interactions as the universe cooled, could have produced topologically stable soliton configurations such as monopoles, strings, and domain walls. These configurations are created when a gauge or Higgs field settles into its vacuum state during the cooling process. Due to the existence of degeneracies in the minimum of the potential of the field, the universe may develop regional differences in the value of the vacuum state of the field which may not respect the symmetry of the original physical interactions thus producing these sorts of objects. Monopoles, which are point-like defects, and strings, which are 1-dimensional line-like objects, have been investigated quite thoroughly (see Ref. 1 for a review) whereas domain walls, which are sheet-like defects, have not been so scrutinized due to their cosmological undesireablity. Domain walls arose, for instance, in studies of CP violation from spontaneous symmetry breaking but these walls, if produced at high temperatures, would greatly perturb the cosmic microwave background beyond the current observed limits: Due to surface tension, domain walls with finite surface area contract and are unstable to collapse on relatively short time scales (little more than the light travel time across the bubble). The stable infinite walls would be conformally stretched by the universal expansion and their energy densities would fall off much slower than those for matter and radiation and thus the infinite walls would then shortly come to dominate the energy density of the universe. The inhomogeneities of the wall induce anisotropies in the microwave background of the same order of magnitude, $\delta T/t \sim \delta\rho/\rho$ and one finds that walls heavier than 10^{-2} GeV cause the universe to be much more inhomogeneous than could be accommodated by the constraints from the microwave background radiation.[2]

There has been, however, recent activity in the investigation of domain walls due to a number of proposals which circumvent this difficulty. One possibility is a biasing of the field potential. This was originally suggested by Zel'dovich,[2] but more recently, in looking for an explanation for the creation of so-called nontopological solitions or soliton stars,[3,4] Frieman, Gelmini, Gleiser, and Kolb[5] have considered the situation in which infinite walls are destablized due to biasing of the potential. The resulting vacuum pressure causes the

walls to intersect, breaking them up into finite bubbles which do not disappear as usual – they are supported by pressure from particles trapped inside and are thus stablized from total collapse. These configurations persist and may form a large fraction of the mass density of the Universe.

Hill, Schramm, and Fry[6] (further discussions are given in Ref. 7) have proposed that certain physics models can lead to the formation of domain walls after the decoupling of the microwave background, thus avoiding the constraint associated with the domain walls formed at higher temperatures. Though of low energy density, these walls possess nonetheless a large density contrast so that they can immediately grow nonlinearly, leading to the possibility that such "soft" domains walls may provide an ideal source of fluctuations for structure formation. In addition, it has been proposed[8] that a single such wall may be responsible for the bulk motions seen of our local galaxies.

Although little is know so far about the wall dynamics upon which these theories rely upon so much, one can outline the forces which influence domain wall evolution. The surface tension of the walls cause irregularities in the walls to straighten out. This causes vacuum bags to collapse and disappear and causes oscillations in small-scale irregularities in larger walls. These motions are subject to the cosmological expansion which damps motion on the Hubble scale. The walls may also experience interactions due to particle and gravitational radiation and also due to friction with particles.

As these forces come to play on a wall, one would like to know such things as the typical wall velocity, the scaling of the energy density of the walls as the universe expands, and the likelihood of the universe of being eventually dominated by a single wall across the horizon. To answer these questions and motivated by these new proposals, I investigate in detail the possible roles of domain walls in the early universe using a new domain wall simulation program. The approach is along the lines of the early simulations in the study of cosmic strings: To set up the initial configurations of the domain walls, space is divided up into a lattice and each unit cell of the lattice is randomly assigned the value of either ground state of the two-fold degenerate potential. The infinitely thin walls thus formed are then subjected to a curvature term which determines their subsequent evolution; effects due to gravitational and particle emissions and interactions are not taken into account.

This approach is thus complementary to that taken by Press, Ryden, and Spergel[9] in which they evolve the field equations instead.

The structure of the paper is given as follows: In section II, I give a short summary on the basic description and equations of domain walls and elaborate further on the proposals of Frieman *et al.*[5] and of Hill *et al.*[6] In section III, the nature and the details of the computation scheme are discussed and the results of the computation are given in section IV. Finally, I conclude with a discussion of the results with their implications for the domain wall models in section V.

II. A Short Discussion of Domain Walls

Domain walls are created when a discrete symmetry is spontaneously broken. The simplest case involves two-fold symmetry and an example of this is the model with a ϕ^4 potential: $V(\phi) = V_0[\phi^2 - \eta^2]^2$. Its Lagranian is given as

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\lambda(\phi^2 - \eta^2)^2 \tag{2.1}$$

in which ϕ is a real scalar field. The symmetry involved is the reflection $\phi \rightarrow -\phi$ and the minima of the potential V(ϕ) are at $\pm\eta$. Above a critical temperature $T_c \sim \eta$, the field has a zero expectation value. Below this temperature, the field acquires a vacuum expectation value $\langle\phi\rangle = +\eta$ or $\langle\phi\rangle = -\eta$. As one goes from a region of $\langle\phi\rangle = \eta$ to a region where $\langle\phi\rangle = -\eta$, one necessarily encounters a region where $\langle\phi\rangle = 0$: a domain wall of false vacuum. This domain wall is characterized by a width $\delta = \lambda^{-1/2}\eta^{-1}$ and surface energy density $\sigma \sim \lambda^{1/2}\eta^3$. More details can be found in Vilenkin's paper.[1]

In addition to this case, another potential which occurs naturally in the context of some of the high energy physics models is the sine-Gordon potential which has an infinite number of degenerate minima: $V(\phi) = V_0[1/2 + 1/2cos(\pi\phi/\eta)]$. The walls are produced in a manner analogous to the case mentioned previously and the walls themselves are quite similiar with the exception in that the behavior of a ϕ^4 wall and that of a sine-Gordon wall differ when the walls collide: the ϕ^4 walls intercommute whereas the sine-Gordon walls can generally pass through one another. Further investigations of these wall properties have been made by Widrow.[10]

In light of these descriptions, elucidations can be made on the two proposals mentioned previously. In biasing something like the the ϕ^4 potential, one adds in additional terms to skew it so as to produce a true and false vacuum minima and (thus induce vacuum pressure); the skewed potential destablizes the domain walls, causing infinite walls to break up as all of space seeks to be in the true vacuum. Frieman *et al.*[5] add an additional term which introduces interactions with another field. This interaction stablizes the finite bubbles that are produced via the pressure of particles trapped within the bubbles. The objects are thus stable to collapse and may persist, resulting in their interest as candidates of dark matter or as gravitational wells to attract matter and form structure.

In the proposal put forth by Hill, Schramm, and Fry,[6] the theoretically favored potential is the sine-Gordon which appears naturally in their symmetry-breaking scheme. With these sine-Gordon walls one would expect behavior in which wall collisions result in the walls simply passing through one another. Furthermore, since the walls are produced at low temperatures ($\sim 10^{-2}$ eV) in this case, the width of the walls is nonneglible and is comparible to the radius of curvature of the walls.

In the present work, the focus is on walls which result from a potential with two discrete minima such as the ϕ^4 potential; therefore, the wall evolution will be somewhat inexact with regard to the model of Hill *et al.*[6] The walls are created initially when all of space are in either of two degenerate minima and during the wall evolution, walls intercommute whenever they intersect. Furthermore, the simulation is done using the thin wall approximation in which the walls are seen as two-dimensional membranes foliating three-dimensional space and are evolved according to the action analogous to that of the Nambu action for strings.[11] This is in contrast to the possibility of evolving the field equations with no particular attention paid to the presence of walls. Press, Ryden, and Spergel have taken this approach and have taken into account the thickness of the walls.

In the thin wall approximation, the equations of motion can be derived analytically.[12] The motion of the walls is given by the action

$$S = -\sigma \int dV \tag{2.2}$$

which is proportional to the volume of the wall hypersurface in spacetime. In minimizing

the action, one arrives at the Euler-Lagrange equation

$$\frac{\partial}{\partial \xi^\alpha}\frac{\delta L}{\delta x^\mu_{,\alpha}} - \frac{\delta L}{\delta x^\mu} = 0 \tag{2.3}$$

in which the wall is described by a map $x^\mu = x^\mu(\xi^\alpha)$ which takes the coordinates (ξ^0, ξ^1, ξ^2) of a 3-dimensional space and maps them into the wall hypersurface in the 4-dimensional spacetime. In addition, $x^\mu_{,\alpha} \equiv \partial x^\mu/\partial \xi^\alpha, L \equiv -\sigma a^3[-\det(h_{ab})]^{1/2}$ is the Lagrangian of the wall, a is the scale factor, and $h_{ab} = g_{\mu\nu}x^\mu_{,\alpha}x^\nu_{,\beta}$ is the induced metric on the 3-dimensional wall hypersurface. (I have chosen here to work in conformal time $d\tau = dt/a(t)$. Physically, the conformal time measures the comoving distance travelled by light since the time of the formation of the walls.) For a wall with cylindrical symmetry, we can make the gauge choice of the map $x^\mu = (\tau, \rho, \phi, z)$:

$$\tau = \xi^0, \quad \rho = R(\xi^0), \quad \phi = \xi^1, \quad z = \xi^2 \tag{2.4}$$

to get the result

$$\ddot{R} + 3\frac{\dot{a}}{a}\dot{R}[1 - \dot{R}^2] = -\frac{[1 - \dot{R}^2]}{R} \quad . \tag{2.5}$$

This shows that the acceleration of a wall segment with a curvature of characteristic radius R is driven by an amount inverse to this radius (times a relativistic factor) and is damped by the expansion by a term consisting of the expansion factor $\dot{a}/a$ times the velocity of the wall times another relativistic factor. This can be slightly rewritten

$$\ddot{R} + 3(\frac{d\ln a}{d\ln \tau})\frac{1}{\tau}\dot{R}[1 - \dot{R}^2] = -\frac{[1 - \dot{R}^2]}{R} \quad . \tag{2.6}$$

The factor $(d\ln a/d\ln\tau)$ is equal to 2 for a matter dominated universe and 1 for a radiation dominated universe. For most of the runs, this quantity was set to 2 but as we shall see, this change in the expansion rate made no significant difference in the final results.

The wall simulation utilizes equation (2.6) to evolve the domain walls. The walls themselves do not possess cylindrical symmetry on a global scale but locally, the wall curvature can be fitted to a circle and thus equation (2.6) can be applied to determine its acceleration.

The gauge choice made in (2.4) assumes the gauge condition

$$\frac{\partial x^\mu}{\partial \xi^0}\frac{\partial x^\mu}{\partial \xi^\alpha} = 0 \tag{2.7}$$

with $\alpha = 1, 2$ which means that the velocity of the wall at a particular point must always be perpendicular to the wall surface. As this is an additional constraint to the motion already described by (2.5), this condition (2.7) must be continually enforced during the simulation run.

III. Computer Simulation of Domain Walls

The simulation of the domain walls is done with only two spatial dimensions (it assumes translational invariance in the third dimension). The movement of the walls is driven by its curvature and retarded by the universal expansion and in some instances, influenced by vacuum pressure.

The initial wall configuration is set down as follows. All of space is divided up into a lattice (a triangular lattice in this case) and is given periodic boundary conditions, resulting in a toroidal topology. The configuration of ground states right after a phase transition is created by giving each triangular region a vacuum expectation value of either $+\eta$ or $-\eta$. The boundaries between the areas with the $+\eta$ vacuum and the areas with the $-\eta$ vacuum are then the domain walls.

Whether or not one encounters an infinite wall depends upon the probablity of being in one of the minima (say $+\eta$ for which the probability is p_+). In two dimensions, from probability $p_+ = 1$ down to $p_+ = p_c$, p_c some critical probability, the $+\eta$ regions percolate and form an infinite region; from probability $p_+ = p_c$ down to $p_+ = 1 - p_c$, neither the $+\eta$ regions nor the $-\eta$ regions percolate and only finite walls develop; from probability $p_+ = 1 - p_c$ down to zero probability, the $-\eta$ regions percolate. Since the percolation of the $+\eta$ and $-\eta$ regions do not occur for a common set of values for p_+, one may not expect any infinite walls to develop; however, in the scheme employed here, infinite walls do develop around $p_+ = p_c$ as walls are defined as the boundary around the $+\eta$ regions whose connectedness is defined by triangles sharing a common side. The $+\eta$ triangles connected only at a vertex are not considered to be connected and thus, it is possible to have infinite domain wall even though only the $+\eta$ region is considered to have percolated. In three dimensions, p_c is less than 0.5 so that between $1 - p_+ = p_c$ and $p_+ = p_c$, both the

$+\eta$ and $-\eta$ regions percolate and form infinite domain walls. (For complete details on the theory of percolation, see Ref. 13). In the present simulation, the ground state values are assigned to the centers of the triangles and these points form the verticies of a hexagonal lattice for which the critical percolation probablity is $p_c = 0.698$.

This pattern of assigning entire triangles vacuum values of $\pm\eta$ assumes a correlation length roughly the size of the triangle. The correlation length can extend in size out to the horizon but can also be much smaller, determined by the details of the microphysics. Recent work involving a general study of the emergence of topological defects in phase transitions has been done by Hodges.[14]

The configuration is then released from the lattice and is allowed to evolve freely with the dynamics of the evolution being driven by the curvature of the walls. The wall evolution is performed by keeping track of a number of points on the wall, namely those that formed the vertices of the triangles in the initial configuration. These points start with zero initial velocity but quickly develop velocities a few tenths of c due to the curvature of the walls. This curvature is determined by taking the inverse of the radius of the circle which passes through the a point of interest and the point's two nearest neighboring points. With the radius of the circle determined, the acceleration of the point is given by equation (2.6) and the determinations of the new velocity and new position follow. Since no special attempt is made to increase the order of the accuracy of this determination, the new position is accurate to 2nd order in the time step.

There are a number of computational details concerning the time step, the gauge condition, and the wall intersections which require further explicating. The time stepping of the program is governed in the following way. A potential value for the next time step is computed for all points on all of the walls according to the equation

$$dt_i \leq \frac{\epsilon R_i}{v_i} \tag{3.1}$$

in which ϵ is a dimensionless parameter. This is basically a requirement that the movement of the point not overshoot the circle with which its acceleration was computed. Then all of the values of dt_i are compared and the smallest is chosen as the next time step.

This requirement on the time step can cause the computation to bog down if the

radius of even one point becomes small relative to its velocity. Thus, to facilitate the computation, points on the walls are removed if they get too close together. Furthermore, if a wall bubble is well on its way to collapse – as indicated by a small surface area and large velocities – then the entire bubble is removed from the computation if its surface area dips below a given threshold. The surface area of a wall (which is actually just the length of the wall in the simulation) is computed by adding up straight-line distances between the points which are being evolved. The surface area is proportion to the energy density as the wall has constant rest-mass energy-density.

The gauge condition expressed by equation (2.7) is explicitly enforced by the program. When the new direction of acceleration of the point is to be determined, the velocity from the previous time step is adjusted so as to assume this new direction but with the same speed so that the full value of this velocity is put into equation (2.6) to compute the magnitude of the acceleration. This enforcement of the gauge condition alters the direction of the velocities by less than a fifth of a degree on average.

The possibility of wall intersections is taken care of with an algorithm to detect possible intersections and to intercommute walls which have been determined to have intersected. Wall intersections are detected in a manner similar to that of Albrecht and Turok[15] for their string simulations. At every time step, the two-dimensional space of wall configurations is divided into a lattice of squares. The program then goes through wall by wall to place the wall points into their appropriate boxes. As these points are put into a box, the program catalogues the portions of walls which pass through the box. If there are other points in the box when a point is put into the box, then the point is checked for intersections with wall segments formed by those other points.

The detailed checking occurs as follows: The current point is evolved according to its present velocity for the duration of 2.5 time steps. Then the program goes through all of the points already in the box and checks all the wall segments formed by these points. These wall segments are likewise evolved and if the trajectory of any of the wall segments and that of the point cross paths, an intersection is to be anticipated. If an intersection is determined, then the intercommutation of the walls is performed by removing the point that intersects the wall segment and reconnecting its two nearest neighboring points to the

two points which had formed the wall segment.

There are several sources of systematic uncertainties. One is that the scheme for computing the radius of curvature at a point is somewhat flawed. As stated before, to compute the radius of curvature, a circle is drawn through the point in question and through its two neighboring points. This scheme works well enough except when the two neighboring points are located fairly close to one another with respect to the distance separating them from the point in question (the three points thus forming a narrow triangle). One would expect that the point would be subjected to a large acceleration due to its displacement with respect to its neighbors; however, a circle drawn encompassing these points will not have a small radius of curvature.

Secondly, properly measuring the surface areas of the walls has a number of difficulties. For one thing, points are removed at various times during the simulation, resulting in a sudden reduction in the total surface area. In addition, the scheme to detect intersections is not perfect. In string simulations, missed intersections would not be of any great consequence as long as their numbers were small but in wall simulations involving vacuum pressure, such could be disasterous. A missed intersection could result in a wall bubble being pinched off with its inside inverted to the outside. This means that, although the bubble may be traveling in a part of space that is the true vacuum, the bubble will continue to expand. If the bubble does not eventually contract, then it poses problems for properly measuring the surface areas of the walls. Some of the simulations had to be cut off when these difficulties arose.

An attempt at a three-dimensional simulation involved using a 14-sided (8 hexagons, 6 squares) polyhedron as the unit cell (analogous to the triangle as the unit cell for the two-dimensional case) which corresponded to BCC lattice with $p_c = .245$. This configuration was chosen so that any point would have at least 3 nearest neighbors but no more than 4; this allowed the extension of the method of computing acceleration. In this case, the acceleration was computed by drawing a sphere through 4 points or drawing 2 spheres colinear with the point for which the acceleration was being computed for the case of 5 points. However, it was found that points tended to converge together, causing the time step, which was regulated in a similar fashion to equation (3.1), to become smaller and

smaller. Unlike the two-dimensional case, it was not trivial to remove points to speed the simulation along; with the requirement that all points have 3 or 4 nearest neighbors, no general point removal scheme could be found and the effort was abandoned.

However, I will mention in passing that there were certain interesting hints gleaned from the runs that were managed in the three-dimensional simulation. In the two-dimensional case, the cosmological expansion damped out small-scale motion, allowing for motion on large scales to take place; this was not what was observed in the three-dimensional case. Small-scale oscillations were not damped out and thus, not much coherent large-scale motion developed, leading to a much slower decrease in the energy density was compared to the two-dimensional simulation. It is not apparent that these effects were not due to numerical problems but this may be an indication that a properly done three-dimensional simulation may not yield the same results seen for the two-dimensional simulation here nor for the simulations done elsewhere, a problem also encountered in the study of cosmic strings where there have been some disagreements which have originated in the treatment of string curvature on very small scales.

IV. Results of the Simulation

Most of the runs were performed on a 64×32 triangular lattice. The conformal time was initially set to a value of 1 and were run until a time of $\tau = 40$ or until the walls had disappeared. For most of the runs, the factor $(d\ln a/d\ln\tau)$ in equation (2.6) was set to 2 for a matter-dominated universe and the probability p_+ was set to 0.7, essentially the value of the critical percolation probability, p_c. At this value, one sees infinite domain walls and a distribution of smaller walls. For three dimensions, infinite domain walls can be achieved with a $p_+ = 0.5$.

Typical wall configurations of a run are shown in Figs. 1a-1c. The walls are shown at times of $\tau = 3$ at the beginning of a run with the walls just becoming fully developed; $\tau = 7$ in the middle of the scaling of the wall; and $\tau = 18$ towards the end of the run when the walls are about the size of the lattice box itself. In this case, there were no infinite walls. In the simulation runs without vacuum pressure, the walls tended to avoid

one another and thus no wall collisions were seen. The velocity distribution of the walls were measured at these same times and are shown in Figs. 2a-2c.

The result of seven runs is compiled in terms of the comoving surface area per comoving volume in Fig. 3 and in terms of the wall velocities in Fig. 4. The dip seen in Fig. 3 at about the time of $\tau = 2$ is due to the simultaneous removal of a number of similarily sized triangular wall bubbles. From Fig. 3, one sees a linear relationship between the comoving surface area per comoving volume and the conformal time. The slope of this relationship was measured between the times of $\tau = 4$ (when the walls have been scaling for awhile) and $\tau = 10$ (before the wall radii become on the order of the lattice box) and the slope obtained is 1.06 ± 0.11. A slope of -1 simply expresses the fact that there is only one wall on average is left to be found within a hubble distance after the surface tension has straightened out large walls and has shrunk small walls. Given that the walls move with some velocity v close to c, the volume is $(vt)^3$ and the one wall in that volume has surface area $(vt)^2$. The energy density thus goes as $\rho \sim 1/(vt)$ and thus in conformal time this goes as τ^{-1}.

From Fig. 4, one sees a constant velocity of about $0.25c$ during the scaling behavior of the walls. This is somewhat smaller than the average velocity of $0.4c$ seen by Press *et al.*[9]

A number of runs were done varying the settings of some of the basic parameters. Setting the factor $(d\ln a/d\ln\tau) = 1$ for expansion in a radiation-dominated universe made no detectable difference. The only role the expansion seemed to have on the wall motions was to damp away small scale oscillations, thus allowing bulk motions to occur.

A number of runs were done varying the probability p_+ of being in the $+\eta$ ground state. Fig. 5 shows the results of running at probablities of 0.5, 0.6, 0.7, 0.8, and 0.9. The general trend is that as one goes away from $p_+ = 0.7$, the slope steepens due to the fact that the walls become smaller and thus one expects them to smooth out and collapse on a shorter time scale.

Eight runs were done with the addition of vacuum pressure and the results are depicted in Fig. 6 for the comoving surface area per comoving volume. The runs were done with values of the vacuum pressure of 0.0, 0.5, 0.8, 1.0, 1.1, 1.2, 1.5, and 2.0 (in units of the

inverse of the correlation length). Some of the runs had to be cut off due to computational difficulties that were described in the previous section but a simular series of runs on a smaller lattice (32×16) managed to avoid those difficulties and showed the surface areas dropping off rapidly. All of the runs were begun without vacuum pressure and without collisions so that the walls would have a chance to straighten themselves out; the vacuum pressure was immediately introduced at $\tau = 1.3$.

From Fig. 6, we see that initially the walls scale as before, going down as τ^{-1} in surface area and velocity measurements show the walls having a constant average velocity of about $0.3c$; at some certain time which depends upon the vacuum pressure the velocities go up and the walls collapse. This can be understood from the fact that the walls have an average radius of curvature which goes as $R = (\tau - \tau_0 + R_0)$ in which R_0 is the initial average curvature. This value is initially larger than the vacuum pressure so the walls scale as before. The vacuum pressure may shrink down some walls faster than before but may also retard the straightening out of some of the walls, leading to a zero net effect. However, as the value of the average radius of curvature drops below that of the vacuum pressure, the main contribution to the wall acceleration comes from vacuum pressure which forces the wall velocity to higher and higher values; the infinite walls intersect and fragment and the fragments quickly disappear under the force of the vacuum.

V. Discussion and Conclusion

The basic picture that emerges is that the wall density goes down as τ^{-1} (and as $a^{-3/2}$), producing a single wall that extends across a horizon volume that is essentially devoid of any wall bubbles. During their evolution, infinite walls tend to straighten out and irregular bubbles tend to become more circular; all the while, the walls do not intersect unless vacuum pressure is applied. These observations are simular to those seen in the the studies of Press *et al.*[9] and confirm their basic findings in two dimensions. The two studies took different approaches and had differences in the thicknesses of the walls evolved but the basic conclusions which come out of the studies seem to be the same. Press *et al.* go on further to investigate the departure of the slope of the surface area from -1 but

this deviation does not show up in this study and indeed, the uncertainties here are large enough to mask such an effect.

The implication for domain wall model of Hill, Schramm, and Fry[6] from this is that the amount of matter to be found today in domain walls is very limited. Press *et al.*[9] give an estimate of $\delta T/T \sim \delta\rho/\rho \sim \Omega_{wall}$ which together with the limits on the cosmic microwave background give $\Omega_{wall} \lesssim 10^{-4}$. More specifically, Hill, Schramm, and Widrow[7] derive

$$\delta T/T \sim (1 + \frac{\gamma v}{c}) G\sigma R \tag{5.1}$$

with v the wall velocity, γ the relativistic factor, G Newton's constant, σ the wall surface energy density, and R the scale of wall structures produced. They note that for domain bubbles $R \sim \delta$ (δ being the width of the walls) and the optimistic result of the original paper[6] is reproduced. However, if the resultant structures are a few infinite walls as the simulations of Press *et al.*[9] and this work appear to show, then $R \sim R_H$, R_H the horizon length, leading to the constraint on Ω_{wall} mentioned above. The walls, whose energy density decrease at a rate of $a^{-1.5}$, would have to have been even smaller relative to that of matter – which goes down as a^{-3} – at the time of structure formation, leaving the wall density too small to have attracted much matter to form large-scale structures. Although the present simulation does not explicitly test thick walls as found in the model of Hill *et al.*,[6] the basic agreement of the results of this simulation with those of Press *et al.*[9] show that these conclusions are not particular to the thickness of the walls. Furthermore, in simulations without vacuum pressure, the fact that the walls do not collide with one another means that the wall evolution really does not depend on whether the potential is ϕ^4 or sine-Gordon. However, this is not quite true as bubbles which simply collapse and disappear in the ϕ^4 case may rebound and persist for the longer period of time in the sine-Gordon case, providing seeds for structure formation.[7]

Variations from this basic picture were addressed by these simulations. The simulations show that the additional presence of vacuum pressure gives rise to the situation in which infinite walls are chopped up into bubbles of finite size. This then allows the walls greater leeway in avoiding the microwave constraint but to be of any interest at all to the creation of large-scale structure, these bubbles must be around for more than a expansion

time to begin to accrete matter. Unfortunately, the finite bubbles seen in the simulations collapse with near light speed and do not have a long enough lifetime to be interesting for structure creation. However, if there is any impediment to their collapse such as particle pressure as suggested by Frieman *et al.*,[5] then the lifetimes of these objects will be such to make structure formation possible. Also, as in the case without vacuum pressure, if collapsing bubbles do not immediately disappear but rebound and bounce a few times (as seen by Widrow for the sine-Gordon walls[10]) then this may also prove domain walls a viable process for producing perturbations for structure formation.

A potential with $p_+ \neq p_c$ gives rise to steeper decreases of the energy density due to the fact that smaller wall structures are produced. The present simulation did not have the necessary resolution to see how steep the energy density could be made to go down but a slope of -2 would result in the wall density doing down no slower than the energy density in matter, thus preventing the domain walls from dominating the energy density of the universe.

An important further step that needs to be taken is an investigation into the nature of curvature on very small scales and a three-dimensional simulation along lines of the two-dimensional simulation done here. It is of interest to investigate the nature of curvature on scales smaller than the correlation length as we have seen from the study with vacuum pressure that the behavior of the walls depends on the average curvature. If the walls are highly erratic on very small scales, they will possess a higher average curvature and the results derived by these simulations will not follow: walls will take longer to straighten out and thus, the drop-off in surface area will be even slower than τ^{-1}; and the turn-off from scaling behavior due to vacuum pressure will occur at later times. This general question of the nature of curvature at scales below the correlation length seems to lie at the heart of the controversy surrounding the study of strings.[16] The string model of Bennett and Bouchet[17] utilizes a scheme which keeps track of kinks in strings whereas that of Turok and Albrecht[15] uses a numerical diffusion technique which results in strings that are not in general as highly curved on small scales. Thus, while the two simulation have agreement on string behavior on large scales, there is a divergence in the scaling trends for small strings, resulting in difference results in chopping efficiency, constraints due to

gravitational radiation, etc. Careful studies of small-scale curvature are necessary to truly understand the behavior of both strings and walls.

As I noted in passing, the attempted simulation in three-dimensions seemed to hint at a different behavior (albeit one that is even less favorable for the domain wall models) than that seen for the two-dimensional case: the expansion term did not eliminate small scale oscillations, preventing movement on large scales and resulting in an energy density which scaled much slower with respect to the conformal time. At this point, it is difficult to tell whether this outcome is physically valid, the result of certain simplifications that were made, or the artifact of the initial lattice configuration. Wall thickness and particle emissions were two features left out of this simulation which were addressed by the other simulations; this feature seen in this simulation was perhaps not present in the work of Press *et al.*[9] because of their focus on thick walls and the consequent loss of resolution on small scales. The initial BBC lattice resulted in unit cell configurations which evolved rather awkwardly: for instance, a 2-cluster configuration had a narrow neck which persisted while the essentially unit cell configuration of points on either side of the neck collapsed freely resulting in a small tangle of points on either side of this neck. A workable three-dimensional simulation would be able to reliably indicate the precise features of the two-dimensional simulation attributable to its dimensionality and properly address the question of the behavior of realistic three-dimensional walls.

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Figure Captions

Fig. 1. Wall configuration for a 64×32 lattice at conformal time a) $\tau = 3$; b) $\tau = 7$; and c) $\tau = 18$.

Fig. 2. Wall velocity distribution for a 64×32 lattice at conformal time a) $\tau = 3$; b) $\tau = 7$; and c) $\tau = 18$.

Fig. 3. Wall comoving surface area per comoving volume as a function of the conformal time for 7 runs done on a 64×32 lattice with $p_+ = 0.7$.

Fig. 4. Average wall velocities as a function for the conformal time for 7 runs with $p_+ = 0.7$. The velocity remains constant at about $0.25c$. The dip in velocities at time of $\tau = 2$ occurs because of the simultaneous removal of a number of rapidly collapsing triangular bubbles.

Fig. 5. Wall comoving surface area per comoving volume as a function of the conformal time for 5 runs with p_+ set at 0.9, 0.8, 0.7, 0.6, and 0.5.

Fig. 6. Wall comoving surface area per comoving volume as a function of the conformal time for 8 runs with the vacuum pressure set to 0.0, 0.5, 0.8, 1.0, 1.1, 1.2, 1.5, and 2.0. The surface area turns downward sooner for larger values of the vacuum pressure.

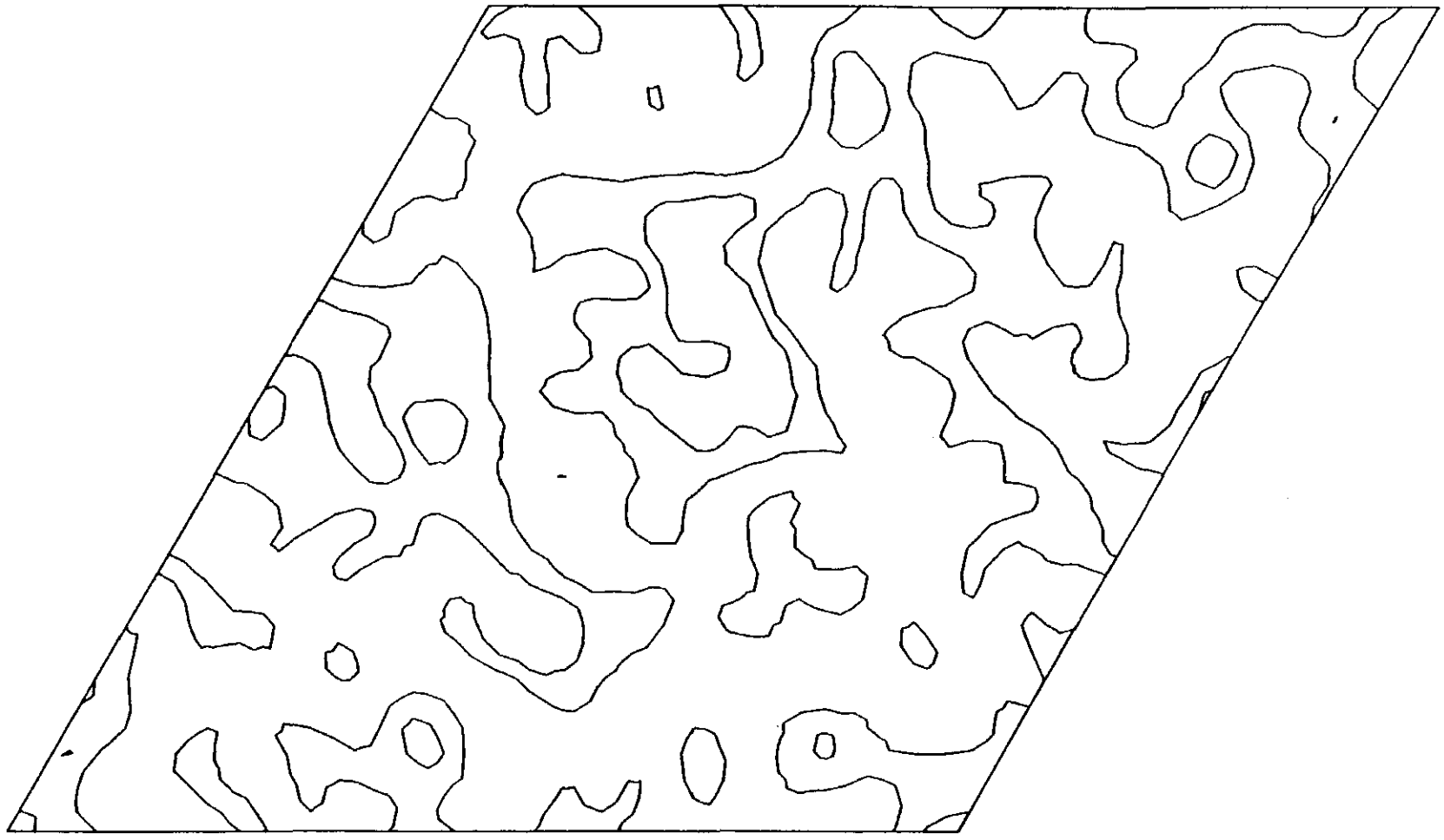


Figure 1a.

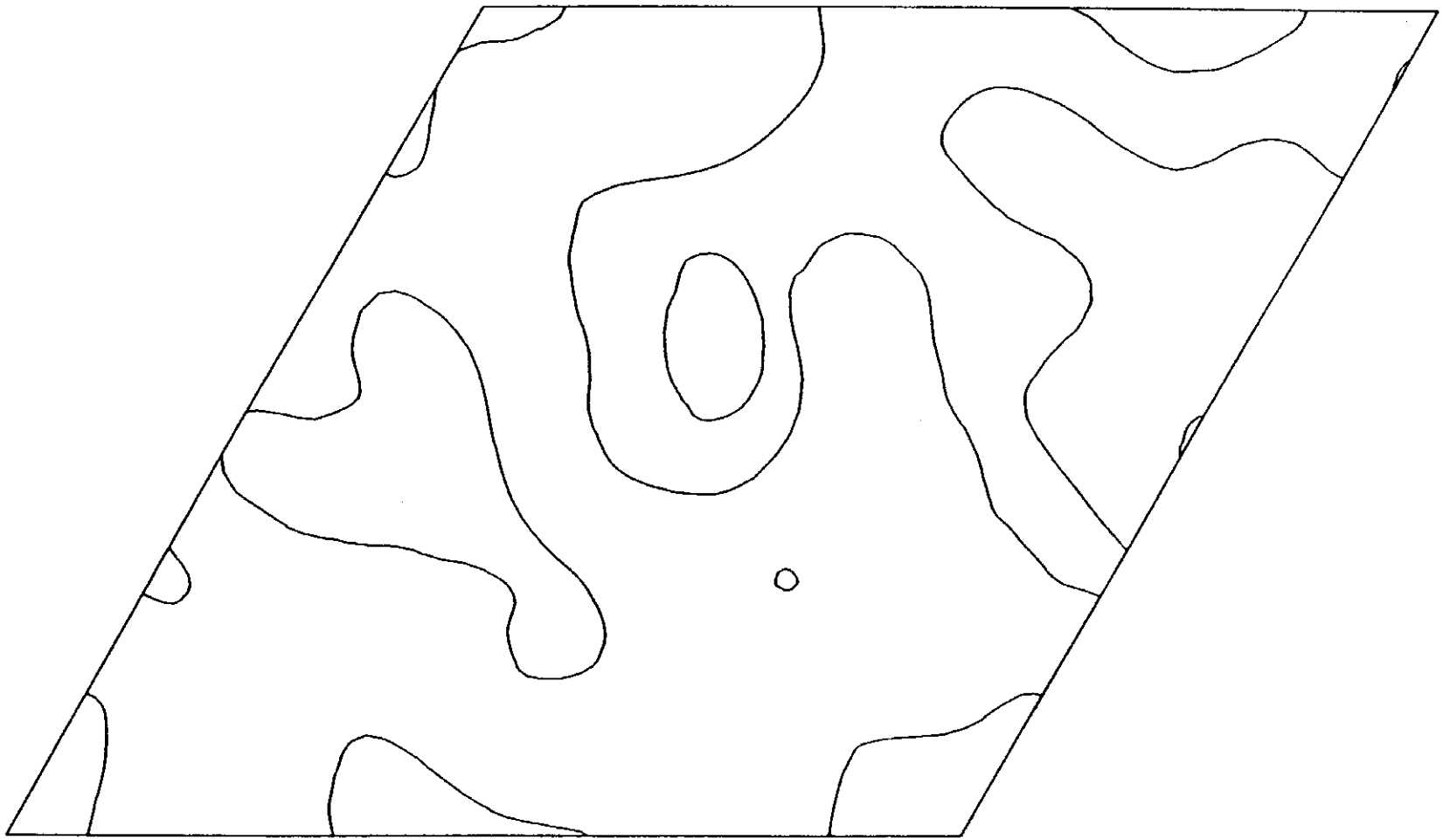


Figure 1b.

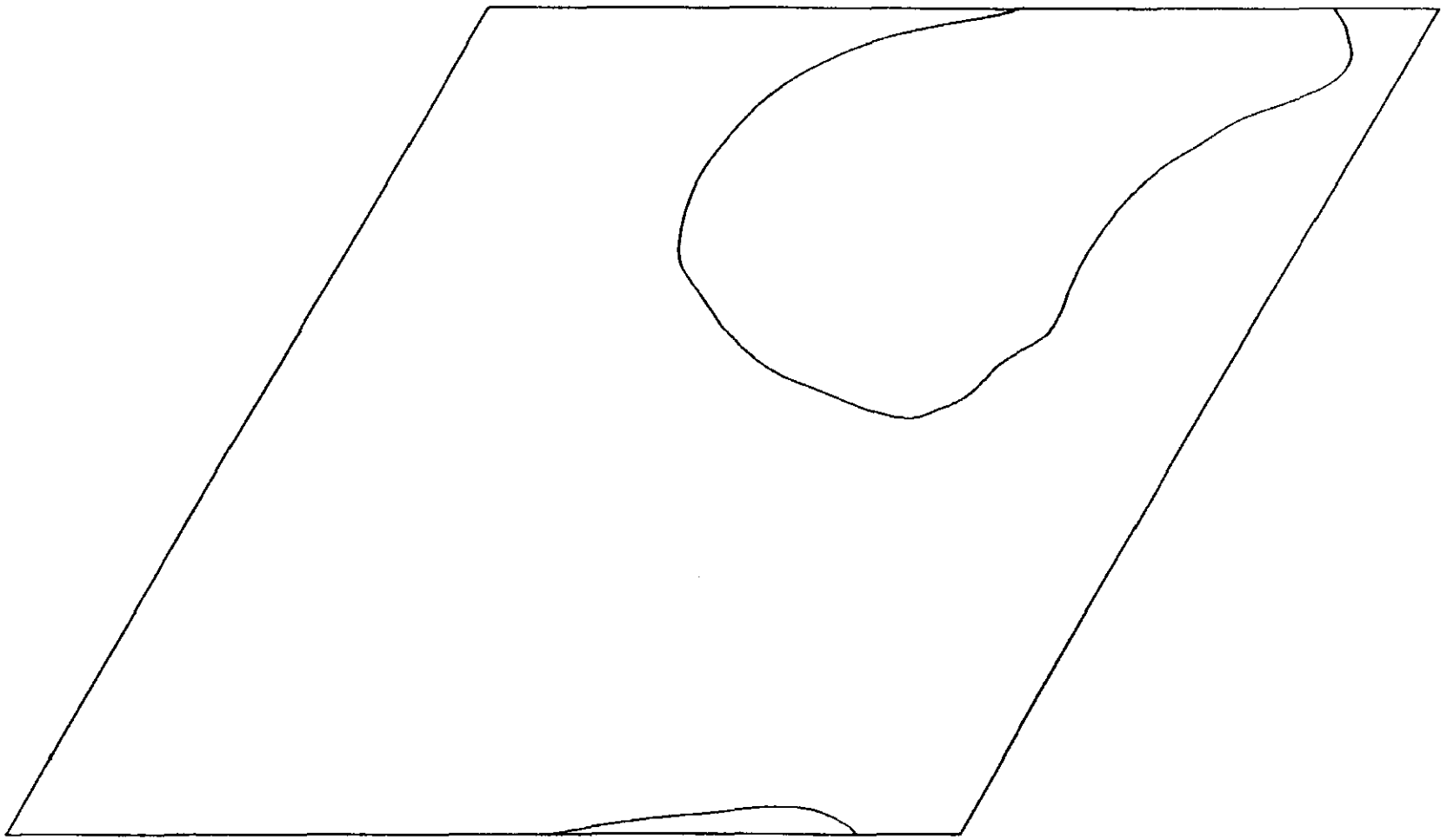


Figure 1c.

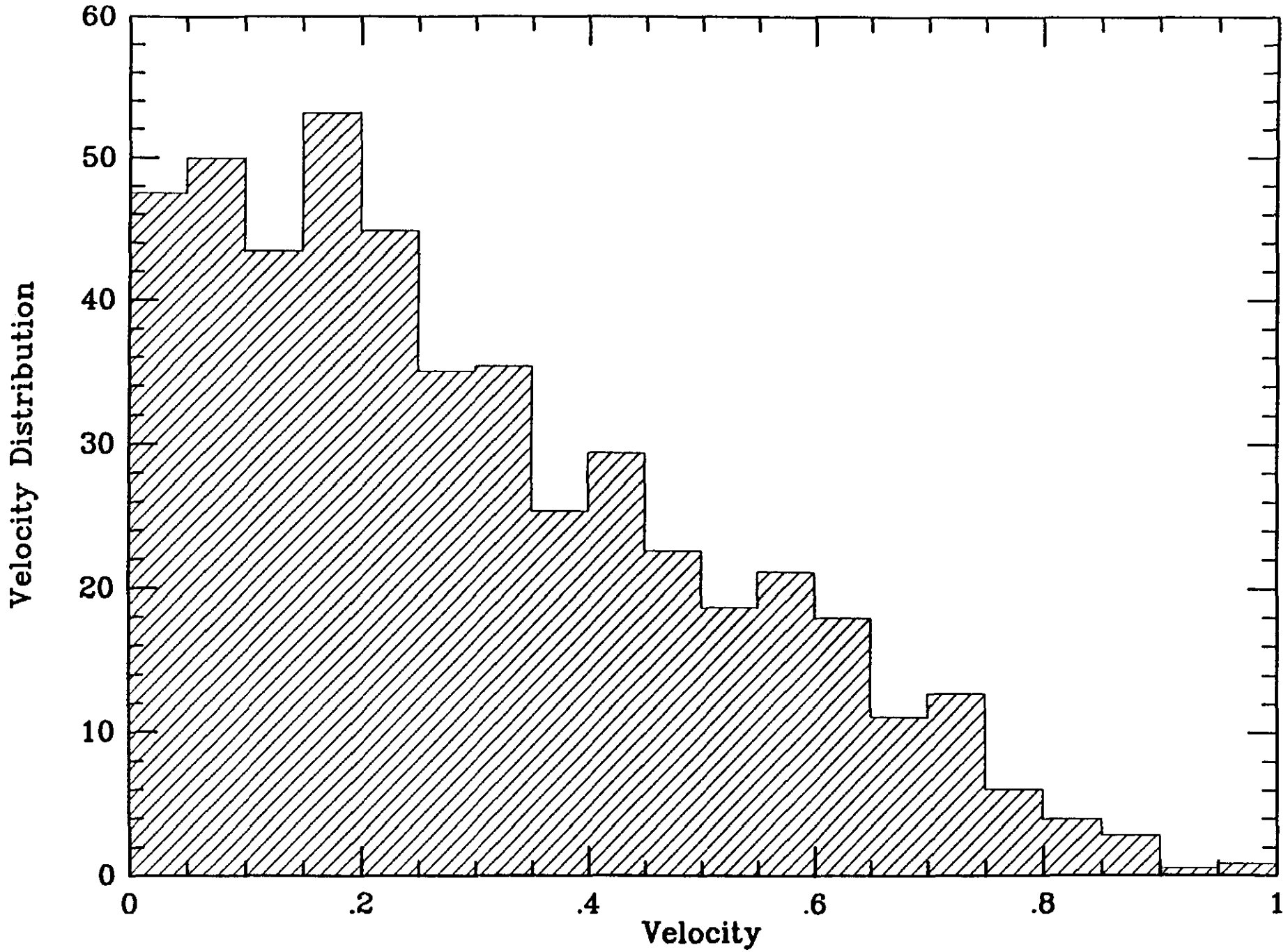
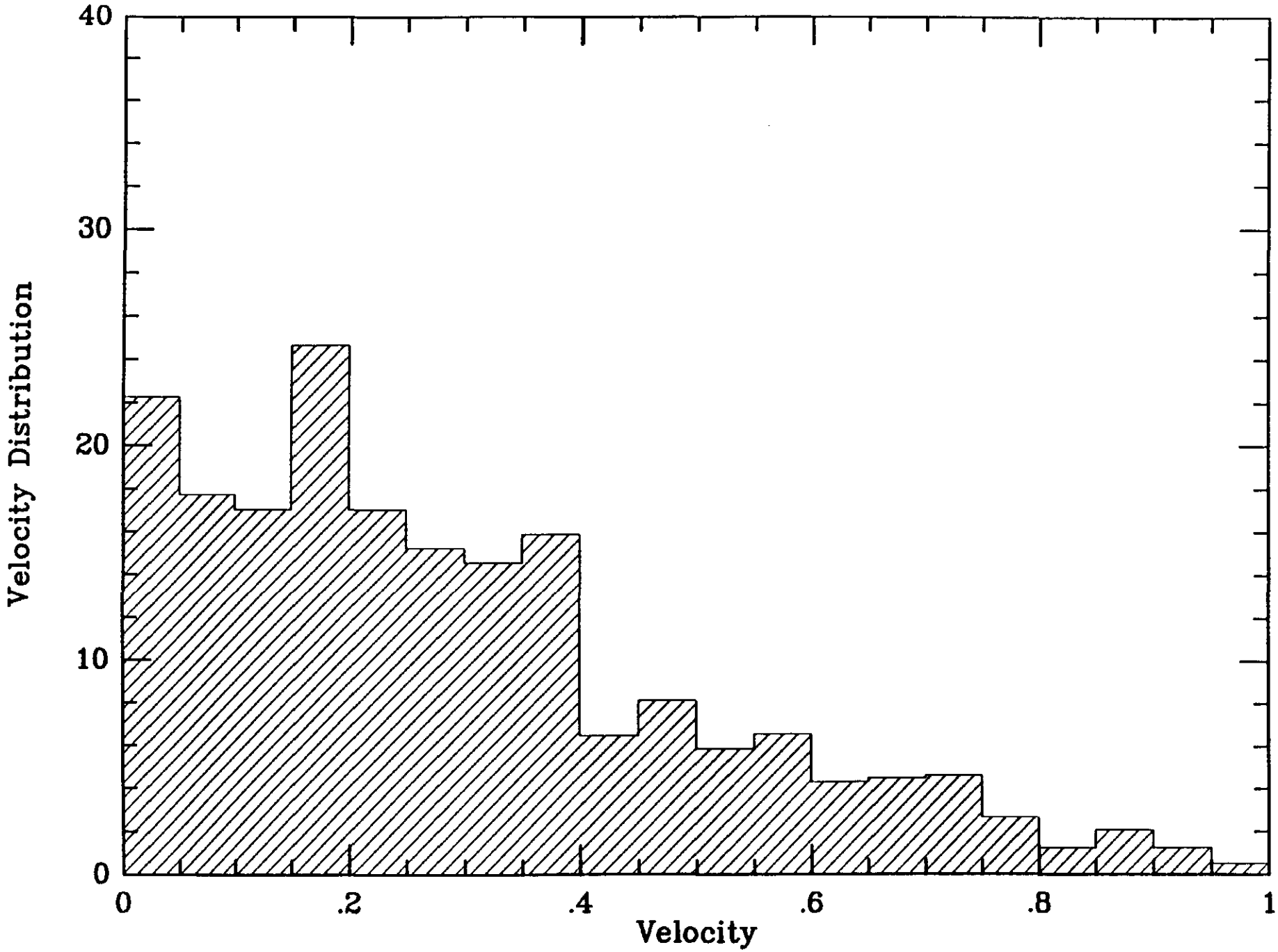


Figure 2a.



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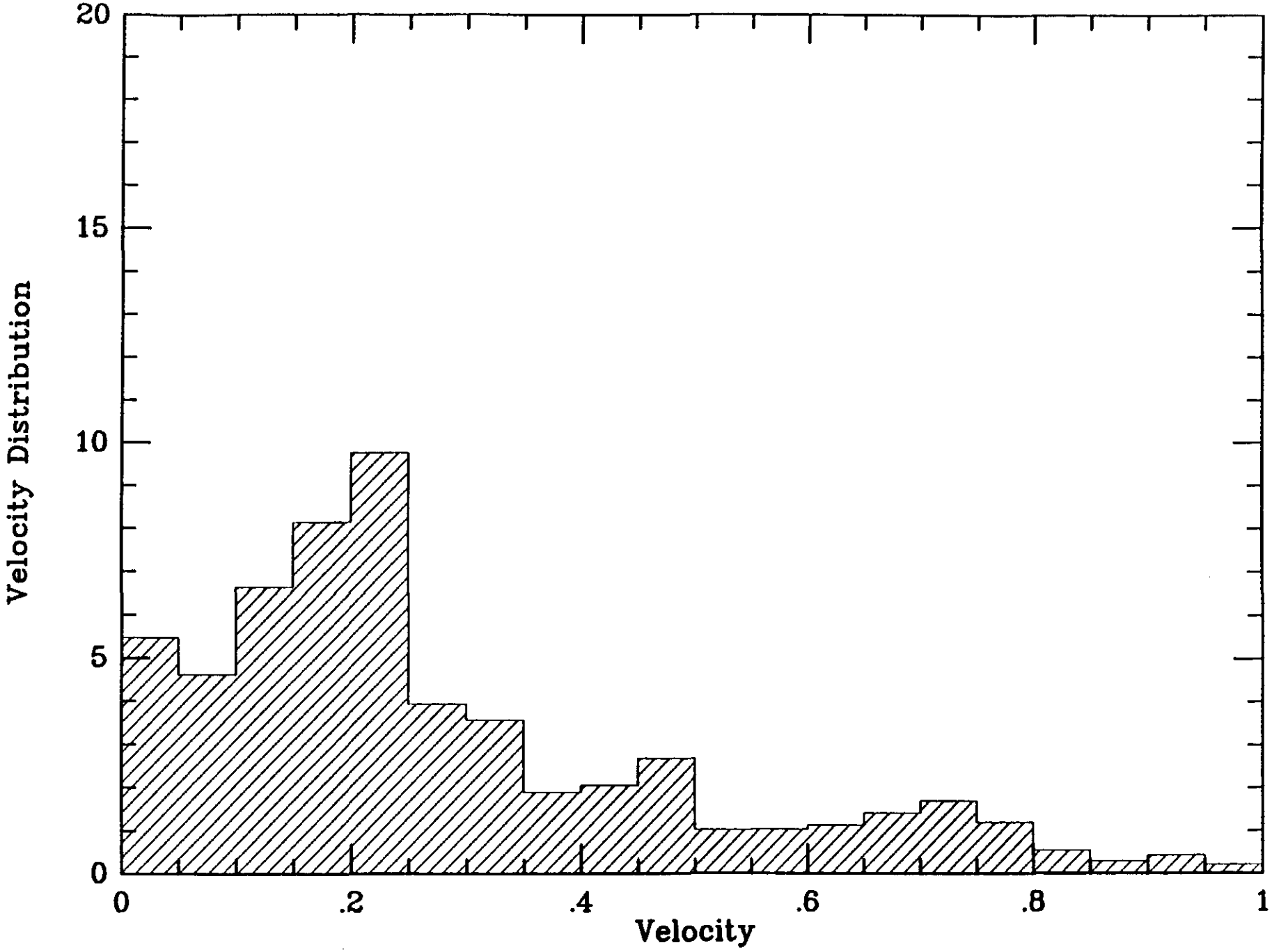

Figure 2c.

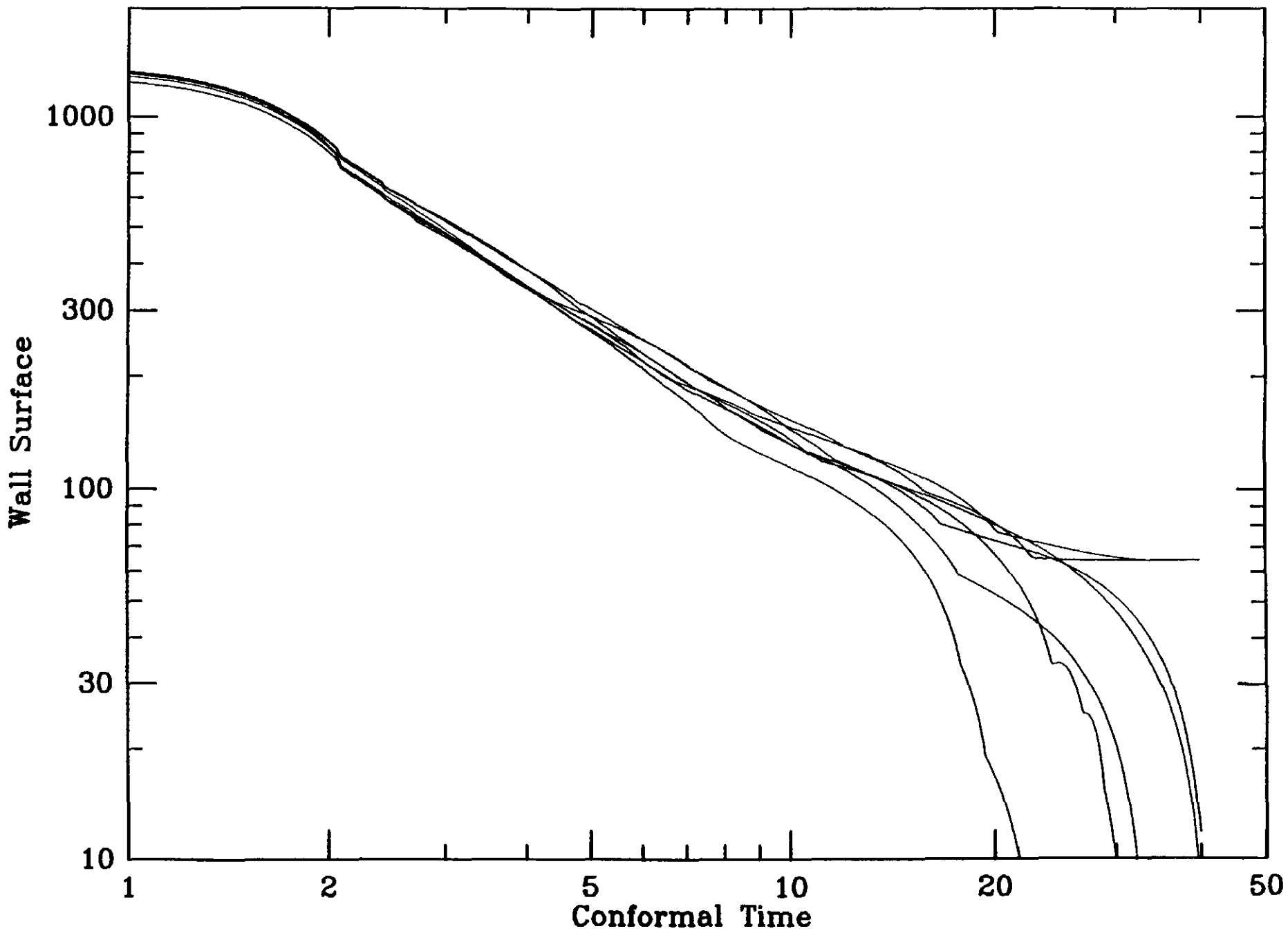

Figure 3.

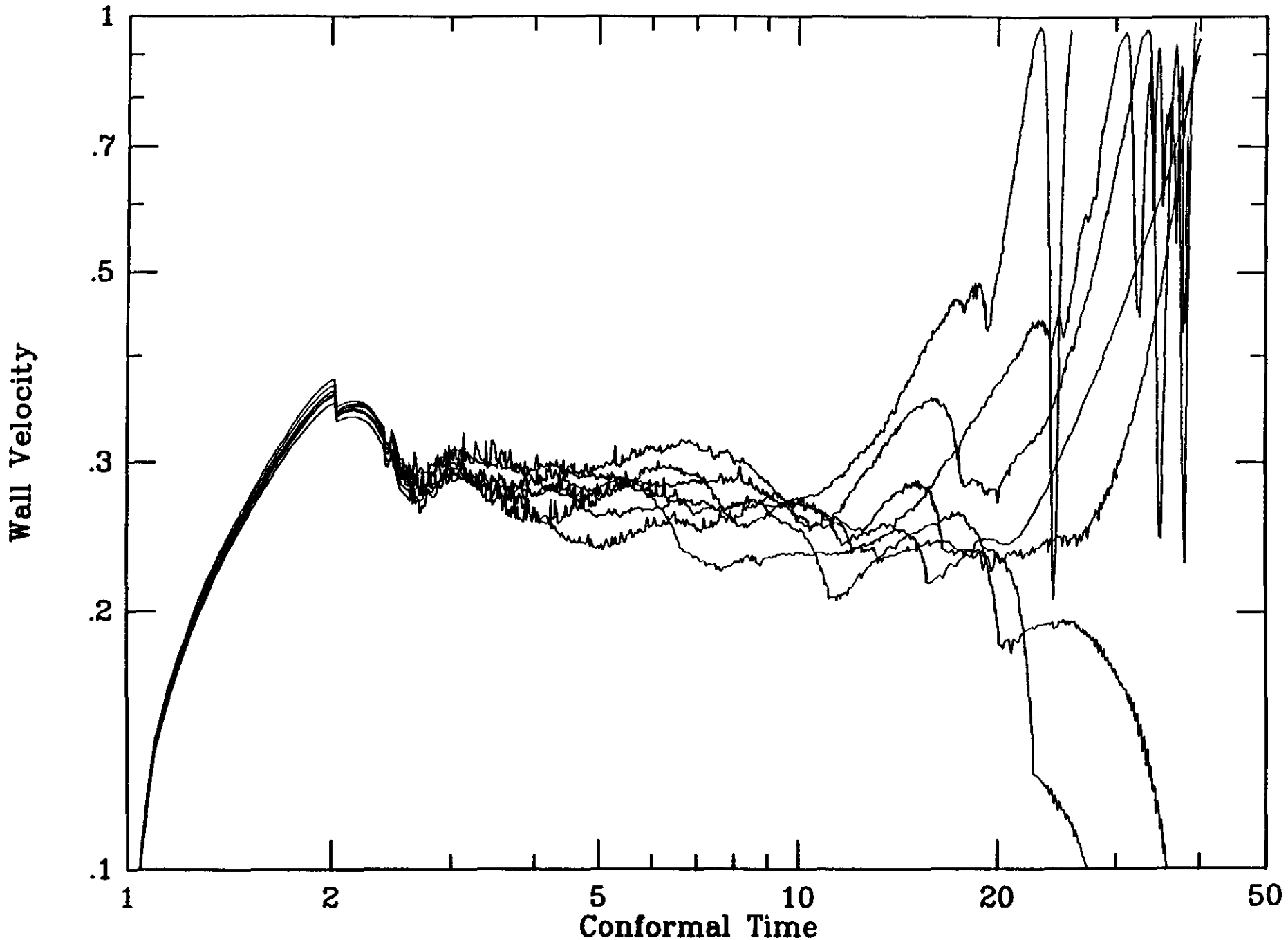

Figure 4.

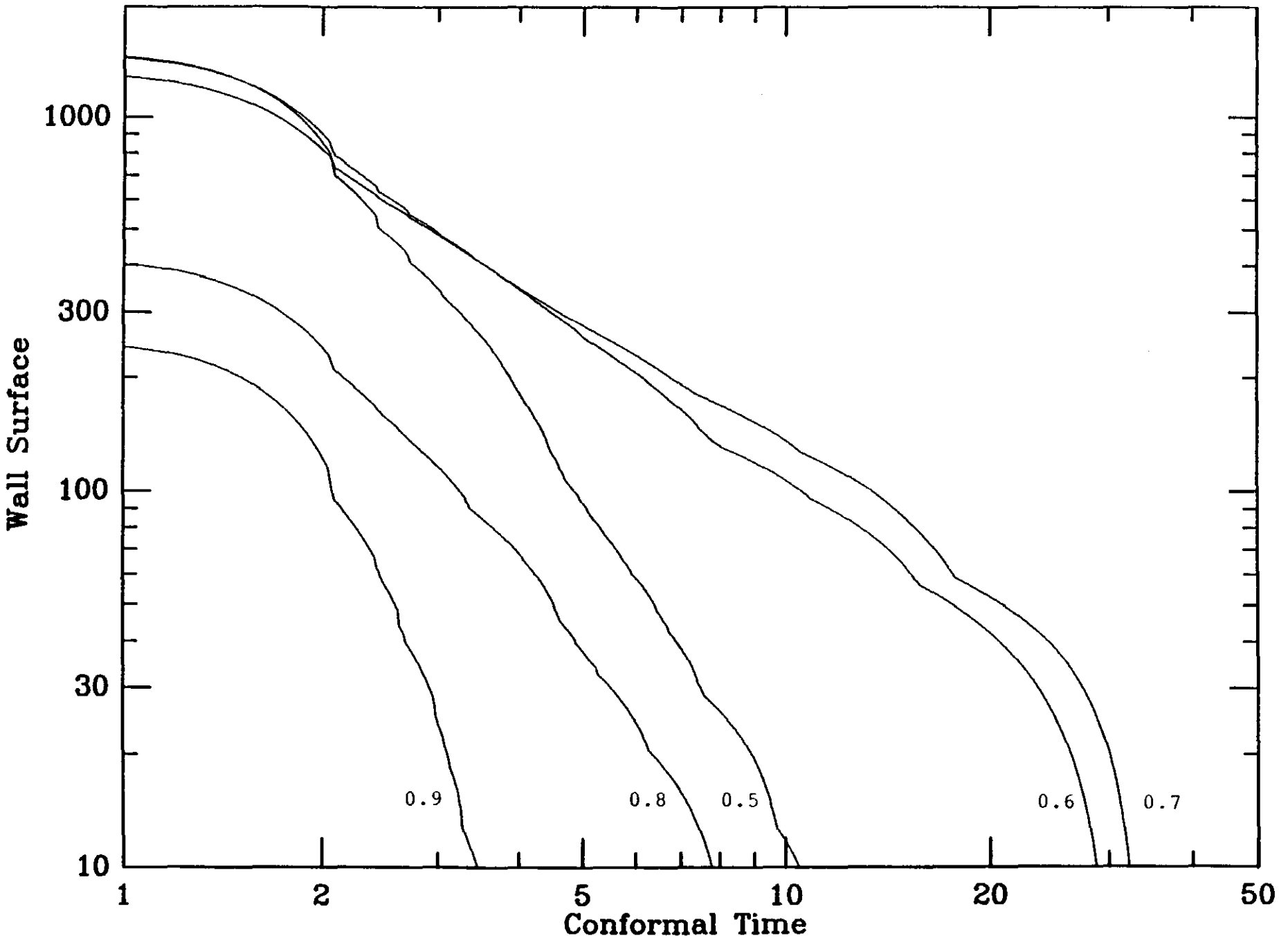
1000
300
100
30
10
Wall Surface
1
2
5
10
20
50
Conformal Time
0.9
0.8
0.5
0.6
0.7

Figure 5.

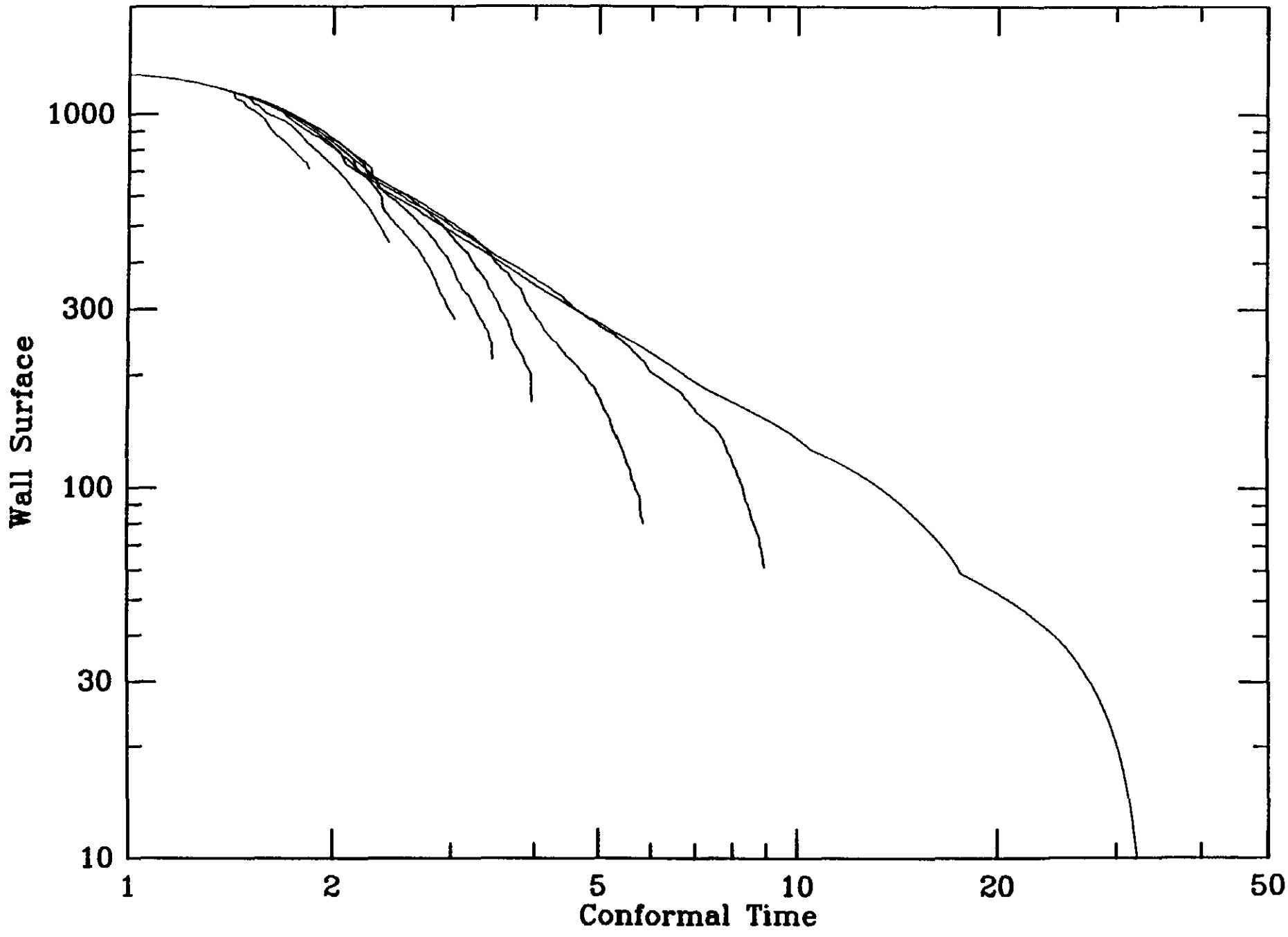

Figure 6.