An $I_c(B,\theta)$ parameterization for $YBa_2Cu_3O_{7-\delta}$ CC Tapes

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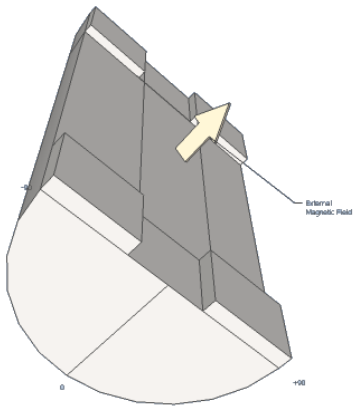
Abstract

Second generation HTS YBCO tapes show strong anisotropic behavior with respect to field orientation. An accurate understanding of how critical current scales with field and field orientation is highly needed in order to precisely include this effect into magnet design. In this document experimental data is presented and a numerical parameterization for Ic(B,θ,4.2K) is shown and discussed.

Angular Holder for In-Field I_c measurements



A setup - designed to change the direction of the c-axis of the tape with respect to the external magnetic field - has been designed for the purpose of investigating anisotropic behavior of YBCO CC tapes. The $YBa_2Cu_3O_{7-\delta}$ CC sample, supported in the middle by G10, is soldered on two copper half cylinders, fixed with screws on the side of the G10 block. A 12mm splice is used on each side of the sample to limit contact resistance and Joule heating at the maximum current level (2kA). The sample holder is placed at the desired angle and tightened to the main current leads. The probe is cooled to liquid helium to perform the measurements.

Superpower SCS4050 $YBa_2Cu_3O_{7-\delta}$ Tape

Second generation HTS YBCO tapes show strong anisotropic behavior with respect to field orientation. For this reason, an accurate understanding of how critical current scales with field and field orientation is needed in order to precisely include this effect into solenoids design. Several short samples have been tested at Fermilab on a rotating sample holder at different angle orientations up to a magnetic field of 15 T.

TABLE I. 2G HTS CONDUCTOR PROPERTIES

Tape Manufacturer	Superpower
2G HTS Tape	SCS4050 [8]
Nominal Width	4 mm
Nominal Thickness	0.1 mm
Stabilizer	Copper (2 x 20 µm)
Substrate	Hastelloy® C276 (50 µm)
YBCO layer thickness	1 µm

Tests have been run within an anti-cryostat equipped with a heater and a needle valve. By continuously regulating the flux of helium within the anti-cryostat and the heater power, the sample temperature could be held steady for the whole duration of the Ic measurement. Using this approach, critical current dependency on sample temperature has been evaluated in a range from 4.2 K up to 33 K. In Figs. 1 through 4 some results have been summarized for a number of field orientations and sample temperatures. Critical current has been defined using a 1µV/cm criterion. The critical current measured in self field conditions at 77K is 87A. θ=0° refers to parallel field to the ab-plane, while θ=90° perpendicular.

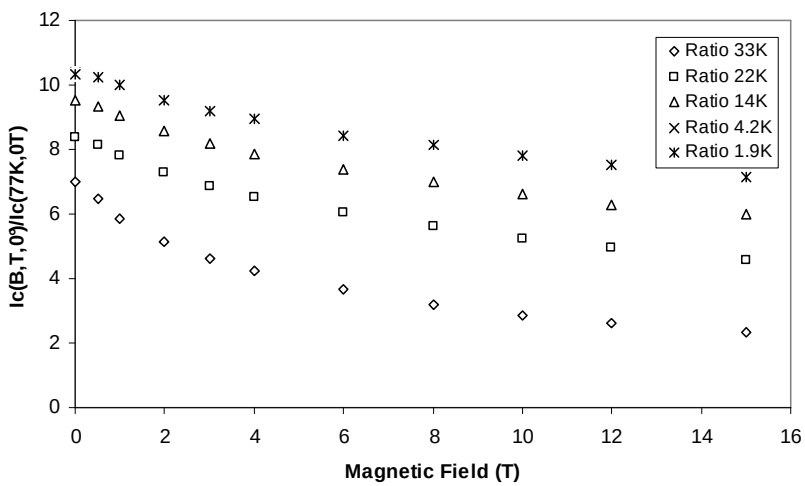


Figure 1 - Ic(B,T)/Ic(0T,77K) vs B(T) for theta=0°

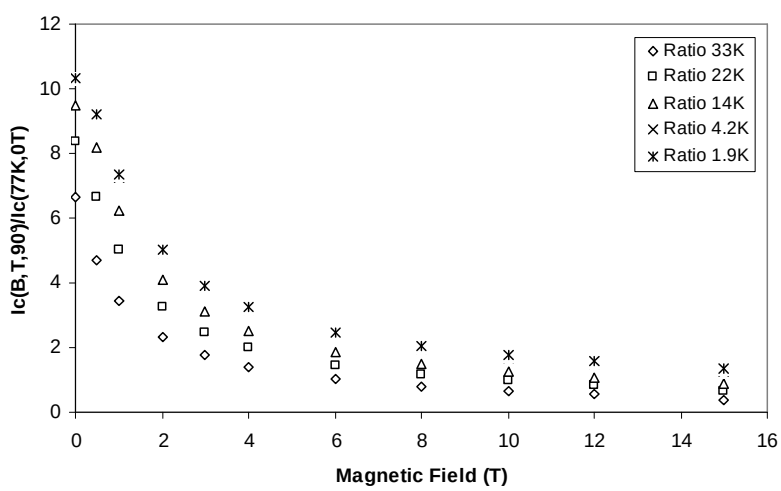


Figure 2 - Ic(B,T)/Ic(0T,77K) vs B(T) for theta=90°

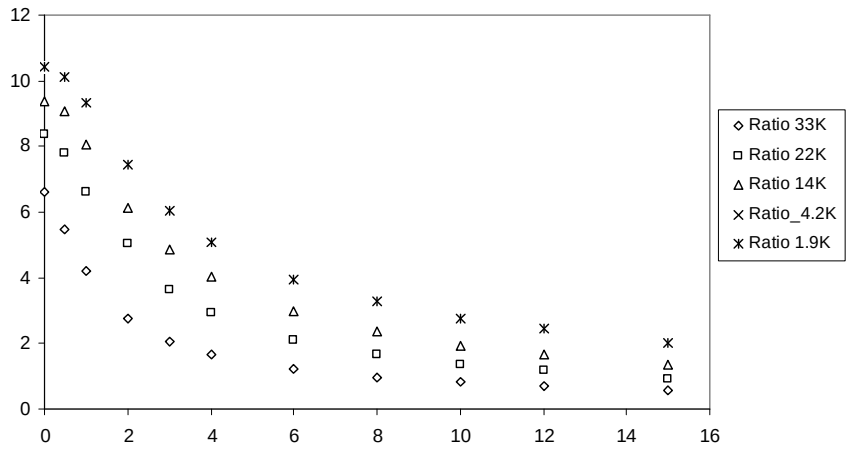


Figure 3 - Ic(B,T)/Ic(0T,77K) vs B(T) for theta=22.5°

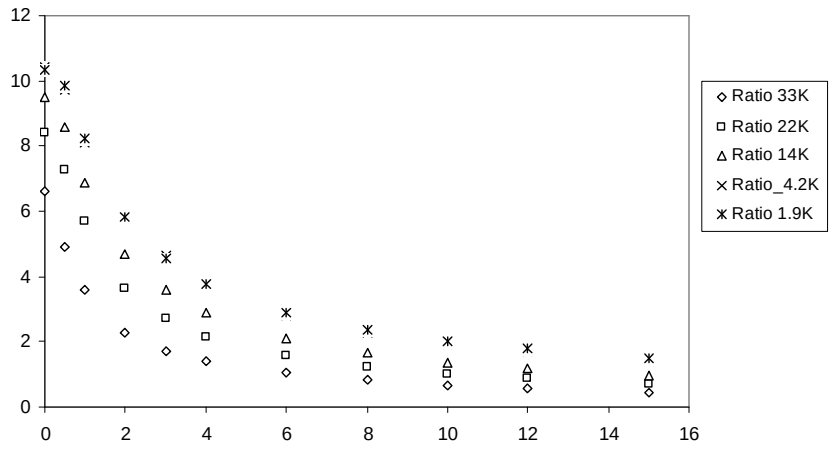


Figure 4 - Ic(B,T)/Ic(0T,77K) vs B(T) for theta=45°

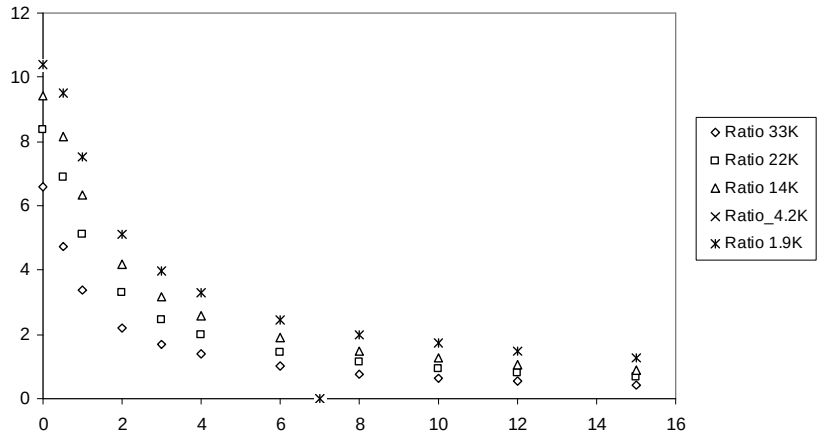


Figure 5 - Ic(B,T)/Ic(0T,77K) vs B(T) for theta=67.5°

In Figure 1 through Figure 5, the ratio between I_c(B,T) and I_c(0T,77K) is plot against the magnetic external field generated by a NbTi/Nb_3Sn external magnet as a function of temperature (from 33K down to 1.9K).

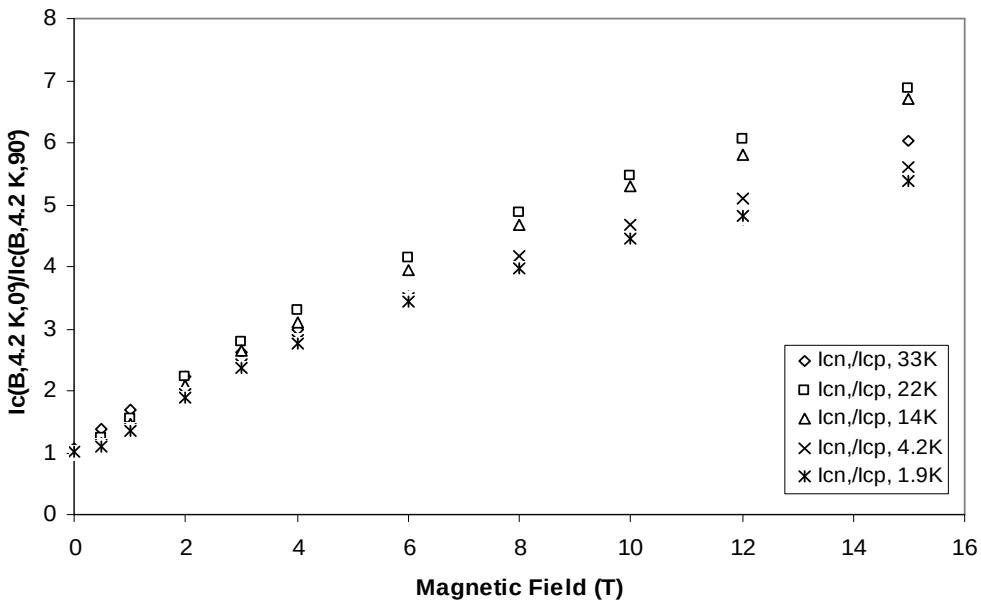


Figure 6 - Anisotropy ratio as a function of field and temperature

$I_c(B,\theta)$ Parameterization in details

Using the data for YBCO tapes from the previous paragraph, a critical current parameterization is herein proposed and fit against experimental data. As first step, only the critical current as a function of field orientation is taken into consideration. For sake of generality, the critical current in helium is scaled on the value obtained in nitrogen under self field conditions. To fit critical current behavior against the angle theta, the following expression has been used.

$$\frac{I_c(\theta)}{I_c(77K,0T)} = \frac{k}{\sqrt{\sin(\theta)^2 + \frac{\cos(\theta)^2}{\varepsilon^2}}} + \left(a(B)\sin(\theta)\right)^2 \qquad [1]$$

Plotting critical current values at 4.2K against field orientation *for several fixed values of magnetic field*, the following set of plots can be obtained. Dots represent I_c experimental data while solid lines represent the above expression fit on each of the available set of data.

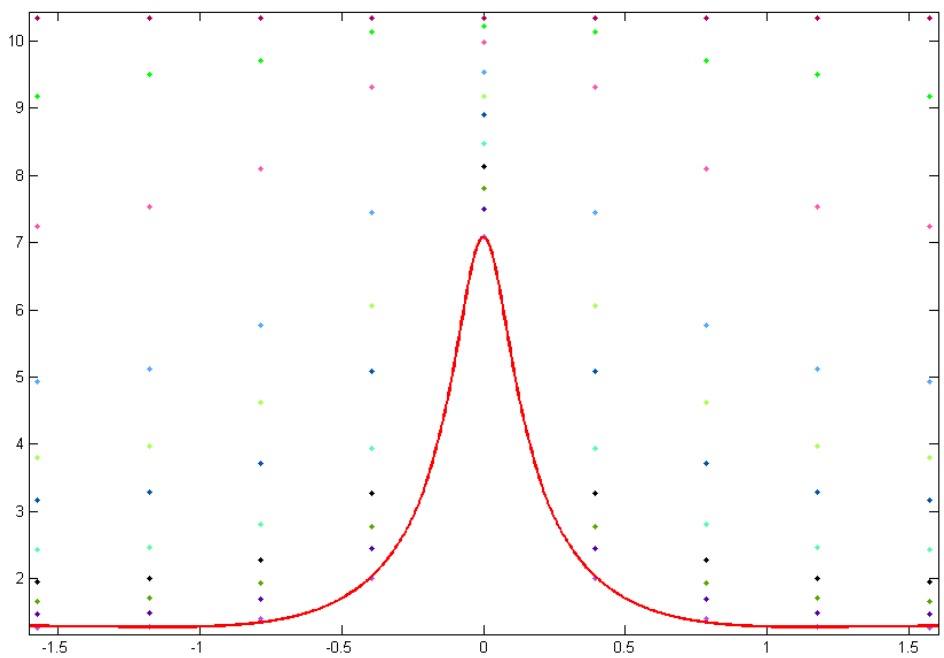


Figure 7 - Ic(15T,4.2K)/Ic(77K,0T) vs θ (rad)

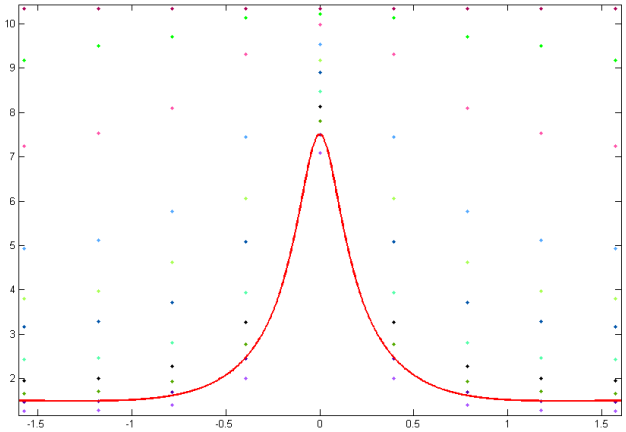


Figure 8 - Ic(12T,4.2K)/Ic(77K,0T) vs θ (rad)

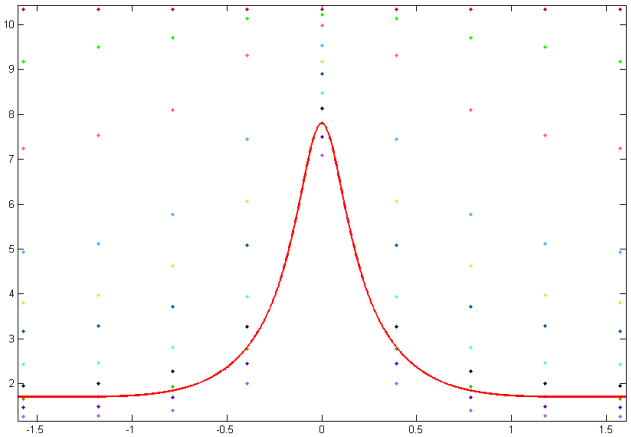


Figure 9 - Ic(10T,4.2K)/Ic(77K,0T) vs θ (rad)

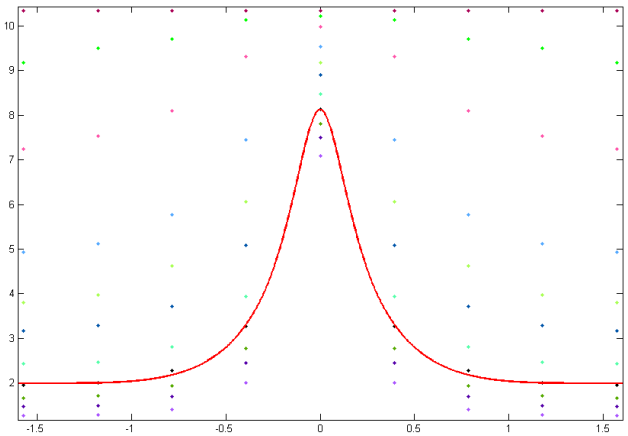


Figure 10 - Ic(8T,4.2K)/Ic(77K,0T) vs θ (rad)

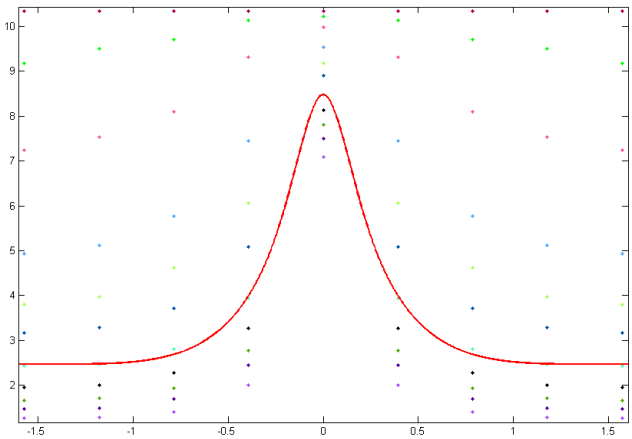


Figure 11 - Ic(6T,4.2K)/Ic(77K,0T) vs θ (rad)

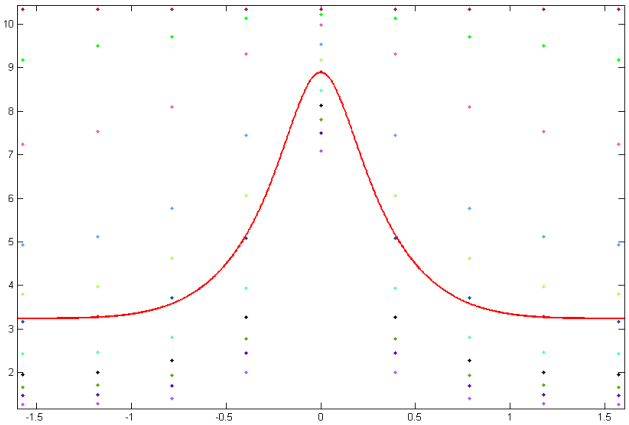


Figure 12- Ic(4T,4.2K)/Ic(77K,0T) vs θ (rad)

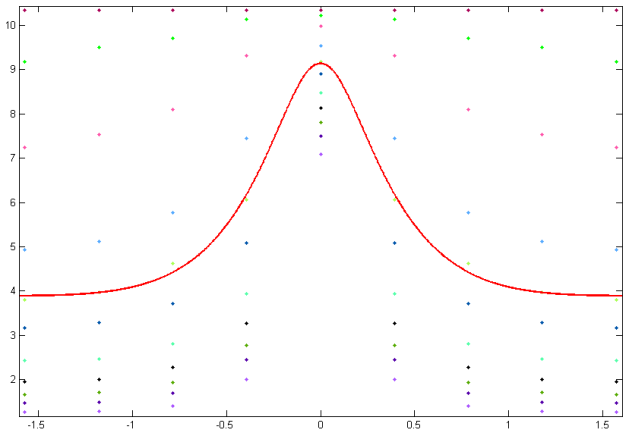


Figure 13 - Ic(3T,4.2K)/Ic(77K,0T) vs θ (rad)

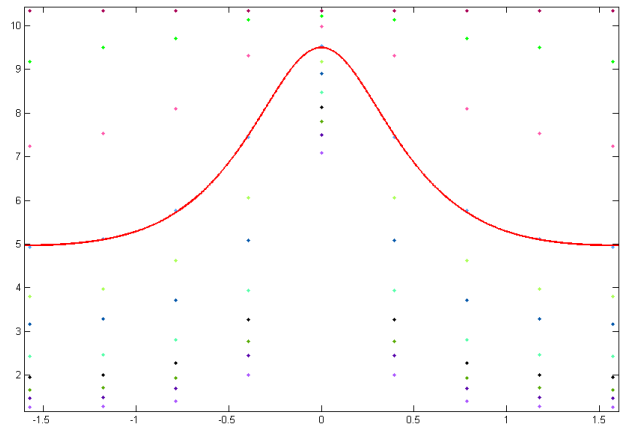


Figure 14 - Ic(2T,4.2K)/Ic(77K,0T) vs θ (rad)

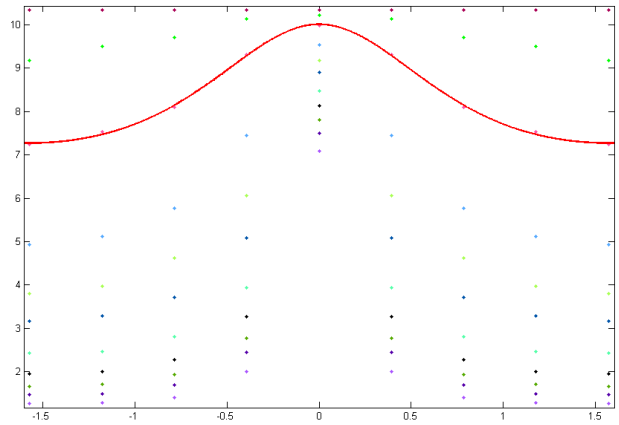


Figure 15 - Ic(1T,4.2K)/Ic(77K,0T) vs θ (rad)

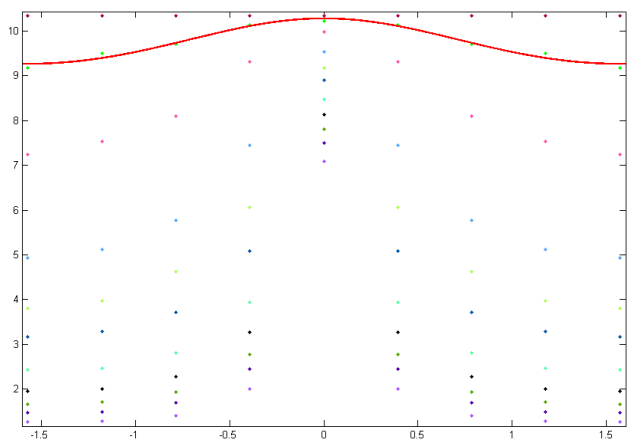


Figure 16 - Ic(0.5T,4.2K)/Ic(77K,0T) vs θ (rad)

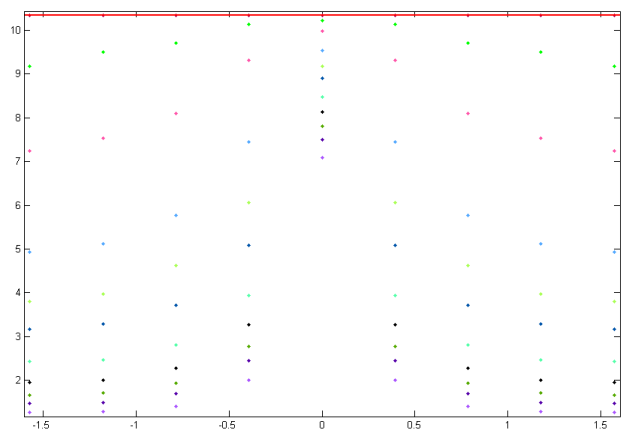


Figure 17 - Ic(0T,4.2K)/Ic(77K,0T) vs θ (rad)

Expression [1] manages to fit each of the datasets *independently* with a reasonable level of accuracy. In order to obtain an expression that describes how critical current scales simultaneously with field orientation and field itself, we can plot the values of the parameters *a, k* and ε as a function of field as resulted by the different regressions run to fit each of the previous plots. In doing so, we obtain the following plots for *a(B), k(B)* and ε *(B).*

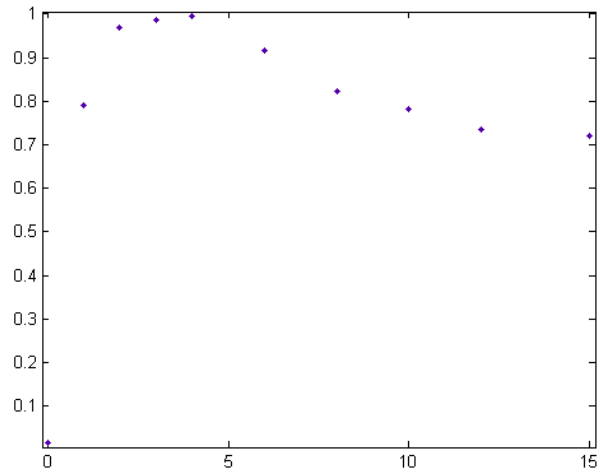


Figure 18 - Coefficient a vs B (T) as obtained from regression

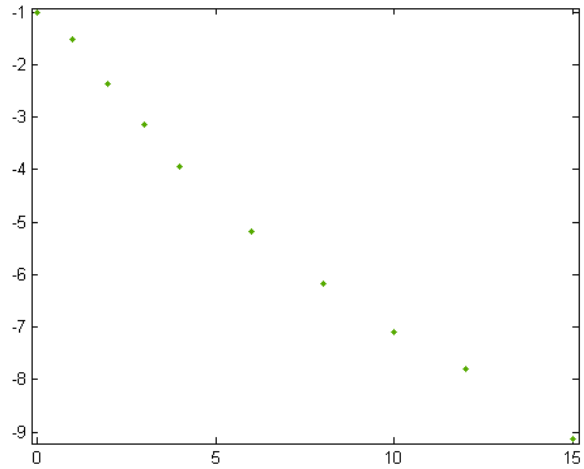


Figure 19 - Coefficient epsilon vs B (T) as obtained from regression

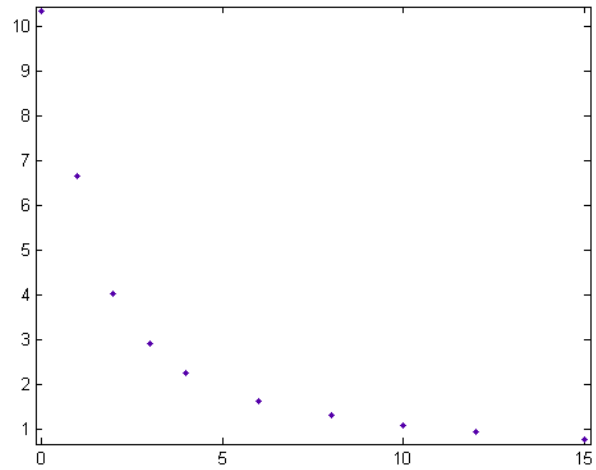


Figure 20 - Coefficient k vs B (T) as obtained from regression

Under parallel field (θ =0), the above general expression now simplifies as follows

$$I_c(\theta=0)=\frac{k(B)}{\sqrt{\frac{1}{\varepsilon(B)^2}}}=k(B)\cdot\varepsilon(B)$$

In order to fit the critical current behavior as a function of field only, we'll try to fit the product $k(B)\cdot\varepsilon(B)$ against actual experimental data $I_c(\theta=0)$.Following this approach, we'll need to fit the curve in Figure 21.

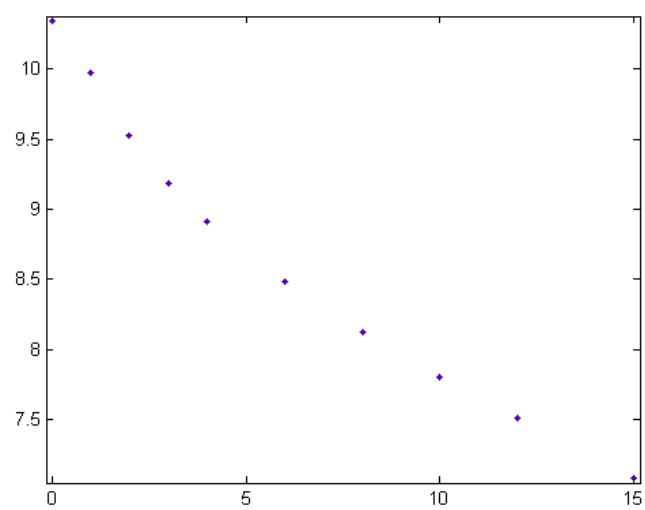


Figure 21 - Ic(4.2K, theta=0) vs B(T)

We can fit the available experimental data for $I_c(\theta=0)$ using the following sum of exponential terms

$$k(B)\cdot\varepsilon(B)=\left[a_1\cdot e^{-\left(\frac{B-b_1}{c_1}\right)^2}+a_2\cdot e^{-\left(\frac{B-b_2}{c_2}\right)^2}+a_3\cdot e^{-\left(\frac{B-b_3}{c_3}\right)^2}\right]$$

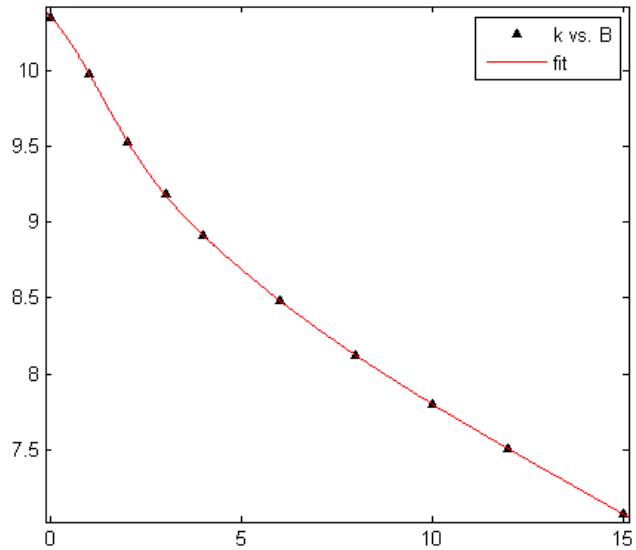


Figure 22 – k*e= Ic(4.2K, theta=0) vs B(T) fitted as a sum of exponential terms

We now need to fit *a(B)* and $\varepsilon(B)$ separately to obtain the full expression for critical current as a function of field and field orientation. We can do this using the following rational expressions

$$a(B) = \frac{\sum_{i=0}^{4} p_i \cdot B^i}{\sum_{i=0}^{3} q_i \cdot B^i}$$

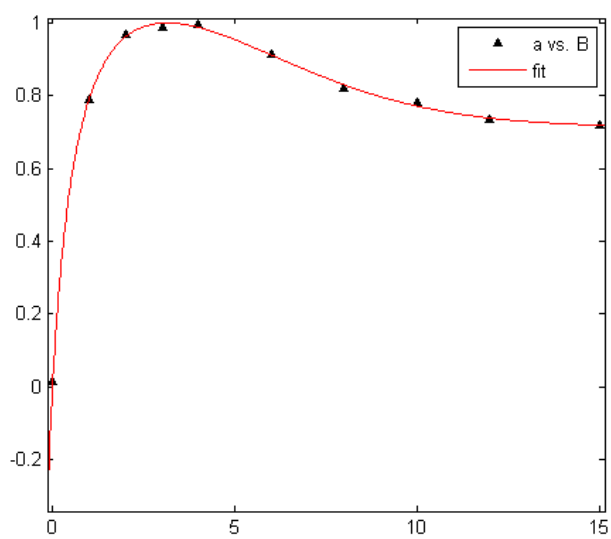


Figure 23 – a vs B(T) fitted as rational expression

$$\varepsilon(B) = \frac{\sum_{i=0}^{3} m_i \cdot B^i}{\sum_{i=0}^{2} n_i \cdot B^i}$$

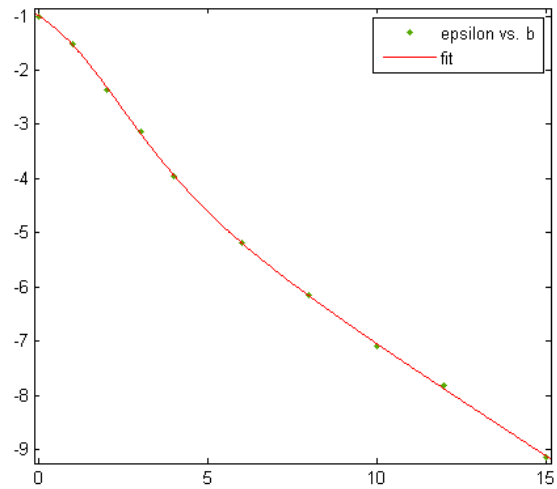


Figure 24 - epsilon vs B(T) fitted as rational expression

Given the previous expression for the product $k(B) \cdot \varepsilon(B)$ and the expression found for $\varepsilon(B)$, we can reformulate $k(B)$ as follows

$$k(B) = \left[a_1 \cdot e^{-\left(\frac{B-b_1}{c_1}\right)^2} + a_2 \cdot e^{-\left(\frac{B-b_2}{c_2}\right)^2} + a_3 \cdot e^{-\left(\frac{B-b_3}{c_3}\right)^2} \right] \cdot \frac{\sum_{i=0}^{3} n_i \cdot B^i}{\sum_{i=0}^{2} m_i \cdot B^i}$$

Substituting the three analytical fitting expressions found for $\varepsilon(B)$, *a(B)* and *k(B)* into the general equation, the following full parameterization for the critical current of YBCO tapes can be obtained.

$$I_c(B,\theta) = \frac{\left[a_1 \cdot e^{-\left(\frac{B-b_1}{c_1}\right)^2} + a_2 \cdot e^{-\left(\frac{B-b_2}{c_2}\right)^2} + a_3 \cdot e^{-\left(\frac{B-b_3}{c_3}\right)^2}\right] \cdot \frac{\sum_{i=0}^{3} n_i \cdot B^i}{\sum_{i=0}^{2} m_i \cdot B^i}}{\sqrt{\sin(\theta)^2 + \frac{\cos(\theta)^2}{\left(\frac{\sum_{i=0}^{3} m_i \cdot B^i}{\sum_{i=0}^{2} n_i \cdot B^i}\right)^2}}} + \left(\frac{\sum_{i=0}^{4} p_i \cdot B^i}{\sum_{i=0}^{3} q_i \cdot B^i} \cdot \sin(\theta)^2\right)^2$$

In Figure 25 the above expression is plotted against some of the available experimental data to show how the analytical expression reproduces critical current values measured on the HTS sample for several incidence angles. (in Figure 25 - theta = 0, 90 and 45 deg)

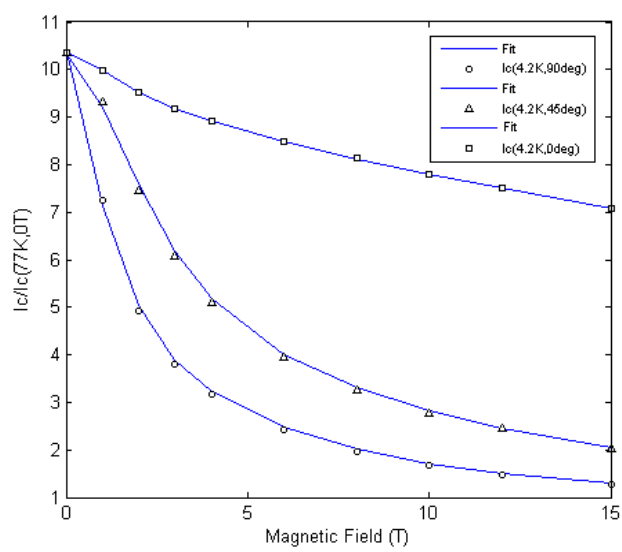


Figure 25 - Fit vs experimental data for several angles and fields

In Figure 26 the final expression is plot for continuous values of magnetic fields and field orientation.

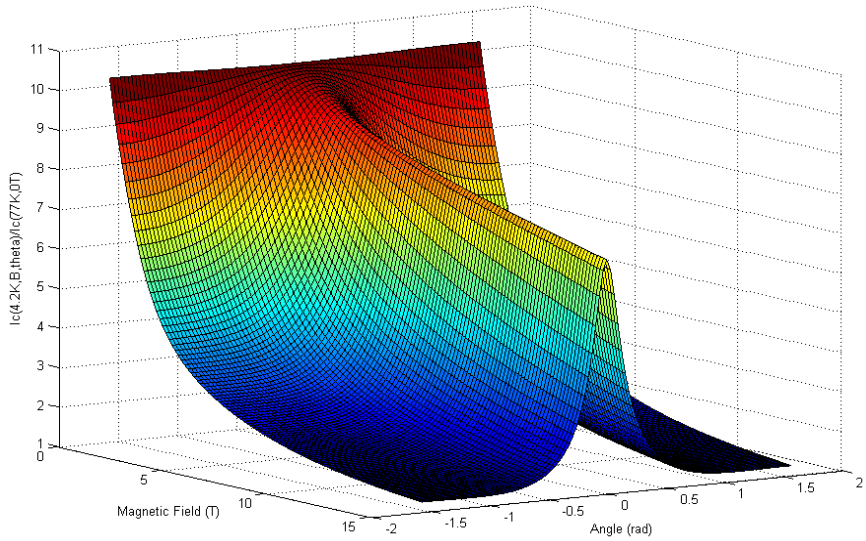


Figure 26 - Critical Current ratio as a function of field and field orientation

As it can be noticed from Figure 26, the shape of the fitted curve is a straight line (same critical current for each field orientation) under self field conditions, and it gets increasingly sharper around 0 radians (parallel field to ab plane) as magnetic field increases.

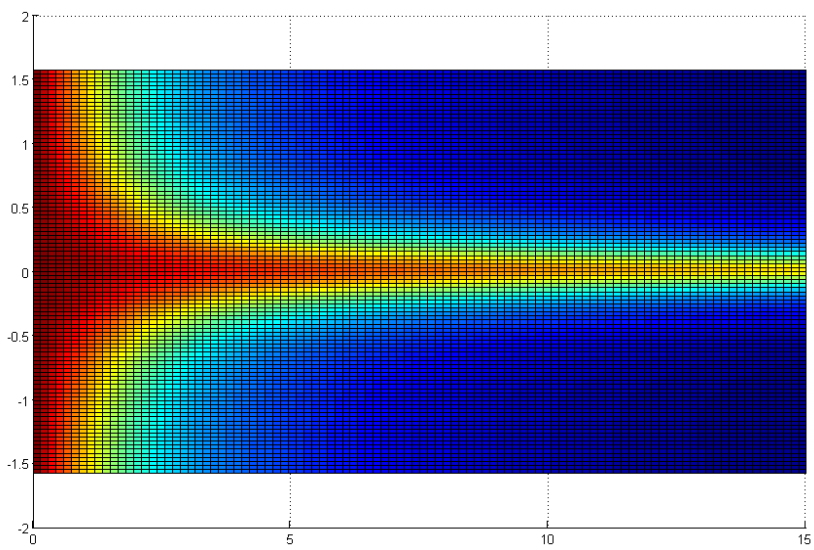


Figure 27 – Critical Current ratio as a function of field and field orientation (Top view)