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Angular Distribution of the J/ψ π° events ON and OFF the $^1P_1$ Resonance

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Abstract

The angular distribution of the J/ψ π° signal in the $^1P_1$ energy region is compared to the distribution of the background events (off resonance and in the $\eta_c^{'}$ region)

Introduction

We are studying the reaction:

$$p\bar{p} \longrightarrow {}^1P_1 \longrightarrow J/\psi + \pi^\circ \tag{1}$$

to be compared with the background :

$$p\bar{p} \longrightarrow J/\psi + \pi^\circ \tag{2}$$

(the J/ψ is detected through its e^+e^- decay).

The formation of the $^1P_1$ as intermediate state in (1), constrains the orbital angular momentum both in the initial and final state. Given the $^1P_1$ quantum numbers:

$$J = 1 \quad P = +1 \quad C = -1,$$

the $p\bar{p}$ initial state must have:

$$P(p\bar{p}) = (-1)^{L+1} = +1 \qquad C(p\bar{p}) = (-1)^{L+S} = -1$$

This implies:

$$L = odd \qquad with \quad S = 0;$$

in particular $L = 1$ to have $J = 1$. If the z-axis is taken along the incident $\bar{p}$ direction, the projection of J along this axis should be:

$$J_z = M = L_z + S_z = 0$$

since $L_z = 0$. Then:

$$|initial>=|JM>=|10>$$

To match the ${}^1P_1$ quantum numbers, the final state $J/\psi\ \pi^\circ$ has an orbital angular momentum $l\ =\ even$:

$$P = P(J/\psi) \times P(\pi^\circ) \times (-1)^l = (-1)(-1)(-1)^l = (-1)^l$$

$$P = (-1)^l = +1 \qquad \Rightarrow \qquad l = even$$

$$C = C(J/\psi) \times C(\pi^\circ) = (-1)(+1) = -1.$$

The angular momentum conservation in the decay:

$${}^1P_1 \longrightarrow J/\psi + \pi^\circ$$

$J_{{}^1P_1} = 1 = l + J_{J/\psi} = l + 1$ constrains the possible values of $l = 0, 2$.
The final state can then be a mixture of a S and D wave:

$$|final>= A\ |Swave> +B\ |Dwave>=|JM>$$

1.

pure S wave

$$|JM>=|10>=A\ Y_0^0\ |10>_{J/\psi}$$

The decay amplitude is then $T = A\ Y_0^0$, yielding a flat angular distribution. Using the helicity formalism language, this corresponds to the case where the 3 helicity states for the J/ψ are equally fed by the ${}^1P_1$ decay: in this case the two indipendent helicity amplitudes $f^1_{\pm1,0}$ and $f^1_{0,0}$ are equal.

2.

pure D wave

$$|JM>=|10>=B\,(\sqrt{(3/10)}\,Y_2^1\,|1-1>_{J/\psi} - \sqrt{(2/5)}\,Y_2^0\,|10>_{J/\psi} + \sqrt{(3/10)}\,Y_2^{-1}\,|11>_{J/\psi})$$

Assuming A=0 the angular distribution is:

$$W(\theta) \propto \frac{2}{5}|Y_2^0|^2 + \frac{3}{10}(|Y_2^1|^2 + |Y_2^{-1}|^2) = \frac{1}{8\pi}(1 + 3\cos^2\theta)$$

This corresponds to have $f^1_{0,0}\ =\ -2\ f^1_{\pm1,0}$

3. **In the general case, both waves can contribute to the angular distribution.**

In the case of reaction (2), where no intermediate state constrains the initial or final angular momentum, we can have:

$$L = odd \quad S = 0 \quad \Rightarrow \quad l = even \qquad (a)$$

$$L = even \quad S = 1 \quad \Rightarrow \quad l = odd \qquad (b)$$

This is easily obtained by conserving P and C quantum numbers:

$$P(p\bar{p}) = (-1)^{L+1} \qquad P(J/\psi\pi^\circ) = (-1)^l$$

$$C(p\bar{p}) = (-1)^{L+S} \qquad C(J/\psi\pi^\circ) = -1.$$

$$J = L + S = l + J_{J/\psi} = l + 1$$

$$M = L_z + S_z = 0 \qquad L = odd, \quad S = 0$$

$$M = L_z + S_z = 0, \pm 1 \qquad L = even, \quad S = 1.$$

To study the background we should consider that in the final state, besides the S and D wave (already discussed for the $^1P_1$ formation, with L = 1 and S = 0), an extra P wave should be taken into account.

$$l = 1 \qquad P\ wave$$

$$J = l + J_{J/\psi} = 1 + 1 = 0, 1, 2.$$

Assuming $L = 0$, for the $p\bar{p}$ state:

$$J = L + S = 1 \qquad M = 0, \pm 1$$

The possible initial state $|JM>$ are then: $|10>$, $|11>$ and $|1-1>$. The corresponding decay amplitudes are:

$$T(|10>) = \sqrt{\frac{1}{2}} Y_1^{-1} |11> - \sqrt{\frac{1}{2}} Y_1^1 |1-1>$$

$$T(|11>) = \sqrt{\frac{1}{2}} Y_1^1 |10> - \sqrt{\frac{1}{2}} Y_1^0 |11>$$

$$T(|1-1>) = -\sqrt{\frac{1}{2}} Y_1^{-1} |10> + \sqrt{\frac{1}{2}} Y_1^0 |1-1>$$

Assuming to sum over the three polarization of the initial state with the J/ψ helicity states equally populated we expect a flat angular distribution, being:

$$T^2(|1 \pm 1>) = \frac{3}{16\pi}(1 + \cos^2\theta)$$

$$T^2(|10>) = \frac{3}{8\pi}\sin^2\theta$$

Assuming $L = 2$ for the $p\bar{p}$ state:

$$J = L + S = 2 + 1 = 1, 2, 3 \qquad M = 0, \pm 1$$

we can have $J = 1, 2$ with $M = 0, \pm 1$, compatible with the P wave ($l = 1$) in the final state.

If $J = 1$ the result is a flat angular distribution (as discussed above).

If $J = 2$, summing over the three polarization of the initial state, we obtain an angular distribution:

$$W(\theta) \propto T^2(|20>) + T^2(|21>) + T^2(|2-1>)$$

$$W(\theta) \propto \frac{1}{4\pi}(2 + 3cos^2\theta)$$

$\|initial>$	$\|final>$	$W(\theta)$
$L \quad S \quad J$	l	
1 0 1	0	$1/4\pi$ (A)
1 0 1	2	$\frac{1}{8\pi}(1 + 3\cos^2\theta)$ (B)
0 1 1	1	$1/4\pi$
2 1 1	1	$1/4\pi$
2 1 2	1	$\frac{1}{4\pi}(2 + 3\cos^2\theta)$ (C)

Table 1: Expected angular distribution as a function of the initial and final quantum numbers

Data analysis

Figure 1 shows the $\cos\theta^*$ of the π° (c.m. frame) as calculated by the fit for data selected by the analysis as $J/\psi\ \pi^\circ$ candidates (see ${}^1P_1$ paper). In particular fig. 1a and 1d are respectively the distribution of the events at the ${}^1P_1$ resonance and off (${}^1P_1$ and $\eta_c^{'}$ energies).

To study if there is any evidence of a preferred distribution of the signal events respect to the background, we used a Monte Carlo that generates the events with the standard CERN package GENBOD.

As discussed before we made the assumption that the helicity states of the J/ψ are equally populated. This results in the further assumption that the final decay $J/\psi \to e^+e^-$ has a flat distribution. Choosing for this decay a $(1+\cos^2\theta^*)$ distribution in the other extreme hypothesis doesn't imply any relevant difference to the final conclusions.

The three distributions A, B and C are used to generate $J/\psi\ \pi^\circ$ events at the c.m. energy of the ${}^1P_1$ (the distributions doesn't change drastically fixing the c.m. energy at the $\eta_c^{'}$ mass - 3610 MeV -).

The geometrical acceptance of our detector was imposed using the following cuts:

$$15^\circ < \theta_{e_{1,2}} < 60^\circ$$

$$2^\circ < \theta_{\gamma_{1,2}} < 68^\circ$$

In figure 2 the continous and dashed line are respectively the data and the MC (normalized to data) distribution for each of the three hypothesis A, B and C.

The following table summarizes the results of the χ^2 calculation, where:

$$\chi^2 = \sum_{i=1}^{10} \frac{(n_i^{exp} - n_i^{th})^2}{\sigma_i^2}$$

with:

n_i^{exp} number of events in the i-th bin

n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin

σ_i poissonian errors of n_i^{exp}

	A	B	C
ON	1.19	0.51	–
OFF	0.42	0.73	0.46

Table 2: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

Hp.: The J/ψ decay has a flat distribution.

1

The following table summarizes the results of the χ^2 calculation, where:

$$\chi^2 = \sum_{i=1}^{10} \frac{(n_i^{exp} - n_i^{th})^2}{\sigma_i^2}$$

with:

n_i^{exp} number of events in the i-th bin

n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin

σ_i poissonian errors of n_i^{exp}

	A	B	C
ON	1.19	0.51	0.68
$-\ln(lik)$	19.85	18.68	18.52
OFF	0.42	0.73	0.46
$-\ln(lik)$	15.97	17.41	16.56

Table 1: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

N.B. $-\ln(lik)$ is the same for all the χ^2 calculations.

2

The following table summarizes the results of the χ^2 calculation, where:

$$\chi^2 = \sum_{i=1}^{10} \frac{(n_i^{exp} - n_i^{th})^2}{\sigma_i^2}$$

with:

n_i^{exp} number of events in the i-th bin

n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin

σ_i errors as $\sqrt{n_i^{th}}$

3

The following table summarizes the results of the χ^2 calculation, taking the formula of the

DPG for bins with few events, where:

$$\chi^2 = \sum_{i=1}^{10} 2(n_i^{th} - n_i^{exp}) + 2n_i^{exp}\ln(n_i^{exp}/n_i^{th})$$

	A	B	C
ON	0.85	0.69	0.59
OFF	0.42	0.75	0.57

Table 2: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

with:
n_i^{exp} number of events in the i-th bin
n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin

	A	B	C
ON	0.90	0.64	0.61
OFF	0.43	0.75	0.56

Table 3: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

Hp.: The J/ψ decay has a distribution $(1+\cos^2\theta^*$.

The following table summarizes the results of the χ^2 calculation, where:

$$\chi^2 = \sum_{i=1}^{10} \frac{(n_i^{exp} - n_i^{th})^2}{\sigma_i^2}$$

with:

n_i^{exp} number of events in the i-th bin
n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin
σ_i poissonian errors of n_i^{exp}

	A	B	C
ON	1.46	0.57	0.67
$-\ln(lik)$	20.34	18.69	18.46
OFF	0.57	0.65	0.48
$-\ln(lik)$	16.63	17.11	16.48

Table 1: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

N.B. $-\ln(lik)$ is the same for all the χ^2 calculations.

The following table summarizes the results of the χ^2 calculation, where:

$$\chi^2 = \sum_{i=1}^{10} \frac{(n_i^{exp} - n_i^{th})^2}{\sigma_i^2}$$

with:

n_i^{exp} number of events in the i-th bin
n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin
σ_i errors as $\sqrt{n_i^{th}}$

	A	B	C
ON	0.94	0.66	0.59
OFF	0.57	0.69	0.55

Table 2: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$

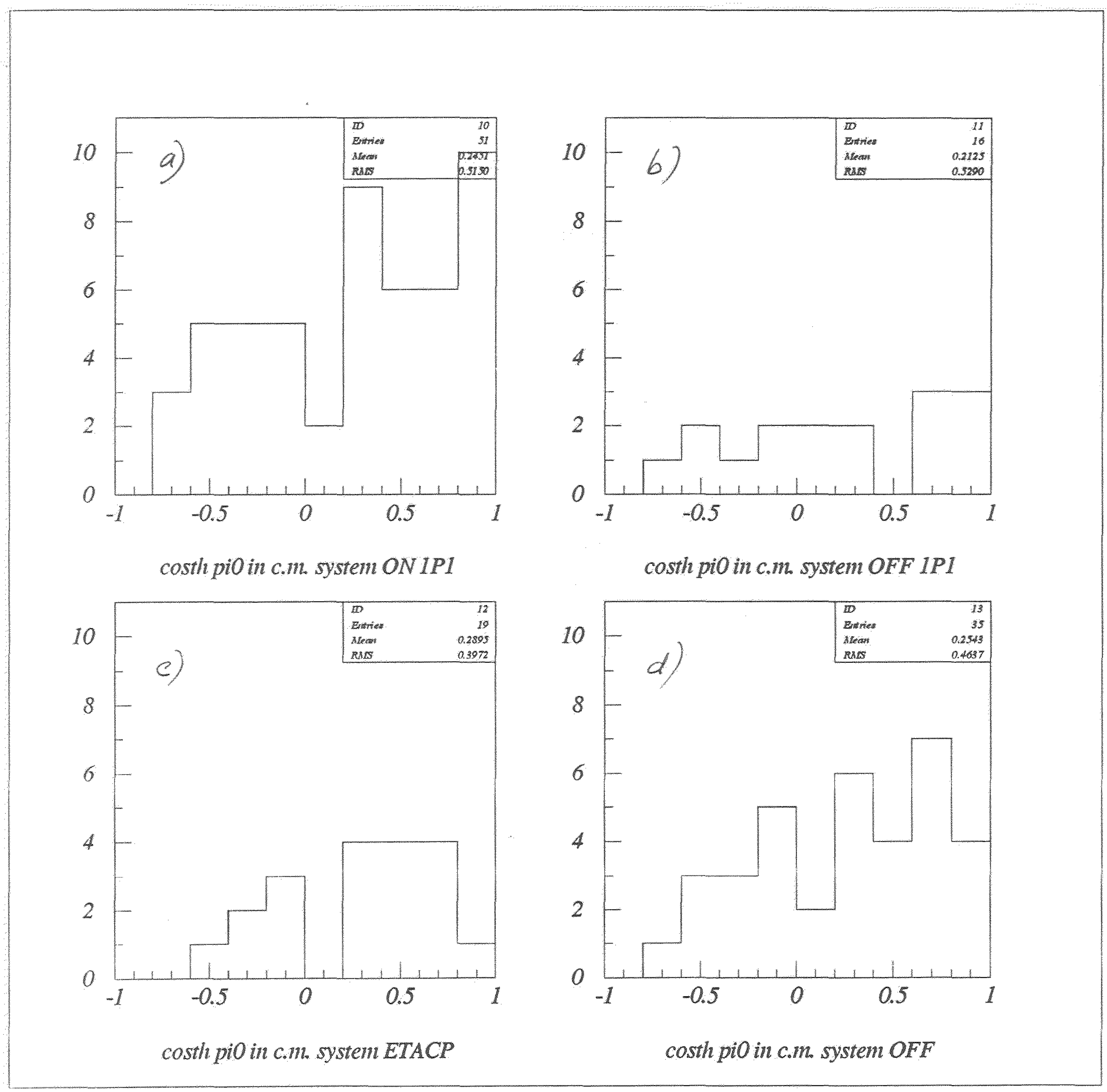
The following table summarizes the results of the χ^2 calculation, taking the formula of the DPG for bins with few events, where:

$$\chi^2 = \sum_{i=1}^{10} 2(n_i^{th} - n_i^{exp}) + 2n_i^{exp} \ln(n_i^{exp}/n_i^{th})$$

with:
n_i^{exp} number of events in the i-th bin
n_i^{th} number of MC events (normalized to $\sum_i n_i = n$) in the i-th bin

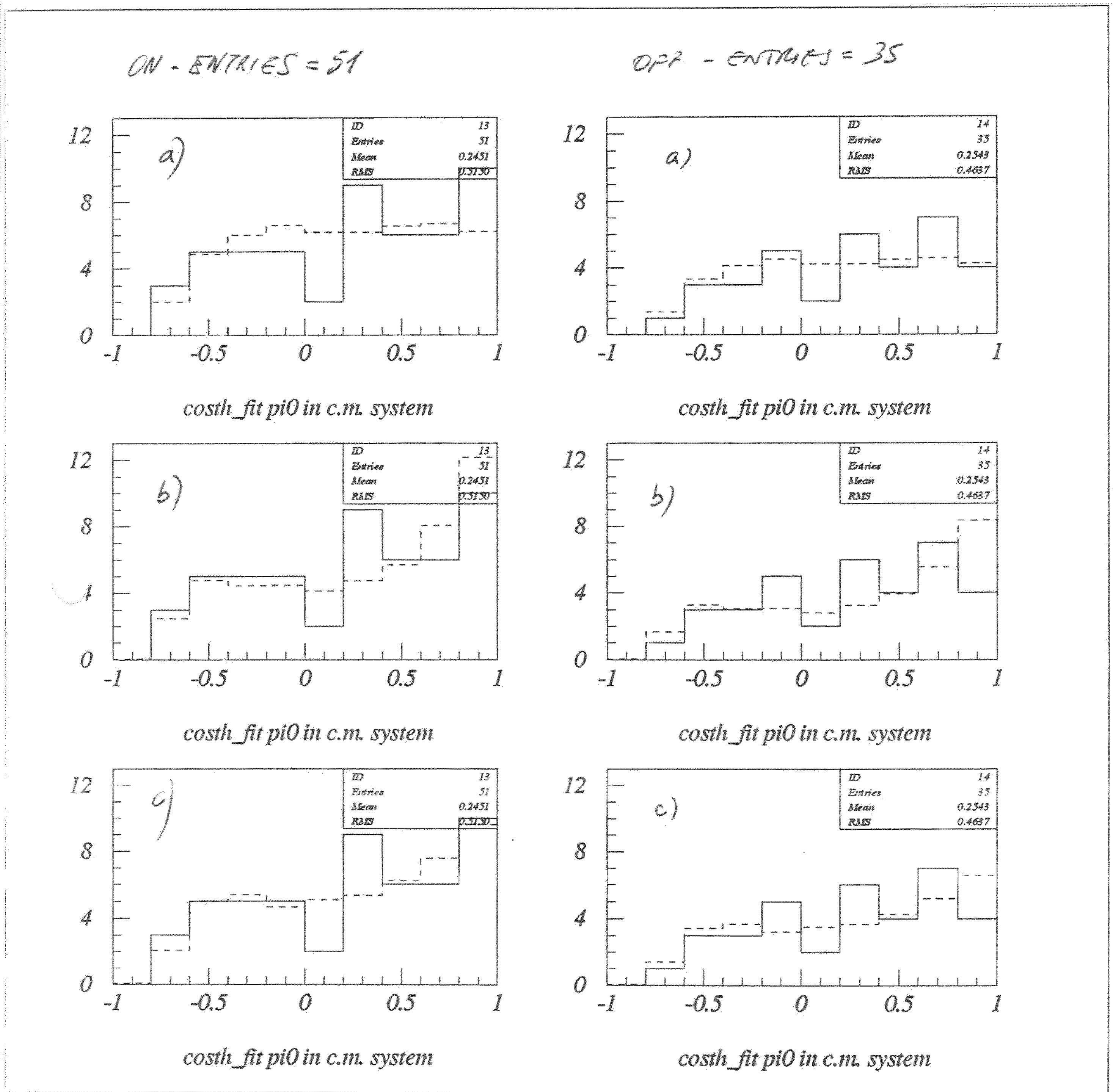
	A	B	C
ON	1.01	0.65	0.59
OFF	0.58	0.68	0.54

Table 3: $\chi^2/\#\ d.o.f.$; $\#\ d.o.f. = N_{bin} - 1 = 10 - 1$



a)
ID 10
Entries 51
Mean 0.2451
RMS 0.5150
costh pi0 in c.m. system ON 1P1
b)
ID 11
Entries 16
Mean 0.2125
RMS 0.5290
costh pi0 in c.m. system OFF 1P1
c)
ID 12
Entries 19
Mean 0.2895
RMS 0.3972
costh pi0 in c.m. system ETACP
d)
ID 13
Entries 35
Mean 0.2543
RMS 0.4637
costh pi0 in c.m. system OFF

FIGURE 1



ON - ENTRIES = 51
OFF - ENTRIES = 35
a)
b)
c)
ID 13
Entries 51
Mean 0.2451
ID 14
Entries 35
Mean 0.2543
RMS 0.4637
costh_fit pi0 in c.m. system

FIGURE 2