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Longitudinal phase space manipulation at medium and high energies: application to energy-recovery linacs

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1 Introduction

The longitudinal phase space evolution in an energy recovering linac (ERL) requires special attention. To zeroth order, after it has been used (e.g. participated to the free-electron laser (FEL) process), the beam has to be re-injected with the proper decelerating phase. This requires the recirculation loop to be $\sim \left(n+\frac{1}{2}\right)\lambda_{rf}$, where n is an integer and λ_{rf} is the wavelength associated to the radio-frequency (rf) accelerating system. To higher order, the longitudinal phase space correlations between an electron relative longitudinal coordinate [1] z and its fractional momentum offset δ needs to be properly chosen such as minimizing the final fractional momentum spread after deceleration. In this note we first follow the approach presented in Reference [1] and consider the simple case of linear longitudinal manipulation, we then build on this simple treatment and consider the effects of longitudinal space charge (LSC). We also address possible issues pertaining to coherent synchrotron radiation (CSR).

2 Linear longitudinal dynamics

We first discuss the requirement on the decelerating phase φ' and consider an electron being accelerated by the accelerating voltage V_{rf} and phase φ. The electron energy gain is $\Delta\mathcal{E} = eV_{rf}\cos\varphi$. Proper energy recovery requires the

[1] In our convention $z > 0$ corresponds to the bunch tail

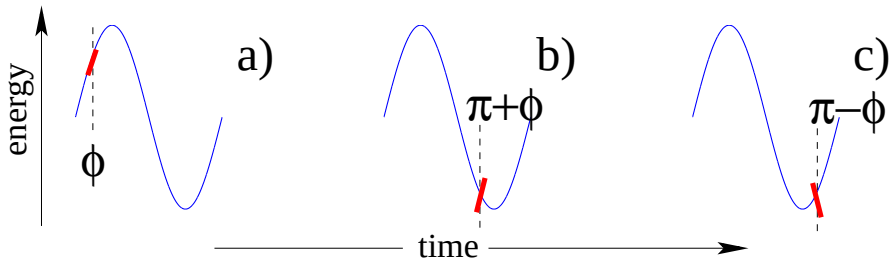

Fig. 1. Illustration of the possible phase choices for deceleration. The beam is accelerated off-crest (a), and presented at the entrance of the linac after an isochronous recirculation transport (b) and a reflecting transport (c). The red line represent the bunch w.r.t. the rf-wave (tail is on the right in our sign convention).

energy loss of the electron being decelerated to be opposite to the energy gain of the accelerated electron. This requirement alone imposes two values for the decelerating phase $\varphi'_{\pm} = \pi \pm \varphi$.

Let's now consider the requirement on the linear correlation between z and δ. The energy gain for an electron with longitudinal coordinate z_A is $\Delta\mathcal{E}_A(s) = eV_{rf}\left[\cos(kz_A + \varphi) - \cos\varphi\right]$ where $k \doteq 2\pi/\lambda_{rf}$. The corresponding fractional momentum spread is $\delta_A(s) \doteq \Delta\mathcal{E}_A(s)/\mathcal{E}_A$ where $\mathcal{E}_A = \mathcal{E}_0 + eV_{rf}\cos\varphi$. The linear correlation imparted in $(z_A,\, \delta_A)$ is $r_A \doteq \frac{d\delta_A}{dz_A}\big|_{z_A=0} = -\frac{eV_{rf}}{\mathcal{E}_A}\sin\varphi$. After acceleration the beam propagates through a recirculation loop characterized by its overall momentum compaction R_{56}. When no source of energy spread is taken into account, the longitudinal motion due to the non-isochronicity of the loop may alter the longitudinal phase space correlation. The correlation coefficient downstream of the recirculation loop is thus given by $r_{D^-} = \frac{r_A}{1+r_A R_{56}}$. Similarly to the acceleration, the deceleration induces a linear correlation given by $r_D = \frac{eV_{rf}}{\sin}\varphi'$, where $\mathcal{E}_D$ is the energy after deceleration. The main goal of the overall longitudinal phase space manipulation is to reduce the fractional momentum spread of the decelerated beam. Ideally the cancelation is achieved, to first order, when

$$r_{D^-}\frac{\mathcal{E}_{A^-}}{\mathcal{E}_D} + r_A = 0. \tag{1}$$

Practically it is instructive to consider the two cases associated to the possible decelerating phase values $\varphi'_{\pm}$. Case (1): the recirculation loop is isochronous $R_{56} = 0$ and Eq. 1 is verified for $\varphi' = \phi + \pi$. Case (2): the recirculation loop is set-up as a reflector ($r_A R_{56} = -2$ or $z \to -z$) and Eq. 1 requires $\varphi' = \pi - \varphi$. These two cases are illustrated in Fig. 1. Case (2) is generally undesired since it implies the bunch is over-compressed and collective effects such as CSR might be detrimental to the beam quality (see below).

We now include in the model the effects of longitudinal space charge (LSC) on the correlation coefficient. This is motivated by recent observations at the

JLab 10 kW FEL [2]. The topic has already been addressed in Ref. [3]. Given the on-axis longitudinal electric field associated to LSC [4], we derive the induced linear correlation in the longitudinal phase space

$$r_{LSC} \doteq \left.\frac{d\delta_{SC}}{dz}\right|_{z=0} = \frac{g_0 \hat{I}}{I_A \sigma_z^2 \gamma} \begin{cases} \frac{L}{\gamma^2} & \text{for a drift} \\ \frac{L}{\gamma_0 \gamma} & \text{in an accelerating section} \end{cases} \tag{2}$$

The longitudinal charge distribution is assumed to be a Gaussian with rms length σ_z. L is the length of the considered section, g_0 is a geometric factor [2], I_A=17 kA is the Alfèn current and $\hat{I} = |Q|c/(\sqrt{2\pi}\sigma_z)$ is the peak current within the bunch (Q is the bunch charge). The Lorentz factors γ and γ_0 correspond respectively to the energy at which Eq. 2 is evaluated and to the initial energy when acceleration is considered [3]. LSC introduces a positive correlation in our convention: the tail (resp. head) of the bunch loses (res. gains) energy. When the full recirculation is taken into account the total LSC-induced change of correlation is

$$r_{LSC,tot} \simeq \frac{g_0 \hat{I}}{I_A \gamma_A} \left[2 \frac{L_{linac}}{\sigma_{z,linac}^2 \gamma_0 \gamma_A} + \frac{L_{recirc}}{\sigma_{z,recirc}^2 \gamma_A^2} \right], \tag{3}$$

where $\sigma_{z,\{linac,recirc\}}$ are the average bunch length in the accelerating (or decelerating) and recirculation sections, and $L_{linac,recirc}$ are the length of the sections. Considering the numerical values experimentally achieved at the JLab 10-kW FEL [2], $\sigma_z \simeq 0.5$ mm, $Q = 135$ pC, $\gamma = 160$ assuming $g_0 = 3$ and taking a drift length $L = 130$ m, we obtain $r_{LSC} \simeq 2.3$ m^{-1}. The latter value translates into a needed change of $\sim 5°$ in the accelerating section phase in order to maintain the same correlation as the one that would be achieved without accounting for LSC (we assume $\mathcal{E}_A = 80$ MeV and $eV_{rf} = 71$ MeV). Because of the significant impact of LSC, the matching condition Eq. 1 has to be modified by changing r_A into $r_A + r_{LSC,tot}$.

Practically the recirculation transport also provides local manipulation of the longitudinal phase space. For instance a local dispersion bump conveniently located compresses the bunch. In this case the upstream accelerating linac has to be operated off-crest. Introducing the magnetic compressor momentum compaction $R_{56,BC}$, the rms bunch length after compression is related to the initial bunch length σ_{z_0} and relative fractional momentum spread σ_{δ_0} via

[2] $g_0 = 1 + \frac{1}{2}\log\left(\frac{b}{a}\right)$ for a transverse uniform bunch of radius a propagating in a perfectly conducting pipe of radius b.

[3] we assume the acceleration is of the form $\gamma(s) = \gamma_0 + \gamma' s$, where $\gamma' = d\gamma/ds$

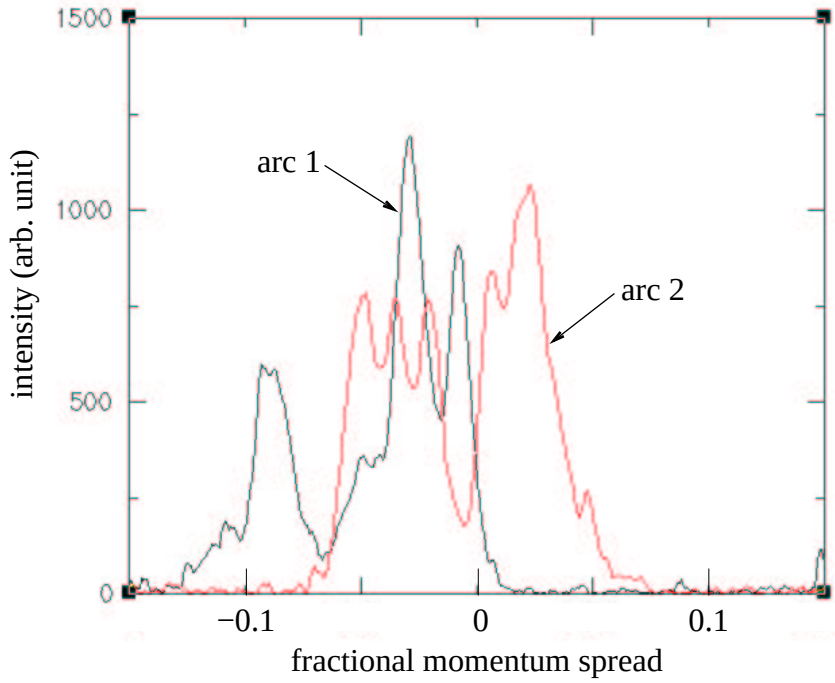

Fig. 2. Fractional momentum spread of a 60 pC bunch measured in the middle of the two 180° arcs of the IR-Demo [measurement by P. Piot (July 1999)]. The origin of the horizontal axis is arbitrary.

$$\sigma_{z,BC} = \left[\mu^2\sigma_{z_0}^2 + \left(R_{56,BC}\sigma_{\delta_0}\frac{\mathcal{E}_0}{\mathcal{E}_A}\right)^2\right], \tag{4}$$

where $\mu \doteq 1 + r_A R_{56,BC}$. The remaining longitudinal lattice downstream of the bunch compressor area (e.g. after a wiggler) is set-up to provide a momentum compaction opposite to the bunch compressor ($R_{56,R} = -R_{56,BC}$) thereby cancelling the total momentum compaction of the recirculation loop (since $R_{56} = R_{56,R} + R_{56,BC}$). Because the minimum achievable bunch length is $R_{56,BC}\sigma_{\delta_0}\frac{\mathcal{E}_0}{\mathcal{E}_A}$, compressing the bunch to its maximum should be avoided, especially at high energy, due to possible detrimental collective effects.

In the JLab IR-Demo, the bunch was going through three longitudinal waists during the recirculation [5], and despite the relatively low energy ($\mathcal{E} = 40$ MeV) and modest uncorrelated energy spread $\mathcal{E}\sigma_{\delta_0} \simeq 5$ keV), significant energy modulation at 60 pC was observed; see Fig. 2.

3 Nonlinear effects and cures

The linear approach in the previous Section breaks as soon as sources of momentum spread dilutions, e.g. induced by the FEL process, are included. The

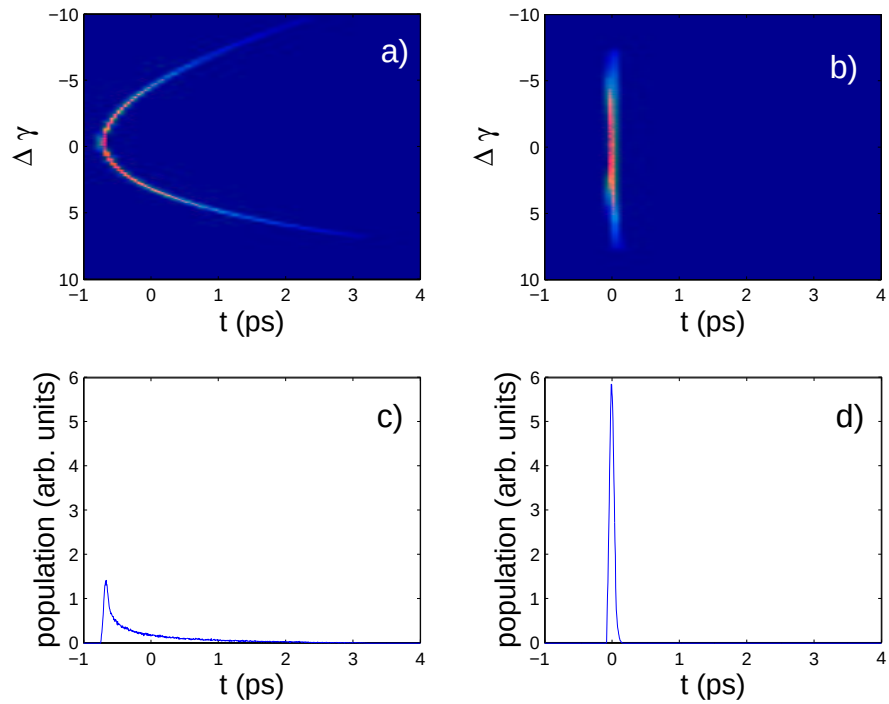

Fig. 3. Longitudinal phase space after compression without (a) and with (b) linearization of the longitudinal phase space prior to the bunch compressor. The profile (c) and (d) respectively corresponds to the charge density associated to (a) and (b).

resulting large fractional momentum spread (and thus the longer bunch after deceleration) requires the previous treatment to include second and possibly higher order terms in z. Nonlinear effects should indeed be included starting from upstream of the injector especially since in common present injector designs the bunch is kept long in order to mitigate transverse emittance growth due to space charge. Then, during acceleration in the linac, the longitudinal phase space accumulates nonlinear correlations introduced by the cosine-like dependence of the accelerating field ($kz \ll 1$ is not satisfied). The fractional momentum spread, including second order in z, is

$$\delta_A(z_A) = r_A z_A + t_A z_A^2 + \mathcal{O}(z_A^3), \tag{5}$$

where $t_A = \frac{1}{2}\frac{d^2\delta_A}{dz^2}$. When the beam is compressed using magnetic compression the longitudinal coordinate becomes $z_a \to z_{BC} = z_A + R_{56,BC}\delta_A + T_{566,BC}\delta_A^2$ ($T_{566,BC}$ is the second-order momentum compaction), and the final rms bunch length becomes

$$\sigma_{z,BC} = \left[\mu^2\sigma_{z_0}^2 + \nu^2\langle z_A^4\rangle + 2\mu\nu\langle z_A^3\rangle + R_{56,BC}^2\langle\delta_{A,u}^2\rangle + T_{566,BC}^2\langle\delta_{A,u}^4\rangle\right]^{1/2}, \tag{6}$$

where $\nu \doteq t_A R_{56,BC} + r_A^2 T_{566,BC}$ and $\delta_{A,u}$ is the stochastic fractional momentum spread. The distortion imposed by the second order effects impacts the

charge density profile [6] and results in a larger rms bunch length (see Fig. 3). The second order contribution on the bunch compression can be suppressed provided $\mu = 0$. At medium or high energies [4] two possible alternatives for canceling μ are (1) design a bunch compressor with proper $R_{56,BC}/T_{566,BC}$ ratio, or (2) locally linearize the longitudinal phase space by using a harmonic rf accelerating section operated on a decelerating phase [8,9]. The first solution requires a bending system that provides the same sign for $R_{56,BC}$ and $T_{566,BC}$. This requirement can be implemented e.g. with FODO-type arc that includes sextupoles, but cannot be met with a standard four-dipoles chicane. In the case of local linearization of the phase space, the harmonic rf section allows an independent control of the coefficient r_A and t_A [10]. In the latter case the second order correlation coefficient $t_A = -ek^2/\mathcal{E}_A\,(V_{rf}\cos\varphi + V_{rf,m}m^2\cos\varphi_m)$ (where $m \in N$ is the harmonic number) can be zeroed by a proper choice of the operating parameters $V_{rf,m}$ and φ_m of the harmonic accelerating section. The introduction of a harmonic rf section is generally preferred since it has the advantage, compared to a tailored dispersive section, not to introduce coupling between the longitudinal and transverse phase spaces. In the case of ERLs operating at the TESLA frequency ($f = 1.3$ GHz), a third harmonic accelerating cavity ($f = 3.9$ GHz) has been developed at Fermilab [11] and will soon be installed in the injector [12] of the TESLA vacuum ultraviolet FEL at DESY.

If the compression is set-up to perform linearly, e.g. by using a higher order harmonic section, the final fractional momentum spread after deceleration is

$$\delta_D = \delta_W\left(\frac{\mathcal{E}_W}{\mathcal{E}_D} + R_{56,R}r_D\right) + \delta_W^2\left(r_D T_{566,R} + t_D R_{56,R}^2\right) \doteq \bar{\mu}\delta_W + \bar{\nu}\delta_W^2, \quad (7)$$

where $R_{56,R}$ and $T_{566,R}$ are the parameters associated to the recirculation transport downstream of the bunch compressor, the subscript $_W$ refers to the parameters after the wiggler, and r_D and t_D are the linear and quadratic correlations induced by the deceleration. Since the path length is properly set to have $\varphi' = \pi + \varphi$, we have the relations $(r_D, t_D) = -(r_A, t_A)\frac{\mathcal{E}_A}{\mathcal{E}_D}$. The condition $\bar{\mu} = 0$ is satisfied de facto when $R_{56,BC} = -R_{56,R}$ and provided the energy loss induced by the FEL process is small compared to $\mathcal{E}_A$. The condition $\bar{\nu} = 0$ requires $T_{566,R} = -t_A T_{566,BC}$.

[4] At non-relativistic energies, nonlinear compression can be used to linearize the phase space [7]

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