FERMI NATIONAL ACCELERATOR LABORATORY

TEVEWWG/top 2005/02
FERMILAB-TM-2323-E
CDF Note 7727 V2
DØ Note 4892
hep-ex/0507091

Combination of CDF and DØ Results on the Top-Quark Mass

The CDF Collaboration, the DØ Collaboration, and the Tevatron Electroweak Working Group[1]

Abstract

The results on the measurements of the top-quark mass, based on the data collected by the Tevatron experiments CDF and DØ at Fermilab during Run-I from 1992 to 1996 and Run-II since 2001, are summarized. The combination of the published Run-I and preliminary Run-II results, taking correlated uncertainties properly into account, is presented. The resulting preliminary world average for the mass of the top quark is: $M_{\mathrm{top}} = 172.7 \pm 2.9$ GeV/c^2, where the total error consists of a statistical part of 1.7 GeV/c^2 and a systematic part of 2.4 GeV/c^2.

Compared to the combination prepared for the LP 2005 conference, this combination for the EPS-HEP 2005 conference includes additional published Run-I and preliminary Run-II measurements.

[1]WWW access at `http://tevewwg.fnal.gov`
The members of the TEVEWWG who contributed significantly to the analysis described in this note are: J. F. Arguin (arguin@fnal.gov) F. Canelli (canelli@fnal.gov), R. Demina (demina@fnal.gov), I. Fleck (fleck@fnal.gov), D. Glenzinski (douglasg@fnal.gov), E. Halkiadakis (evah@fnal.gov), M. W. Grünewald (mwg@fnal.gov), A. Juste (juste@fnal.gov), T. Maruyama (maruyama@fnal.gov), A. Quadt (quadt@fnal.gov), E. Thomson (thomsone@fnal.gov), C. Tully (tully@fnal.gov), E. W. Varnes (varnes@fnal.gov), D. O. Whiteson (danielw@fnal.gov).

1 Introduction

The experiments CDF and DØ, taking data at the Tevatron proton-antiproton collider located at the Fermi National Accelerator Laboratory, have made several direct experimental measurements of the pole mass, M_{top}, of the top quark t. The published measurements [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] are based on about 100 pb^{-1} of Run-I data (1992-1996) while the results from the analyses of about 320 pb^{-1} of Run-II date are preliminary [13, 14, 15, 16, 17, 18, 19]. They utilize all decay topologies[2] arising in $t\bar{t}$ production given by the leptonic or hadronic decay of the W boson occurring in top-quark decay: the di-lepton channel (di-l) [1, 2, 3, 4, 5, 13, 16, 17], the lepton+jets channel (l+j) [6, 7, 8, 9, 10, 14, 15, 18, 19], and the all-jets channel (all-j) [11, 12]. The lepton+jets channel yields the most precise determination of M_{top}. The recently presented preliminary measurements in this channel by the CDF and DØ collaborations [15, 19] are based on large Run-II data sets with well controlled systematic uncertainties, each yielding a top quark mass precision similar to or better than the previous Run-I world average [20].

This note reports on the combination of these measurements, using the five published Run-I measurements [2, 3, 5, 7, 10, 11] combined earlier [20] and in particular including the most recent preliminary Run-II measurements from CDF [15, 16] and DØ [19]. The combination takes into account the statistical and systematic uncertainties as well as the correlations between systematic uncertainties, and replaces previous combinations [20, 21]. The most precise individual measurements of M_{top} are now the preliminary lepton+jets measurements from Run II. These are $173.5^{+4.1}_{-4.0}$ GeV/c^2 (CDF, [15]) and 169.5 ± 4.7 GeV/c^2 (DØ, [19]). These have weights in the new M_{top} combination of 36% and 33%, respectively.

2 Measurements

The eight measurements of M_{top} to be combined are listed in Table 1. The preliminary Run-II DØ measurement in the lepton+jets channel constrains the jet energy scale from an in-situ calibration, based on the hadronic $W \to qq'$ invariant mass in the $t\bar{t}$ events. The preliminary Run-II CDF measurement in the lepton+jets channel constrains the jet energy scale simultaneously from external studies (calorimeter-track comparisons on E/p from single isolated tracks) as well from an in-situ calibration using the W-boson mass.

For the combination procedure the preliminary Run-II CDF lepton+jets channel is split into two separate measurements with identical central value and fully correlated statistical and systematic errors. Only the jet energy scale uncertainty is uncorrelated. One measurement, $(\mathrm{l+j})_e$, is associated with an energy scale uncertainty of 3.1 GeV from the external calibration,

[2] Decay channels with identification of tau leptons in the final state are presently under study for cross section and branching ratio measurements. They are not yet used for measurements of the top quark mass.

which is fully correlated with the energy scale uncertainty of other results. The other measurement, $(\mathrm{l+j})_i$, is associated with an energy scale uncertainty of 4.2 GeV, estimated to be the contribution from the in-situ calibration, which is uncorrelated with any CDF or DØ result. The combination of these two measurements yields identical central value, statistical error and systematic error, and a total jet energy scale uncertainty of 2.5 GeV, as quoted in [15] for the preliminary Run-II CDF measurement in the lepton+jets channel.

Besides central values and statistical uncertainties, the systematic errors arising from various sources are reported in Table 1. For each measurement, the individual error contributions are combined in quadrature. In order of decreasing importance, the systematic error sources are:

- Jet energy scale (JES): The systematics for jet energy scale include the uncertainties on the absolute jet energy corrections, calorimeter stability, underlying event and relative jet energy corrections. Since the jet energy scale uncertainty is the largest uncertainty in all channels, dominating the overall precision of this combination, the various components of this uncertainty have been studied quantitatively and are grouped into contributions, correlated or uncorrelated between the channels, the two experiments, and the data-taking periods (Run-I, Run-II). Based on studies of the correlations of the JES subcomponents in the Run-II data, also the systematic uncertainties of the Run-I measurements are, retrospectively, split into various components so that the correlations of those subcomponents can be better taken into account. The respective error groups are as follows:

  iJES: The component of the jet energy scale originating from in-situ calibration procedures, here using the $W \rightarrow qq'$ invariant mass in the preliminary Run-II l+j channels from CDF and DØ, is labeled iJES. For DØ Run-II this component includes both the statistical error from the in-situ calibration and the systematic error from a possible additional p_T dependence of the data to Monte Carlo ratio of the jet energy scale. The correlation due to using the same method based on the same assumptions and tested with the same MC is presently estimated to be negligible.

  aJES and bJES: The components of the jet energy scale covering aspects of the b-jet energy scale. The part labeled bJES includes fragmentation, color flow and semi-leptonic b decay fractions, and is treated as fully correlated between all channels of all experiments. The additional part labeled aJES for DØ Run-II contains the uncertainty arising from the detector electromagnetic/hadronic calibration of b-jets versus light-quark jets which was negligible in Run-I due to a different calorimeter read-out. The uncertainties assigned to the Run-I measurements are based on studies of the Run-II data, and subtracted quadratically from the total JES error quoted in the Run-I publications.

  cJES: The correlated part of the remaining, external jet energy scale uncertainty (cJES) from the external calibration includes uncertainties from fragmentation and out-of-cone showering corrections and is correlated between all channels of all experiments.

  dJES: The part of the jet energy scale uncertainty correlated between measurements within the same data-taking period (either Run-I or Run-II) but not between experiments, arising from the calibration data samples.

rJES: The remaining external jet energy scale uncertainty (rJES) summarizes uncertainties mainly from the calorimeter response, relative response of different calorimeter sections, multiple interactions for CDF and contributions from the underlying event. It is treated as correlated between all channels of a given experiment independent of data-taking period, but not between experiments.

- Model for signal (signal): The systematics for the signal model include initial and final state radiation effects, b-tagging bias, dependence upon parton distribution functions as well as variations in Λ_{QCD}.

- Model for background (BG): The background model includes the effect of varying the background fraction attributed to QCD multi-jet production with fake leptons and missing E_T. It also includes the estimates of the effects of varying the fragmentation scale and fragmentation model. In Run-I, the scale was varied from $Q^2 = M_{\mathrm{W}}^2$ to $Q^2 = \langle p_t \rangle^2$ in VECBOS [22] simulations of W+jets production, and ISAJET [23] fragmentation was used instead of HERWIG [24] fragmentation. In DØ Run-II, ALPGEN [25] and PYTHIA [26, 27, 28] are compared, and the scale is varied from the default $Q^2 = M_{\mathrm{W}}^2 + \sum p_{T,j}^2$ to $Q^2 = \langle p_{T,j}^2 \rangle$.

- Uranium noise and multiple interactions (UN/MI): This uncertainty includes uncertainties arising from uranium noise in the DØ calorimeter and from multiple interactions overlapping signal events. CDF includes the systematic uncertainty due to multiple interactions in the uncorrelated part of the JES contribution from external calibration. For DØ in Run-II, the shorter integration time, plus the fact that the in-situ JES calibration includes these contributions, results in this uncertainty becoming negligibly small and therefore not considered for this preliminary result.

- Method for mass fitting (fit): This systematic uncertainty takes into account the finite sizes of Monte Carlo samples used for fitting, impact of jet permutations, and other fitting biases. In the CDF Run-I lepton+jets analysis, the systematic uncertainty due to finite Monte Carlo statistics is included in the statistical uncertainty.

- Monte Carlo generator (MC): The systematic uncertainty on the Monte Carlo generator provides an estimate of the sensitivity to the simulated physics model by comparing HERWIG to PYTHIA or to ISAJET. In the DØ analyses, the systematic uncertainty associated with the comparison of HERWIG to ISAJET (Run-I) or replacing 30% of the ALPGEN $t\bar{t}$ simulation by an ALPGEN $t\bar{t} + jets$ simulation (Run-II) is included in the signal model uncertainty.

Further studies on the systematic errors, in particular on the breakdown of the various contributions to the jet energy scale uncertainties and the use of leading order MC, and the correlations are necessary to achieve better understanding and will be pursued in the future. The described procedure with the quoted numbers represent our current, preliminary understanding of the various error sources and their correlations.

	Run-I published					Run-II preliminary			
	CDF			DØ		CDF			DØ
	all-j	l+j	di-l	l+j	di-l	$(\mathrm{l+j})_i$	$(\mathrm{l+j})_e$	di-l	l+j
Result	186.0	176.1	167.4	180.1	168.4	173.5	173.5	165.3	169.5
Stat.	10.0	5.1	10.3	3.6	12.3	2.7	2.7	6.3	3.0
iJES	0.0	0.0	0.0	0.0	0.0	4.2	0.0	0.0	3.3
aJES	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.9
bJES	0.6	0.6	0.8	0.7	0.7	0.6	0.6	0.8	0.7
cJES	3.0	2.7	2.6	2.0	2.0	0.0	2.0	2.2	0.0
dJES	0.3	0.7	0.6	0.0	0.0	0.0	0.6	0.8	0.0
rJES	4.0	3.4	2.7	2.5	1.1	0.0	2.2	1.1	0.0
Signal	1.8	2.6	2.8	1.1	1.8	1.1	1.1	1.5	0.3
MC	0.8	0.1	0.6	0.0	0.0	0.2	0.2	0.8	0.0
UN/MI	0.0	0.0	0.0	1.3	1.3	0.0	0.0	0.0	0.0
BG	1.7	1.3	0.3	1.0	1.1	1.2	1.2	1.6	0.7
Fit	0.6	0.0	0.7	0.6	1.1	0.6	0.6	0.6	0.6
Syst.	5.7	5.3	4.9	3.9	3.6	4.6	3.5	3.6	3.6
Total	11.5	7.3	11.4	5.3	12.8	5.3	4.4	7.3	4.7

Table 1: Summary of the eight measurements of M_{top} performed by CDF and DØ. All numbers are in GeV/c^2. Note that the preliminary Run-II CDF measurement in the lepton+jets channel is split into two measurements in order to treat the correlations of the jet energy scale uncertainties properly. For each measurement, the corresponding column lists experiment and channel, central value and contributions to the total error, namely statistical error and systematic errors arising from various sources defined in the text. Overall systematic errors and total errors are obtained by combining individual errors in quadrature.

3 Combination

In the combination, the error contributions arising from different sources are uncorrelated between measurements. The correlations of error contributions arising from the same source are as follows:

- uncorrelated: statistical error, fit error, iJES error;
- 100% correlated within each experiment: rJES error, UN/MI error;
- 100% correlated within each experiment for the same data-taking period (either Run-I or Run-II): aJES, dJES;
- 100% correlated within each channel: BG error;
- 100% correlated between all measurements: bJES error, cJES error, signal error, MC error.

Note that the jet energy scale uncertainty from the in-situ calibration (iJES) in the preliminary Run-II lepton+jets measurements from CDF and DØ are treated as uncorrelated with any other measurement. All uncertainties except iJES are treated fully correlated between the two preliminary Run-II CDF lepton+jets measurements. The resulting matrix of global correlation coefficients is listed in Table 2.

The measurements are combined using a program implementing a numerical χ^2 minimization as well as the analytic BLUE method [29, 30]. The two methods used are mathematically equivalent, and are also equivalent to the method used in an older combination [31], and give identical results for the combination. In addition, the BLUE method yields the decomposition of the error on the average in terms of the error categories specified for the input measurements [30].

4 Results

The combined value for the top-quark mass is:

$$M_{\mathrm{top}} \;=\; 172.7 \pm 2.9 \text{ GeV}/c^2\,, \qquad (1)$$

where the total error of 2.9 GeV/c^2 contains the following components: a statistical error of 1.7 GeV/c^2; and systematic error contributions of: total JES 2.0 GeV/c^2, signal 0.9 GeV/c^2, background 0.9 GeV/c^2, UN/MI 0.3 GeV/c^2, fit 0.3 GeV/c^2, and MC 0.2 GeV/c^2, for a total systematic error of 2.4 GeV/c^2.

		Run-I published					Run-II preliminary			
		CDF			DØ		CDF			DØ
		all-j	l+j	di-l	l+j	di-l	$(\text{l+j})_i$	$(\text{l+j})_e$	di-l	l+j
CDF-I	all-j	1.00								
CDF-I	l+j	0.32	1.00							
CDF-I	di-l	0.19	0.29	1.00						
DØ-I	l+j	0.14	0.26	0.15	1.00					
DØ-I	di-l	0.07	0.11	0.08	0.16	1.00				
CDF-II	$(\text{l+j})_i$	0.04	0.12	0.06	0.10	0.03	1.00			
CDF-II	$(\text{l+j})_e$	0.34	0.53	0.29	0.29	0.11	0.45	1.00		
CDF-II	di-l	0.18	0.26	0.17	0.17	0.10	0.06	0.30	1.00	
DØ-II	l+j	0.02	0.07	0.03	0.07	0.02	0.07	0.08	0.03	1.00

Table 2: Matrix of global correlation coefficients between the measurements of M_{top}.

The χ^2 of this average is 6.40 for 7 degrees of freedom, corresponding to a probability of 49%, showing that all measurements are in good agreement with each other which can also be seen in Figure 1. The pull of each measurement with respect to the average and the weight of each measurement in the average are reported in Table 3. Note that the weight of the CDF-I lepton+jets measurement is close to zero. In general, this situation can occur if the correlation of two measurements of a physical quantity is close to the ratio of their errors. In case the correlation coefficient r becomes equal to the error ratio, the weight of the less precise measurement becomes zero as it does not improve the combination. While a weight=0 means that the lower accuracy measurement is ignored, a negative weight implies that this particular result does contribute to lowering the variance of the final answer. In our combination, the CDF-I lepton+jets measurements and the externally calibrated CDF-II lepton+jets measurement turn out to be 53% correlated (see Table 2), while the Run-II measurement has almost half the error of the Run-I measurement (see Table 1). See Reference [29] for further discussion of small or even negative weights.

In addition, a combination of the eight measurements in three physical observables, the top quark mass in the di-lepton channel, $M_{\text{top}}^{\text{di-l}}$, the lepton+jets channel, $M_{\text{top}}^{\text{l+j}}$, and the all-jets channel, $M_{\text{top}}^{\text{all-j}}$, has been performed. The results of this combination, obtained with a χ^2 of 2.64 for 5 degrees of freedom corresponding to a probability of 76%, are shown in Table 4. Please note that those results differ from a naive combination, where only the one measurement in the all-jets channel is considered for $M_{\text{top}}^{\text{all-j}}$, the five measurements in the lepton+jets channel are considered for $M_{\text{top}}^{\text{l+j}}$ and the three measurements in the dilepton channel are combined into $M_{\text{top}}^{\text{di-l}}$. In our combination the correlations between systematic uncertainties

	Run-I published					Run-II preliminary			
	CDF			DØ		CDF			DØ
	all-j	l+j	di-l	l+j	di-l	$(\mathrm{l+j})_i$	$(\mathrm{l+j})_e$	di-l	l+j
Pull	+1.19	+0.51	−0.48	+1.67	−0.34	+0.18	+0.24	−1.11	−0.86
Weight [%]	+1.1	+0.1	+1.1	+18.6	+2.1	+18.6	+17.2	+8.1	+33.2

Table 3: Pull and weight of each measurement in the average. See Reference [29] for a discussion of negative weights.

of all measurements, also the ones made in different channels, are properly taken into account in a global fit and can change the central fit values and errors of the result, yielding smaller errors on the three combined results than three independent combinations.

Parameter	Value	Correlations		
$M_{\mathrm{top}}^{\mathrm{all-j}}$ [GeV/c^2]	184.9 ± 10.9	1.00		
$M_{\mathrm{top}}^{\mathrm{l+j}}$ [GeV/c^2]	173.5 ± 3.0	0.22	1.00	
$M_{\mathrm{top}}^{\mathrm{di-l}}$ [GeV/c^2]	165.0 ± 5.8	0.14	0.30	1.00

Table 4: Summary of the combination of the eight measurements by CDF and DØ in terms of three physical quantities, the top quark mass in the di-lepton, lepton+jets and all-jets channel.

5 Summary

A preliminary combination of measurements of the mass of the top quark, M_{top}, from the Tevatron experiments CDF and DØ is presented. The combination includes five published Run-I measurements and three preliminary Run-II measurements. Taking into account statistical and systematic errors including their correlations, the preliminary Tevatron and thus the world-average result is: $M_{\mathrm{top}} = 172.7 \pm 2.9$ GeV/c^2. The mass of the top quark is now known with an accuracy of 1.7%.

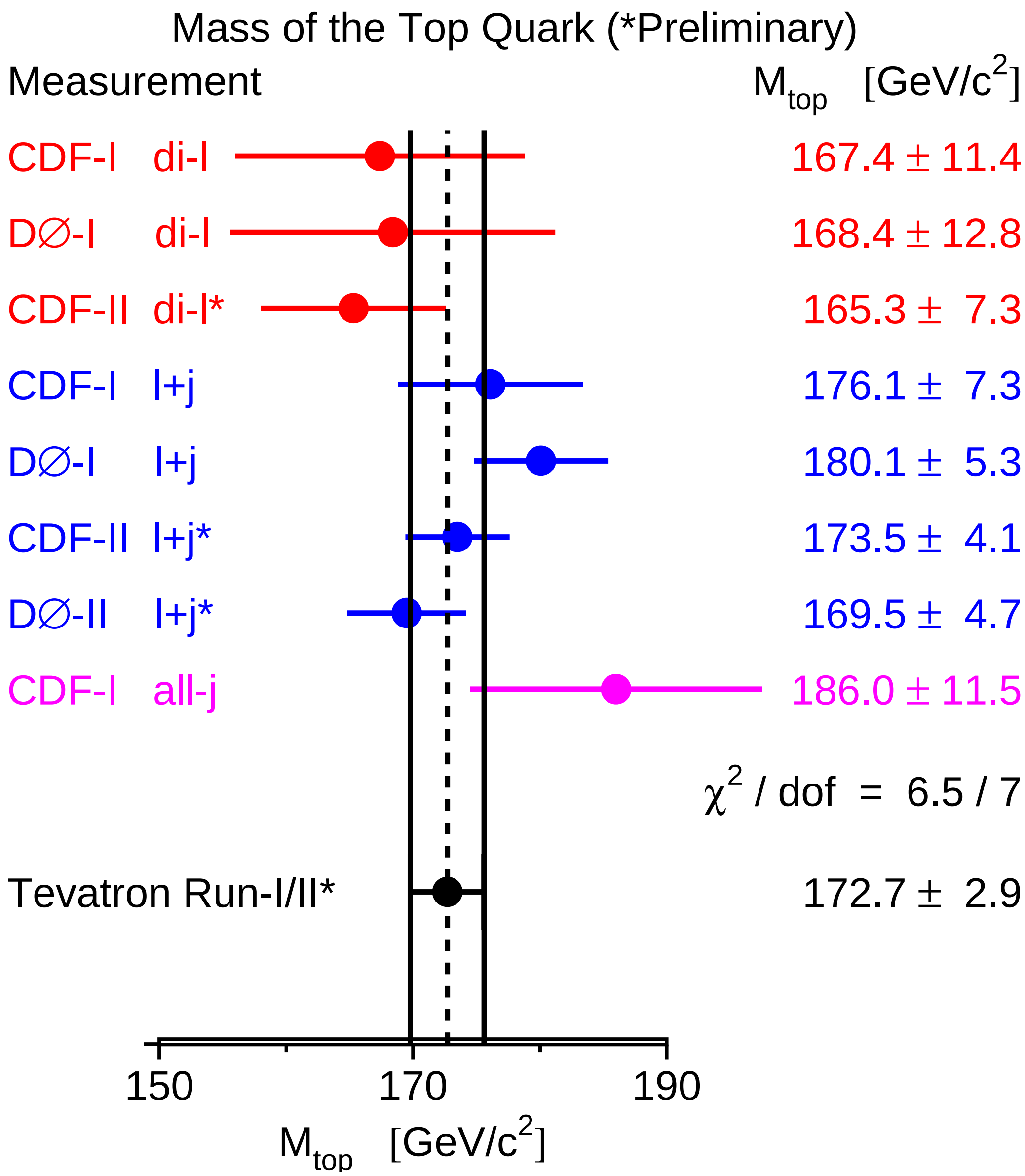

Figure 1: Comparison of the measurements of the top-quark mass and their average.

References

[1] The CDF Collaboration, F. Abe *et al.*, *Measurement of the top quark mass and $t\bar{t}$ production cross section from dilepton events at the Collider Detector at Fermilab*, Phys. Rev. Lett. **80** (1998) 2779, `hep-ex/9802017`.

[2] The CDF Collaboration, F. Abe *et al.*, *Measurement of the top quark mass with the Collider Detector at Fermilab*, Phys. Rev. Lett. **82** (1999) 271, `hep-ex/9810029`.

[3] The CDF Collaboration, F. Abe *et al.*, *Measurement of the top quark mass with the Collider Detector at Fermilab*, Erratum: Phys. Rev. Lett. **82** (1999) 2808, `hep-ex/9810029`.

[4] The DØ Collaboration, B. Abbott *et al.*, *Measurement of the top quark mass using dilepton events*, Phys. Rev. Lett. **80** (1998) 2063, `hep-ex/9706014`.

[5] The DØ Collaboration, B. Abbott *et al.*, *Measurement of the top quark mass in the dilepton channel*, Phys. Rev. **D60** (1999) 052001, `hep-ex/9808029`.

[6] The CDF Collaboration, F. Abe *et al.*, *Measurement of the top quark mass*, Phys. Rev. Lett. **80** (1998) 2767, `hep-ex/9801014`.

[7] The CDF Collaboration, T. Affolder *et al.*, *Measurement of the top quark mass with the Collider Detector at Fermilab*, Phys. Rev. **D63** (2001) 032003, `hep-ex/0006028`.

[8] The DØ Collaboration, S. Abachi *et al.*, *Direct measurement of the top quark mass*, Phys. Rev. Lett. **79** (1997) 1197, `hep-ex/9703008`.

[9] The DØ Collaboration, B. Abbott *et al.*, *Direct measurement of the top quark mass at DØ*, Phys. Rev. **D58** (1998) 052001, `hep-ex/9801025`.

[10] The DØ Collaboration, V. M. Abazov *et al.*, *A precision measurement of the mass of the top quark*, Nature **429** (2004) 638, `hep-ex/0406031`.

[11] The CDF Collaboration, F. Abe *et al.*, *First observation of the all hadronic decay of $t\bar{t}$ pairs*, Phys. Rev. Lett. **79** (1997) 1992.

[12] The DØ Collaboration, V. M. Abazov *et al.*, *Measurement of the top quark mass in all-jet events*, Phys. Lett. **B606** (2005) 25, `hep-ex/0410086`.

[13] The CDF Collaboration, D. Acosta *et al.*, *Measurement of the Top Quark Mass using the Minuit Fitter in Dilepton Events at CDF*, CDF/ANAL/TOP/PUB/7239;
The CDF Collaboration, D. Acosta *et al.*, *Measurement of the top mass using full kinematic template method in dilepton channel at CDF*, CDF note 7194;
The CDF Collaboration, D. Acosta *et al.*, *Measurement of the Top Quark Mass using the Neutrino Weighting Algorithm on Dilepton Events at CDF*, CDF/ANAL/TOP/PUBLIC/7303.

[14] The CDF Collaboration, D. Acosta *et al.*, *Top Mass Measurement in the Lepton+Jets Channel using a Multivariate Template Method*, CDF/ANAL/TOP/PUBLIC/7102; The CDF Collaboration, D. Acosta *et al.*, *Top Quark Mass Measurement using the Dynamic Likelihood Method in the lepton plus jets channel at CDF RunII*, CDF/ANAL/TOP/PUBLIC/7056; The CDF Collaboration, D. Acosta *et al.*, *Updated Top Quark Mass Measurement in the Lepton+Jets Channel Using the Jet Probability Algorithm*, CDF note 7637.

[15] The CDF Collaboration, D. Acosta *et al.*, *Measurement of the Top Quark Mass using the Template Method in the Lepton plus Jets Channel With In Situ $W \to jj$ Calibration at CDF-II*, CDF/ANAL/TOP/PUB/7680.

[16] The CDF Collaboration, D. Acosta *et al.*, *Measurement of the Top Quark Mass in the Dilepton Channel using the Leading-Order Differential Cross-Section at CDF II*, CDF/ANAL/TOP/PUB/7718.

[17] The DØ Collaboration, V.M. Abazov *et al.*, *Measurement of the Top Quark Mass in the Dilepton Channel*, DØ-note 4725-CONF.

[18] The DØ Collaboration, V.M. Abazov *et al.*, *Measurement of the Top Quark Mass in the Lepton+Jets Channel using DØ Run II Data: The Low Bias Template Method*, DØ-note 4728-CONF The DØ Collaboration, V.M. Abazov *et al.*, *Measurement of the Top Quark Mass in the Lepton+Jets Channel using DØ Run II Data*, DØ-note 4574-CONF.

[19] The DØ Collaboration, V.M. Abazov *et al.*, *Top Quark Mass Measurement with the Matrix Element Method in the Lepton+Jets Final State at DØ Run II*, DØ-note 4874-CONF.

[20] The CDF Collaboration, the DØ Collaboration and the Tevatron Electroweak Working Group, *Combination of CDF and DØ results on the top-quark mass*, `hep-ex/0404010`.

[21] The CDF Collaboration, the DØ Collaboration and the Tevatron Electroweak Working Group, *Combination of CDF and DØ results on the top-quark mass*, `hep-ex/0507006`.

[22] F. A. Berends, H. Kuijf, B. Tausk, and W. T. Giele, *On the production of a W and jets at hadron colliders*, Nucl. Phys. **B357** (1991) 32.

[23] F. E. Paige and S. D. Protopopescu, *ISAJET: A Monte Carlo event generator for pp and $\bar{p}p$ interactions*, BNL Reports 38034 and 38774 (1986) unpublished.

[24] G. Marchesini *et al.*, *HERWIG: A Monte Carlo event generator for simulating hadron emission reactions with interfering gluons. Version 5.1 - April 1991*, Comput. Phys. Commun. **67** (1992) 465.

[25] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau, and A. D. Polosa, *ALPGEN, a generator for hard multiparton processes in hadronic collisions*, JHEP **07** (2003) 001, `hep-ph/0206293`.

[26] H.-U. Bengtsson and T. Sjostrand, *The Lund Monte Carlo for hadronic processes: PYTHIA version 4.8*, Comput. Phys. Commun. **46** (1987) 43.

[27] T. Sjostrand, *High-energy physics event generation with PYTHIA 5.7 and JETSET 7.4*, Comput. Phys. Commun. **82** (1994) 74.

[28] T. Sjostrand *et al.*, *High-energy-physics event generation with PYTHIA 6.1*, Comput. Phys. Commun. **135** (2001) 238, `hep-ph/0010017`.

[29] L. Lyons, D. Gibaut, and P. Clifford, *How to combine correlated estimates of a single physical quantity*, Nucl. Instrum. Meth. **A270** (1988) 110.

[30] A. Valassi, *Combining correlated measurements of several different physical quantities*, Nucl. Instrum. Meth. **A500** (2003) 391.

[31] The Top Averaging Collaboration, L. Demortier *et al.*, *Combining the top quark mass results for Run 1 from CDF and DØ*, FERMILAB-TM-2084 (1999).