A Discussion of Bunched-Beam Stochastic Cooling

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Abstract

The analysis of Herr and Mohl[1] is used as a basis for a discussion of bunched beam cooling in the Fermilab recycler and the Tevatron. Differences between the two cooling regimes are discussed. Criteria discussed in that paper imply the failure of stochastic cooling in the Tevatron while permitting the success of stochastic cooling in the Recycler. These "predictions" are in agreement with observations.

Bunched beam cooling evaluations

In order to further understand bunched-beam cooling we will review the analyses of Herr and Mohl,[1] which are based in part on the first bunched-beam cooling observations (in the CERN ICE ring).

The current due to a single particle (indexed by i) seen at a pickup can be expanded as :

$$I_i = e\,\delta(\phi_i) = \sum_{h=0}^{\infty} I_h = \frac{e\omega_0}{2\pi}\left[1 + 2\sum_{h=1}^{\infty}\cos(h\phi_i)\right] \qquad [1]$$

Due to synchrotron oscillations:

$$\phi_i = \omega_0 t + \frac{\Delta\omega_i}{\omega_s}\sin(\omega_s t + \alpha_i)$$

$$\omega_i = \omega_0 + \Delta\omega_i \cos(\omega_s t + \alpha_i)$$

Here, ω_0 is the baseline angular revolution frequency ($\beta_0 c/R$), ω_s is the angular synchrotron frequency

$$\omega_s = \sqrt{\frac{eV'\alpha_p c}{2\pi R\beta E_0}} = \omega_0\sqrt{\frac{eV'\alpha_p R}{2\pi\beta^3 c E_0}} \qquad [2]$$

From which the single particle current can be rewritten as:

$$I_i(t) = \frac{e\omega_0}{\pi}\,\Re\left[\frac{1}{2} + \sum_{h=1}^{\infty}\exp\left(j\,h\left[\omega_0 t + \Delta\phi_i \sin(\omega_s t + \alpha_i)\right]\right)\right] \qquad [3]$$

Here, $\Delta\phi_i = \Delta\omega_i/\omega_s$,

One uses the expansion in Bessel functions J_k:

$$\exp\left[j(x\sin(y))\right] = \sum_{k=-\infty}^{\infty} J_k(x)\, e^{jky}$$

to obtain an expression for the hth harmonic I_h of the expansion for $I_i(t)$ can be written as:

$$I_h = \frac{e\,\omega_0}{\pi}\,\Re\left\{\sum_{k=-\infty}^{\infty} J_k(h\,\Delta\phi_i)\, e^{j(h\omega_0 t + k\omega_s t + k\alpha_i)}\right\} \qquad [4]$$

The above expression is valid for $h \geq 1$. For h=0, I_i also has $I_0 = e\omega_0/(2\pi)$. A complete expression for the current can be written as:

$$I(t) = \sum I_i = \frac{e\omega_0}{\pi}\sum_i \left[\frac{1}{2} + \sum_{h=1}^{\infty} \Re\left\{\sum_{k=-\infty}^{\infty} J_k(h\,\Delta\phi_i)\, e^{j(h\omega_0 t + k\omega_s t + k\alpha_i)}\right\}\right] \qquad [5]$$

In stochastic cooling only the harmonics h that are within the cooling bandwidth are included. (For the recycler, the longitudinal cooling systems cover two bands: 0.5 to 1.0 GHz, and 1.0 to 2.0 GHz, which means harmonics from h = 5600 to 2.25×10^4.) Around each of the harmonics h, we obtain a sequence of sidebands (see fig. 1) in ±k spaced $\pm k\omega_s$ about $h\omega_o$.

The above expression can be compared with expressions for coasting beam cooling:

$$\phi_i = \omega_i t + \alpha_i = \omega_0 t + \Delta\omega_i t + \alpha_i$$

where $\Delta\omega_i = \dfrac{\alpha_p}{\beta^2}\,\omega_0\,\dfrac{\delta E}{E_0}$

$$I_i(t) = \frac{e\omega_i}{\pi}\,\Re\left[\frac{1}{2} + \sum_{h=1}^{\infty} \exp\left(jh\left[\omega_0 t + \Delta\omega_i t + \alpha_i\right]\right)\right] \qquad [6]$$

Eq.5 is the basis for the discussion in Herr and Mohl. Their discussion includes the following key points:

1. The central bands (k=0) add up linearly with intensity (if $\Delta\phi$ is small enough that $J_0(h\Delta\phi_i) \cong 1$), so that instead of Schottky noise ($I_{h,k} \propto \sqrt{N}$), we obtain the full bunch current ($I_{h,k} \propto N$).
2. The k > 0 (and k <0) sidebands at a given harmonic h add up rms due to the phase factor $e^{ik\alpha I}$. However sidebands with the same k but different h have the same phase factor and a similar weighting factor $J_k(h\ \Delta\phi_i)$, provided $\Delta\phi_i$ is relatively small. Thus the k > 0 harmonics also have a noise power that adds coherently.
3. Around each harmonic h we have a line spectrum in the subharmonics k where each sideband has a width of $\sim k\,\Delta\omega_s$, and an enhanced height of $\Gamma_k \cong \omega_s/(k\,\Delta\omega_s)$, rather than a continuous band (for $k < \omega_s/\Delta\omega_s$).

These conditions differ from the coasting beam case, where the hth harmonic of the current is:

$$I_h = \sum_i e\,\frac{\omega_i}{\pi}\,\Re\left[e^{j(h\omega_i t + h\alpha_i}\right] \qquad [7]$$

Here the independence of different harmonics is assured by the phase factor $e^{jh\alpha i}$.

The above conditions, particularly 1 and 2, imply noise terms that increase as particle intensity N and these can overwhelm the Schottky noise that increases as $\sqrt{N}$.

Fermilab bunched beam cooling experience

Prior to the Recycler, the Fermilab experience was that bunched beam cooling did not work, or at least not very well. A transverse cooling system was constructed for the Tevatron Collider, and was designed to cool Tevatron bunches during collision conditions. The system provided very little or no cooling, with the pickup signals dominated by coherent oscillations.[2, 3]

The failure of bunched-beam cooling in the collider may be associated with the suppression of frequency spread by the rf bunching. Stochastic cooling requires mixing of the particles. The rf bunching forces the beam particles to circulate at the baseline oscillation frequency, oscillating about that central frequency at the synchrotron frequency. One source of particle mixing would be synchrotron frequency spread, and the failure of bunched-beam cooling may be associated with inadequate synchrotron frequency spread.

The Fermilab Recycler was not initially designed for bunched beam stochastic cooling. Cooling was to occur within barrier buckets.[4] However the barrier bucket rf system can also provide a linear bunching ramp, and beam was cooled within this linear rf bucket. It was found that the emittance cooling rate for a linear-rf bunch was similar to that for a barrier-bucket bunch. In fact, the bunching with linear rf causes the bunch length to decrease while the momentum spread decreases, allowing the longitudinal emittance to be cooled to smaller values than can be obtained with barrier bucket.[5]

The Recycler has a linear rf buncher, which has, to first order, no synchrotron frequency spread. Therefore there should be no frequency mixing in synchrotron oscillations and it is difficult to see where the mixing required for stochastic cooling is obtained. It is somewhat surprising that the cooling rate with linear rf bunching is nearly as good as with barrier-bucket rf, which has a relatively large synchrotron frequency spread.

Discussion

The parameter regimes for the cooling systems are dramatically different, and the great difference between these parameters determines these variations. The criteria discussed in the Herr and Mohl paper place the beams in entirely separate parameter regimes.

In Tevatron bunched-beam cooling, the rf frequency is ~53MHz, the circumference is 6.28km, the rf harmonic h_{rf} is 1113. At an rf voltage of ~1MV and E_0 =1000GeV, the synchrotron tune is ~0.0005, or the synchrotron frequency is ~25Hz. The rms bunchlength was ~0.5m and that would imply that the parameter $\delta\phi$ is ~0.5/1000 or 5×10^{-4}. The cooling system covered 4 to 8 GHz (h= 8×10^4 to 16×10^4).

In the recycler the rf voltage is ~1kV, and the rf harmonic is close to one. At the August 2004 experiment values (V'=0.417 kV/μs),we find a synchrotron tune of ~6×10^{-6}, or a synchrotron frequency of ~0.5Hz. The rms energy spread was cooled from δE = 4.5MeV to $\delta E \cong$ 2.8MeV. The parameter $\delta\phi$ is ~0.3. The rms bunch length is ~200m, or 400× larger than in the Tevatron; this difference, which is associated with the difference in bunching frequencies, is the key difference between the two systems.

A critical comparison point, according to Herr and Mohl, is the relative intensity of the random Schottky noise (intensity $I_{h,k} \propto \sqrt{N}$) and the coherent signal ($I_{C,h,c} \propto N$) caused by the dynamics of the entire beam undergoing synchrotron oscillations about the reference revolution frequency.

The dominance of the central band (k=0) and the noise concentrations in the sidebands may in principle be avoided by working at frequencies much higher than the bunch frequency. This requires that:

$$h\,\Delta\phi >> 2\pi$$

This criterion is only marginally met in the Tevatron ($h\Delta\phi$ ~40 to 80), but is met by a large factor in the recycler ($h\Delta\phi \cong 1600$ to 5000).

We consider the large argument expansion for the Bessel functions in eq. 5:

$$J_k(x) = \sqrt{\frac{2}{\pi x}}\,\cos\left(x - \frac{\pi}{2}k - \frac{\pi}{4}\right). \qquad [8]$$

The cosine like behavior of J_k at large x introduces a randomizing phase of the form $e^{ih\Delta\phi}$.

For large h, the fourier amplitudes vary as:

$$I_h \propto \frac{N}{(h\,\Delta\phi)^2},$$

provided that the distribution is reasonably smooth, as it will be for a Gaussian distribution, or any variation likely to occur in cooling.

Qualitatively, for the Schottky noise ($I_h \propto (N)^{1/2}$) to dominate over the bunch harmonics ($I_h \propto N/(h\,\Delta\phi)^2$), we need :

$$h\,\Delta\phi > \sqrt[4]{N}\,. \qquad [9]$$

For the Fermilab examples, $(N)^{1/4}$ is ~500 for the Tevatron and ~740 for the recycler, and $h\Delta\phi$ is ~50 for the Tevatron and ~5000 for the Recycler. The criterion of eq. 9 is <u>not met</u> for the Tevatron but <u>is met</u> for the Recycler.

Mixing for Stochastic Cooling

A significant difficulty exists in that the intrinsic synchrotron frequency spread is small, and in the limit of linear synchrotron oscillations that frequency spread is zero. In that limit particles will remain associated with each other and not be mixed sufficiently for stochastic cooling. However, this suppression would require that particle motions remain linear over many synchrotron oscillations, and relatively small perturbations can shift particle motions sufficiently to obtain effective mixing.

In second order, there are particle frequency spreads due to the transverse motion (p_x, p_y) and the longitudinal momentum spread ($(\delta p/p)^2$ terms). The lowest order change due to transverse motion is:

$\delta\omega_{rms} \cong -\frac{\left\langle p_x^2 + p_y^2 \right\rangle}{2p^2}\omega_0$, where $\frac{\left\langle p_x^2 \right\rangle}{p^2} \cong \frac{\varepsilon_{x,N,rms}}{\gamma\langle\beta_x\rangle}$, with a similar expression for p_y. For the RR, $\varepsilon_{x,N,rms}$ was 0.5×10^{-6}m, γ=9.53, $\langle\beta_x\rangle$ =~18m. At these parameters, $\delta\omega_{rms} \cong -2.9\times10^{-9}\ \omega_0$.

Longitudinal momentum spread also changes revolution frequency in second order, with:

$\delta\omega_{rms} \cong -\frac{3\left\langle(\delta p)^2\right\rangle}{2\gamma^2 p^2}\omega_0$, (assuming $\partial(\gamma_t)/\partial p = 0$). At $\delta p/p=3\times10^{-3}$, $\delta\omega_{rms} \cong -1.5\times10^{-9}\ \omega_0$.

These two effects spread the (rms) revolution frequencies by ~3×10^{-9}, with a larger effect from the transverse velocity spread. While these second order effects are small, they may be sufficient to mix particle motion over synchrotron periods. A minimal mixing requirement is the particle positions be randomized in position by $\delta z \cong c/(2\pi W) \cong 0.032$m in a synchrotron period (~1s). The second-order frequency spread of 3×10^{-9} mixes z-positions by ~1m in a synchrotron period, which could be enough to satisfy the mixing requirements.

Another source of particle frequency variations is diffusion.[6] Diffusion processes such as intrabeam scattering and beam-gas scattering randomize particle position and energies over time. In ref. [6] it is suggested that diffusive processes, such as intrabeam scattering (IBS) and beam-gas multiple scattering, can randomize particle energies and positions sufficiently to enable mixing. The mixing time would be the time required for the rms diffusion to move a particle by the sample size; i. e. $\delta z \sim c/(2\pi W)$, where W is the cooling band width (1.5GHz), and good mixing would require that particle trajectories diffuse by the sample size within a synchrotron period. At RR parameters, the IBS scattering, with the synchrotron oscillation rf, is ~1000× the minimum needed to randomize particles by δz within a synchrotron period. Beam-gas scattering is ~20—50× the minimum.

Over a synchrotron period the mixing from intrabeam scattering is similar in magnitude to the second order longitudinal velocity effect, and beam-gas scattering is an order of magnitude smaller. The underlying physics is somewhat different. The velocity variation separation grows linearly in time, while diffusion amplitudes increase as the square root of time.

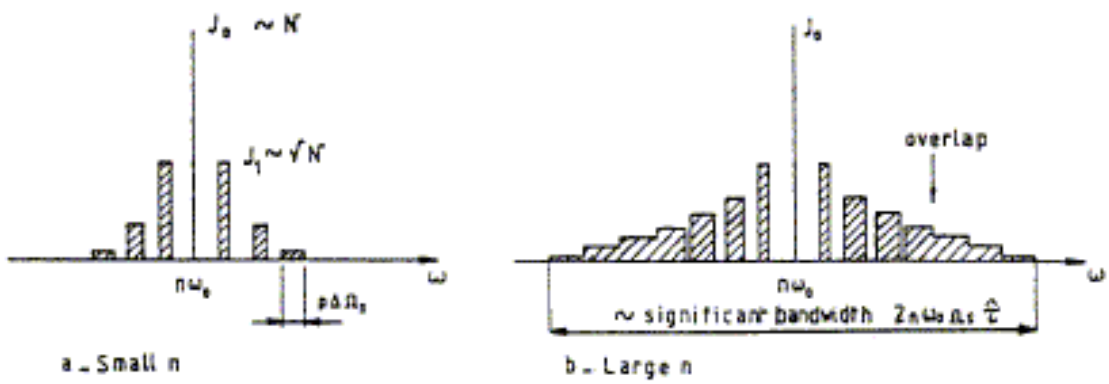
Summary

Bunched beam cooling is discussed and conditions for stochastic cooling in the Recycler and the Tevatron are compared using the criteria developed by Herr and Mohl. Under these criteria bunched beam stochastic cooling is effective in the Recycler but not the Tevatron, in agreement with observations. While the linear rf does not provide mixing, nonlinear effects and stochastic effects (intrabeam scattering and beam-gas scattering) can provide adequate mixing at Recycler parameters. The key difference is that for the Recycler the bunch length is much greater than the cooling wavelength. The beam dynamics is much closer to coasting beam conditions; for example, synchrotron subharmonics are much less coherent.

Table 1:

Parameters for bunched beam cooling				
Parameter	Symbol	Recycler (2004)	Tevatron (1991)	Units
Circumference	$2\pi R$	3320	6280.0	m
Reference Beam Energy	E_0	8.939	900.0	GeV
Revolution frequency	$f_0 = \omega_0/(2\pi)$	89.8	47.8	kHz
Slip Factor	$\alpha_p=1/\gamma^2-1/\gamma_T^2$	0.0085	-0.0028	
Particles per bunch	N	30×10^{10}	6×10^{10}	
rf harmonic	h_{rf}	~1	1113	
Rf Voltage	V_{rf}	0.001	1.0	MV
Synchrotron tune	ν_s	0.000006	0.0005	
Synchrotron frequency	$f_s = \omega_s/(2\pi)$	0.5	25	Hz
Cooling frequencies	W	0.5 to 2.0	4.0 to 8.0	GHz
Energy spread	σ_E	4.5 to 2.8	126	MeV
Rms bunch length	σ_τ	720 to 450	1.7	ns
$\Delta\phi_{rms}$	$\Delta\phi_{rms}$	0.3	0.0005	
Cooling harmonics	h_{min}, h_{max}	5580, 22300	83700, 167400	

Figure 1: schematic view of longitudinal Schottky spectrum of a bunched beam near a harmonic n. (We use h to denote harmonic number in the text.) Synchrotron sidebands about the revolution harmonics n are shown. (from ref. [7])



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