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Pepper-pot Scraper Parameters and Data Processing

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Abstract

Parameters required for the pepperpot scraper are described, and its data processing is proposed.

1 Intentions

The pepper-pot scraper is proposed to be installed under the acceleration section. Accompanied with OTR detectors (optical transition radiation foil) and cameras at several locations downstream, this tool would allow to measure

- Deviations from the axial symmetry and linearity in beam angles;
- Beam temperature;
- Magnetic field at the cathode.

For linear, axially symmetric distribution of electron angles, the holes pattern of the scraper is to be seen undistorted by the camera. In other words, image analysis would allow to measure both quadrupole and non-linear aberrations in the beam angles.

Without temperature, hole images would have sharp boundaries. A finite temperature makes these boundaries smooth. Thus, finite thickness of the hole image boundary is a measure of the beam temperature. Finite size of a pixel grain of the camera sets a limit for smallest measurable temperature.

Magnetic field at the cathode gives rise to the beam angular momentum. That is why the angle between the scraper axes and their image in the camera allows to figure out what is the magnetic field at the cathode.

2 Parameters

The pepper-pot holes are supposed to form a square grid. Their density should be high enough to cover the smallest beam diameter of $\simeq 5$ mm at the scraper location by 5 holes, giving $h \lesssim 1$ mm for the hole-to-hole step. Finite temperature results in widening of the hole image edges. Image of an infinitesimally small point is derived in the Appendix; it can be presented as $\propto \exp\left(-\rho^2/(2\sigma^2)\right)$, where ρ is a distance from the image center, and the width σ depends on the thermal emittance and optical functions. If the beam is round at the scraper, and there are no optical elements between the scraper and detector, the width of the point image is

$$\sigma = \frac{p_T a_g}{p a_0} l \quad ,$$

where p is the total momentum, $p_T = \sqrt{m_e T}$ is the thermal momentum, a_g is the cathode radius, a_0 is the beam radius at the scraper, and l is the scraper - detector distance.

To prevent a loss of the image brightness, the hole has to be wide enough, so that a radius of its image be much larger than the width of its edge σ. This leads to the intensity modulation $\mathbf{I}(x)$ over the hole image as for a half-plane:

$$\int_0^\infty \exp\left(-\frac{(y-x)^2}{2\sigma^2}\right) dy \propto \mathbf{I}(x) = \frac{1}{2}\mathrm{erfc}\left(-\frac{x}{\sqrt{2}\sigma}\right) \quad ; \tag{1}$$

the function $\mathbf{I}(x)$ normalized so that $\mathbf{I}(-\infty) = 0$; $\mathbf{I}(\infty) = 1$. For this intensity distribution, the thermal width x_T can be restored from the intensity function using that

$$\sigma = \sqrt{\pi} \int_{-\infty}^{\infty} \mathbf{I}(x) \left(1 - \mathbf{I}(x)\right) dx \ . \tag{2}$$

Now, let $J(\mathbf{r})$ be measured brightness of a hole image as a function of 2D coordinate $\mathbf{r}$ on the camera screen. Having this function, the temperature can be found from Eq. 2 :

$$\sigma = \frac{\sqrt{\pi}}{L\ J_{\max}^2} \int J(\mathbf{r}) \left(J_{\max} - J(\mathbf{r})\right) d^2\mathbf{r} \ , \tag{3}$$

with L as a length of the image boundary and $J_{\max}$ as the maximal brightness.

For cathode temperature $T = 0.13$ eV, cathode radius $a_g = 3.8$ mm, beam radius at the scraper $a_0 = 5$ mm, momentum $p = 5$ MeV/c, and $l = 1.4$ m, the thermal width $\sigma = 0.055$ mm. To recognize the boundary, this number has to be several times smaller than an image of the hole-to-hole half-distance, minus image of the hole radius, or

$$\frac{p_T a_g}{p a_d} l \ll \left(\frac{h}{2} - r_0\right) \ , \tag{4}$$

where a_d is the beam radius at the detector position, and r_0 is the hole radius. For the case under study, the beam at the detector position is rather wide. Taking $h = 0.8$ mm, $a_d = 10$ mm, $r_0 = 0.125$ mm, gives a sufficiently big factor of $\simeq 10$ for exceeding of the right hand side over the left hand side here. For these parameters, the radius of the hole image $r_0 a_d / a_0 = 0.25$ mm, which is ≈ 5 times higher than the width of its edge σ.

3 Tolerances

3.1 Offsets of hole centers

Optical aberrations lead to stray angles in the beam; let $\delta\theta_s$ be an aberration angle for the beam edge at the scraper position. This angle gives rise to the beam image distortion $\delta a_a = \delta\theta_s l$. Let Δh be a systematic error of the hole position at the edge of the beam spot. This error gives rise to the beam image distortion $\Delta a_h = \Delta h \; a_d/a_s$. To recognize the aberrations, the distortion due to the grid imperfection has to be smaller than the aberrational one, $\Delta a_h \ll \delta a_a$. Taking into account that aberrations at the scraper $\delta\theta_s$ lead to aberrations at the cooler $\delta\theta_c = \delta\theta_s a_s/a_c$, a tolerance for the grid error writes as

$$\Delta h \ll \delta\theta_c l \; a_c/a_d \quad .$$

Assuming $\delta\theta_c = 100$ μrad, $a_c/a_d = 0.5$, gives $\Delta h \ll 70$ μm.

For random errors in the hole positions, the tolerance is relaxed by a factor of $\simeq 1/\sqrt{N}$, where $N \gtrsim 100$ is a number of holes covered by the beam.

3.2 Tilt Errors

Tilt errors $\Delta\alpha$ of the OTR foil and the camera lead to the beam image distortion $\Delta a_\alpha = a_d \Delta\alpha$. To see the aberrations, this distortion has to be smaller than the aberrational one δa_a. This can be expressed as

$$\Delta\alpha \ll \delta\theta_c \frac{l \; a_c}{a_d a_s} \quad .$$

For the same numbers as above, this limits $\Delta\alpha \ll 15$ mrad.

3.3 Axial Errors

The camera might have an error axial angle (erroneous turn over its own axis) $\Delta\phi$. This would lead to an error in the cathode field measurement $\Delta B/B = \Delta\phi/\phi_0$, where ϕ_0 is the beam axial rotation between the scraper and collector. Taking the desirable accuracy $\Delta B/B = 0.01$, and estimating $\phi_0 \simeq 0.3$, this gives a tolerance to the axial angle error of the camera $\Delta\phi \simeq 3$ mrad.

4 Data Processing

After the image of the pepper-pot scraper is digitized, the data have to be processed to find a list of the hole image centers and edge widths

$$\mathbf{D} \equiv \{\{x_1, y_1, \sigma_1\}, \{x_2, y_2, \sigma_2\}, ..., \{x_N, y_N, \sigma_N\}\},$$

where N is the number of holes covered by the beam. For a given hole image, its values have to be found by summation over pixels whose signals $J_{i,\alpha}$ are above 0.5 of a signal at the spot center $J_{i,\alpha} \geq 0.5 \; \widehat{J}_i$:

$$
\begin{aligned}
x_i &= \frac{1}{M_i}\sum_\alpha x_{i,\alpha} \\
y_i &= \frac{1}{M_i}\sum_\alpha y_{i,\alpha} \\
\sigma_i &= \frac{\sqrt{\pi}\Delta S}{L_i\ \widehat{J}_i^2}\sum_\alpha J_{i,\alpha}\left(\widehat{J}_i - J_{i,\alpha}\right)
\end{aligned}
$$

where the subscript i stays for the hole number, subscript α is the pixel number, M_i is the number of pixels with the signal above the half-maximum, L_i is a length of the boundary and ΔS is a pixel square. Assuming the hole to be round, $L_i = \sqrt{4\pi\ M_i\ \Delta S}$. After the data list $\mathbf{D}$ is obtained, the secondary processing is needed to find the beam vortex and the aberrations.

5 Appendix: Image of a point

Here a beam density distribution is calculated at some distance l downstream a scraper with a point-like hole. Trajectories of x-y coupled beam are described in terms of 4 beta-functions β_{1x}, $\beta_{1y}, \beta_{2x}, \beta_{2y}$,2 phases ψ_1, ψ_2, and 2 phase shifts ν_1, ν_2 (see Ref. [1])

$$
\begin{aligned}
x &= \sqrt{2\beta_{1x}I_1}\cos\psi_1 + \sqrt{2\beta_{2x}I_2}\cos(\psi_2 - \nu_2) \\
y &= \sqrt{2\beta_{1y}I_1}\cos(\psi_1 - \nu_1) + \sqrt{2\beta_{2x}I_2}\cos\psi_2\ .
\end{aligned}
\tag{5}
$$

The beta-functions and phases vary along the trajectory: $\beta_{i\alpha} = \beta_{i\alpha}(s)$, $\psi_i = \psi_i(s) = \psi_i(0) + \mu_i(s)$.

For the electron beam under consideration, the first invariant I_1 is determined by the magnetic field at the cathode, and the second invariant I_2 is due to the finite temperature, $I_2 \ll I_1$. Assuming the beam distribution to be constant over the first invariant, and to be thermal over the second invariant, an image of a point-like hole (x_0, y_0) is described as

$$
n(x,y) \propto \int dI_1 d\psi_1 dI_2 d\psi_2 \exp\left(-\frac{I_2}{\varepsilon_2}\right)\ \delta(x(0)-x_0)\ \delta(y(0)-y_0)\ \delta(x(s)-x)\ \delta(y(s)-y)\ .
$$

This integral can be taken for an arbitrary 4D optics. In general case, the result can be expressed in terms of deviations of the observation point (x, y) from a trajectory of a zero-temperature particle $x = \overline{x} + \xi, \quad y = \overline{y} + \eta$, where $\overline{x}$, $\overline{y}$ are given by Eq. 5 with $I_2 = 0$. In these terms, the distribution is described by an exponent of a homogeneous quadratic form:

$$
n(\xi,\eta) \propto \exp\left(-\frac{A\xi^2 + B\xi\eta + C\eta^2}{\varepsilon_2}\right)
$$

The coefficients A, B, C are determined by the optical functions; for a general 4D optics, those expressions obtained with *Mathematica* look cumbersome. For round optics though, with $\beta_{1x} = \beta_{1y} = \beta_{2x} = \beta_{2y} = \beta$, $\psi_1 = \psi_2 = \psi$, $\nu_1 = \nu_2 = \pi/2$, the distribution reduces to a simple formulae:

$$n(\xi, \eta) \propto \exp\left(-\frac{\xi^2 + \eta^2}{2\sigma^2}\right) \quad ; \quad \sigma^2 = 4\varepsilon_2 \beta(s) \sin^2 \mu \quad ,$$

where $\beta(s)$ is the beta-function in the detector location, and μ is the phase advance between the scraper and detector. In a specific case of a drift with the length l,

$$\beta(s) = \beta_0 \left(1 - \frac{l\alpha}{\beta_0}\right)^2 + \frac{l^2}{4\beta_0} \quad ; \qquad \sin^2 \mu = \frac{l^2}{4\beta_0 \beta(s)} \quad ,$$

leading to

$$\sigma^2 = \frac{\varepsilon_2 l^2}{\beta_0} \tag{6}$$

with β_0 as the beta-function at the scraper position.

Emittance of the thermal mode ε_2 can be expressed as $\varepsilon_2 = \varepsilon_T^2/\varepsilon_1$. Here $\varepsilon_1$is the main normal mode emittance, related to the beam radius at the scraper position a_0 as $\varepsilon_1 = a_0^2/\beta_0$. The thermal emittance ε_T is determined by the beam thermal momentum at the cathode $p_T = \sqrt{m_e T}$, the cathode radius a_g and the total momentum p as $\varepsilon_T = a_g p_T/p$. After all these substitutions, Eq. 6 rewrites as

$$\sigma = \frac{p_T a_g}{p a_0} l$$

References

[1] V. Lebedev, A. Bogacz, "Betatron motion with coupling of horizontal and vertical degrees of freedom", e-print JLAB-ACC-99-19.