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BUNCH AREA WITH SLANTING BARRIER VOLTAGES

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Abstract

In order that the bunch area has rounded corners, instead of sharp-edge square pulses, it is proposed that the inner parts of the barrier waves have linearly increasing voltage. The longitudinal emittance or bunch area of a beam supported by these barrier waves is derived.

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During momentum-mining of antiprotons in the Recycler Ring, [1] the cooled core of the antiproton beam is extracted and placed inside individual rf buckets for transfer into the Main Injector. These buckets are established by two square rf waveforms. In the longitudinal phase space, the bunch inside each bucket consists of a central rectangular part flanked by two parabolic edges on each side. The transition from the flat rectangular part to the parabolic part has been considered too *sharp*, especially when the rectangular part becomes small compared with the two parabolic edges. A remedy is to modify the square wave form to one that has a slope. A choice is illustrated in Fig. 1, where "bkt"= 18.935 ns denotes the length of one 52.8-MHz rf bucket. In this way the four sharp corners of the beam will be smoothed. Keeping the slope constant, the peak rf voltage V_0, which can have the maximum of 2 kV, can be lowered when a beam of smaller energy spread is confined. Here, we want to compute the maximum energy and bunch area of the beam stored between two such buckets as a function of the peak rf voltage.

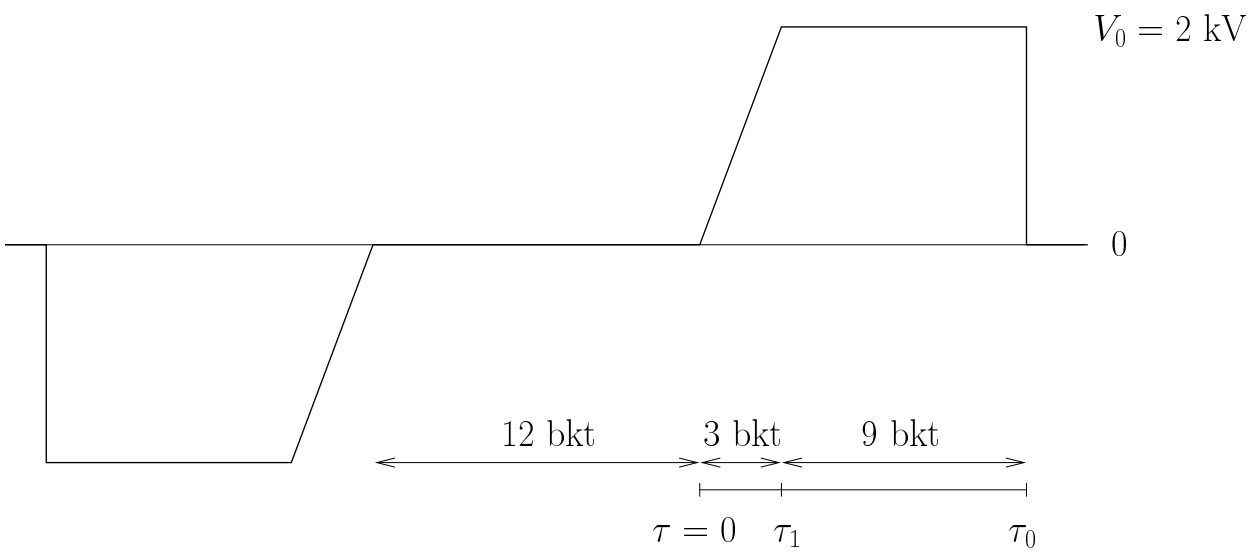

Figure 1: Sketch of barrier wave form designed to produce a bunch that has rounded corners in the longitudinal phase space. The measurements are in bkt, which equals 18.935 ns or the length of a 52.8-MHZ rf bucket. The slope V' of the slanting voltage is always fixed at 2 kV per 3 bkt while the flat-top voltage V_0 is increased in order to accommodate beam with larger energy spread. Thus the time τ_1 when the flat-top voltage is reached is given by $\tau_1 = V_0/V'$ and has the maximum of 3 bkt when the barrier voltage is at its maximum. All other parameters like the distance between the two barrier edges, 12 bkt, and the total barrier depth $\tau_0 = 12$ bkt are held fixed.

The equations of motion governing the motion of a beam particle in the longitudinal

phase space can be written as

$$\frac{d\tau}{dt} = \frac{\eta \Delta E}{\beta^2 E} \ ,$$
$$\frac{d(\Delta E)}{dt} = \frac{eV(\tau)}{T_0} \ , \tag{1}$$

where we have chosen τ, the particle's arrival time at a certain position in the ring, and its energy offset ΔE as the canonical variables. Other parameters of the beam are $E = 8.938$ GeV, the nominal energy, $\beta c = 0.9944c$, the nominal velocity (c is the velocity of light), $T_0 = 11.13$ μs the nominal revolution period, and $\eta = -8.511 \times 10^{-3}$ the nominal slip factor. The reference arrival time τ is chosen to be zero at the beginning of the voltage slope so that the barrier rf waveform becomes

$$V_0(\tau) = \begin{cases} V'\tau & 0 < \tau < \tau_1 \ , \\ V_0 & \tau_1 < \tau < \tau_0 \ , \end{cases} \tag{2}$$

where from Fig. 1, $V' = 2/3$ kV/bkt= 35.208 kV/μs and $\tau_0 = 12$ bkt= 0.22722 μs. The location τ_1 where the voltage changes from slanting to flat depends on the maximum voltage V_0 applied. Therefore $\tau_1 = V_0/V'$ and is equal to 3 bkt= 0.056805 μs when $V_0 = 2$ kV. Combing the two equations of motion, we obtain the differential equation

$$\frac{\eta T_0 \Delta E}{\beta^2 E} d(\Delta E) = eV(\tau) d\tau \ . \tag{3}$$

An integration gives the particle trajectory,

$$-\frac{\eta T_0}{2\beta^2 E}\left(\Delta E_0^2 - \Delta E^2\right) = \int_0^\tau V(\tau) d\tau \ , \tag{4}$$

where ΔE_0 is the maximum energy offset allowed for a given barrier voltage V_0, and it occurs at $\tau = 0$. The particle will be at $\tau = \tau_0$, the deepest penetration of the barrier when $\Delta E = 0$. We therefore have

$$-\frac{\eta T_0}{2\beta^2 E}\Delta E_0^2 = \frac{V'\tau_1^2}{2} + V_0(\tau_0 - \tau_1) \ . \tag{5}$$

To ease writing, it is convenient to introduce

$$\epsilon^2 = -\frac{\eta T_0}{2\beta^2 E}\Delta E^2 \quad \text{and} \quad \epsilon_0^2 = -\frac{\eta T_0}{2\beta^2 E}\Delta E_0^2 \ . \tag{6}$$

Both of them carry the dimension of V-s. Now we can solve for

$$\epsilon_0^2 - \epsilon^2 = \begin{cases} \dfrac{V'\tau^2}{2} & 0 < \tau < \tau_1 \ , \\ \dfrac{V'\tau_1^2}{2} + V_0(\tau - \tau_1) & \tau_1 < \tau < \tau_2 \ , \end{cases} \tag{7}$$

with

$$\epsilon_0^2 = \frac{V'\tau_1^2}{2} + V_0(\tau_0 - \tau_1) \ . \tag{8}$$

To obtain the bunch area, we need to compute the integral of ϵ over τ. The integral from $\tau = 0$ to τ_1 is

$$\int_0^{\tau_1} \epsilon \, d\tau = \frac{\epsilon_0^2}{V'} \sin^{-1}\left(\tau_1 \sqrt{\frac{V'}{2\epsilon_0^2}}\right) + \frac{1}{2}\tau_1 \sqrt{\frac{2\epsilon_0^2}{V'} - \tau_1^2} \ . \tag{9}$$

The integral from $\tau = \tau_1$ to τ_0 is

$$\int_{\tau_1}^{\tau_0} \epsilon \, d\tau = \frac{2}{3}\sqrt{V_0}(\tau_0 - \tau_1)^{3/2} \ . \tag{10}$$

The bunch area can now be expressed as

$$A = 4\sqrt{\frac{2e\beta^2 E}{-\eta T_0}} \left[\epsilon_0 \tau_{\rm rect} + \frac{\epsilon_0^2}{\sqrt{2V'}} \sin^{-1}\frac{\tau_1}{\epsilon_0}\sqrt{\frac{V'}{2}} + \tau_1 \sqrt{\frac{V'}{8}}\sqrt{\frac{2\epsilon_0^2}{V'} - \tau_1^2} + \frac{2\sqrt{V_0}}{3}(\tau_0 - \tau_1)^{3/2} \right] \ . \tag{11}$$

where we have included the rectangular portion of the bunch (of length $2\tau_r = 12$ bkt) as the first term. Since $V_0 = V'\tau_1$, the bunch area can be rewritten in terms of V_0 as

$$A = 4\sqrt{\frac{2e\beta^2 E}{-\eta T_0}} \left[\epsilon_0 \tau_{\rm rect} + \frac{\epsilon_0^2}{\sqrt{2V'}} \sin^{-1}\frac{V_0}{\sqrt{2V'}\epsilon_0} + \frac{V_0}{4\sqrt{2V'}}\sqrt{\frac{2\epsilon_0^2}{V'} - \frac{V_0^2}{V'^2}} + \frac{2\sqrt{V_0}}{3}\left(\tau_0 - \frac{V_0}{V'}\right)^{3/2} \right] \ , \tag{12}$$

where from Eq. (8),

$$\epsilon_0 = \sqrt{\frac{V_0^2}{2V'} + V_0\left(\tau_0 - \frac{V_0}{V'}\right)} \ . \tag{13}$$

The bunch area A and the maximum energy offset ΔE_0 are plotted in Fig. 2 as functions of peak barrier voltage V_0. We see that the maximum bunch area that can be contained in the $V_0 = 2$ kV barriers as illustrated in Fig. 1 is close to $A = 8$ eV-s.

In the absence of the slanting portion of the barrier rf wave, the bunch area reduces to (with $V' \to \infty$)

$$A = 4\sqrt{\frac{2e\beta^2 E}{-\eta T_0}} \left[\epsilon_0 \tau_{\rm rect} + \frac{2}{3} V_0^{1/2} \tau_0^{3/2} \right] \ , \tag{14}$$

where τ_0 is the depth of the square barrier and $2\tau_{\rm rect}$ is the distance between the inner edges of the barriers.

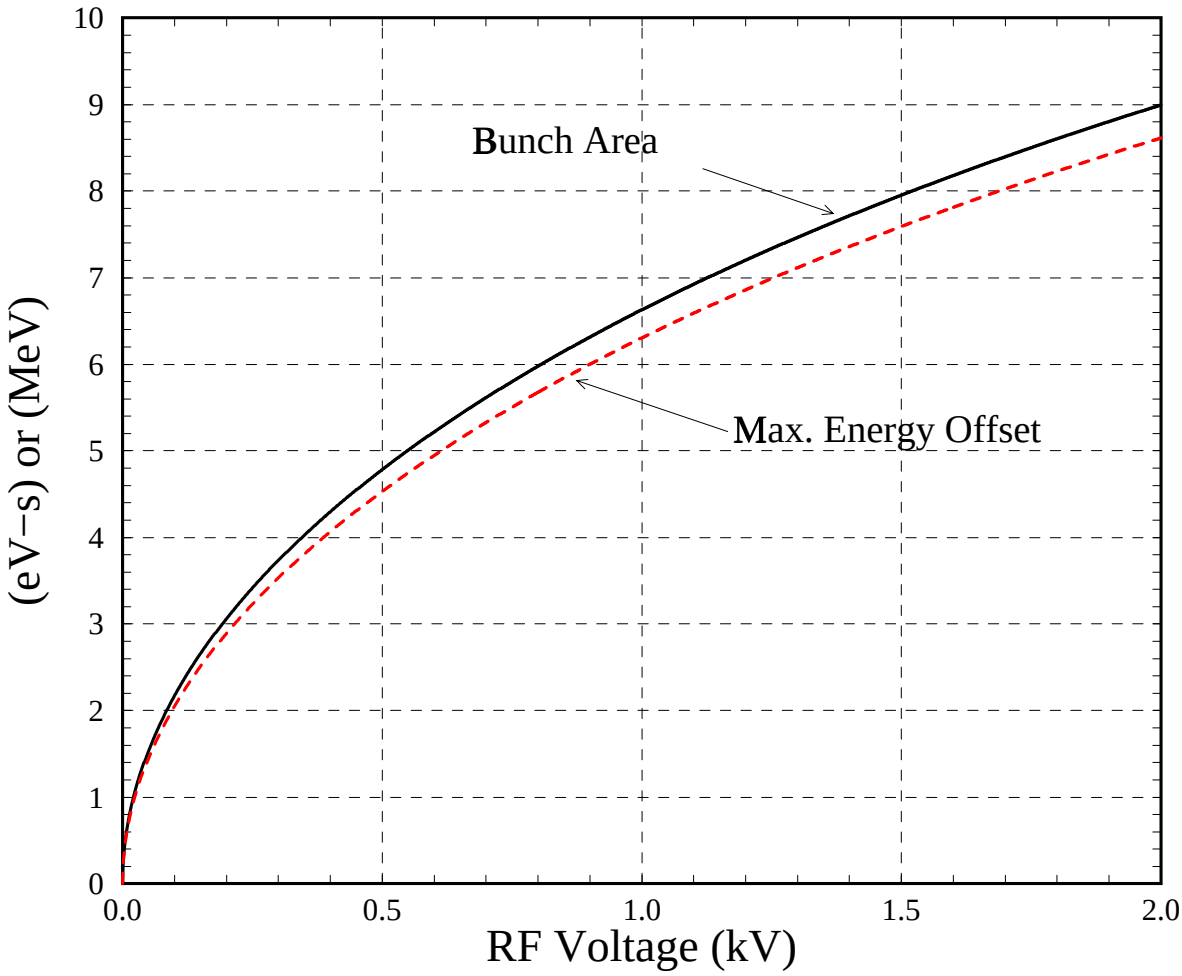

Figure 2: Plot of maximum bunch area and maximum energy offset as functions of the barrier rf voltage in the barrier waveform illustrated in Fig. 1.

We may also wish to compute the bunch area given the bunch length. Thus, if we know of the penetration $\tau_0 - \tau_1$, we can obtain the maximum energy offset or ϵ_0 through Eq. (8). Then the bunch area can be computed using either Eq. (11) or Eq. (12).

References

[1] C.M. Bhat. *Longitudinal Momentum Mining of Beam Particles in a Storage Ring*, Fermilab-Pub-04-077, 2004.