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Fermilab-TM-2205

Comparison of Tevatron C0 and F0 Lambertson Beam Impedance

Jim Crisp and Brian Fellenz

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Introduction

Both the longitudinal and transverse beam impedance measurements for the Tevatron C0 and F0 lambertsons are presented. The C0 lambertsons were designed for circulating beam to travel through the 1" high by 6" wide field region. In the F0 lambertsons, circulating beam passes through the 2.5" high by 4" wide field free region. The more recently designed F0 lambertsons have significantly less impedance than the older C0 lambertsons. Transverse impedance scales as one over the diameter of the aperture cubed. The three C0 style lambertsons were recently removed from the Tevatron. Four of the F0 lambertsons remain. Nine of the F0 style lambertsons are in the Main Injector and three more are required for Numi.

Measured Longitudinal Impedance

The longitudinal impedance is measured with a wire stretched along the beam path. The wire forms a coaxial, or TEM, transmission line. Attenuation is measured with a network analyzer and used to calculate the longitudinal impedance. The characteristic impedance of the 0.010" tin plated copper wire is 331Ω in the 2.5" aperture of the F0 lambertson and 291Ω in the 1" aperture of the C0 lambertson. Resistive L-pads are used to match the wire to 50Ω. The measurement is normalized to a reference pipe, 1.250" ID for the C0 and 2.375" ID for the F0 lambertsons. This normalization corrects for losses in the resistive L-pads, impedance mismatch error, and for the skin effect losses on the wire itself. The method works well below the microwave cutoff frequency of the aperture (1.6GHz in the F0 lambertson).

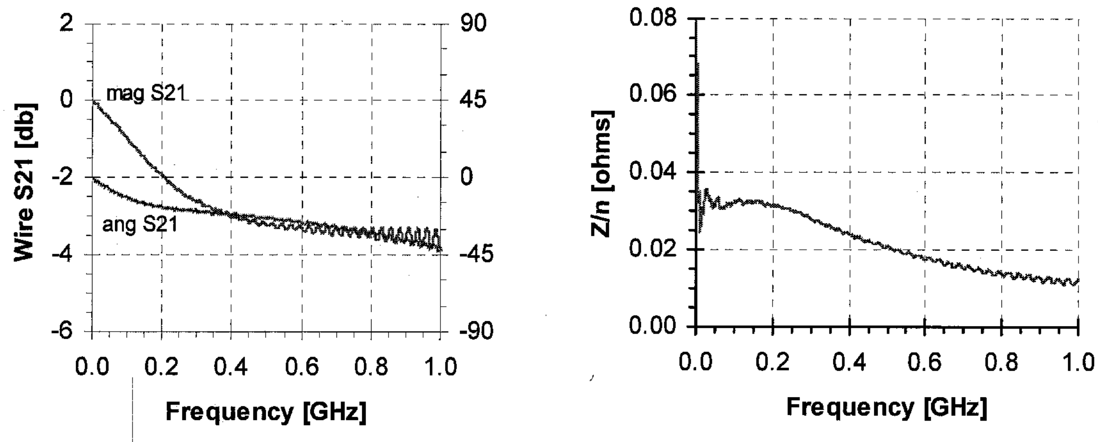
Lambertson magnets do not have conventional vacuum pipes. Both types of lambertsons discussed here are constructed with two sets of laminations. One set resides in a stainless steel vacuum chamber. The 0.0625" wall thickness of the stainless steel is one skin depth at 100KHz. Above this frequency, impedance will not be affected by anything outside of the vacuum. Magnet laminations are electrically connected at their outer edge, by welding them to an index bars or rods. The beam image currents flow over the lamination surfaces to this common connection. The skin depth and resistivity of the lamination material determine the beam impedance at frequencies where the lamination is much smaller than a wavelength. Impedance will be inversely proportional to skin depth and distributed evenly along the length of the magnet.

Longitudinal impedance is derived from the attenuation (S_{21}) measured through the device, figure 1.

$$Z_L = -2Z_0 \ln S_{21} = 2\, Z_0 \frac{S_{21}[db]}{20\log(e)} \qquad \textit{for distributed impedance}$$

$$\frac{Z}{n} = Z_L \frac{\omega_0}{\omega} \qquad \omega_0 = 2\pi \textit{ rotation frequency}$$

C0 lambertson longitudinal impedance



F0 lambertson longitudinal impedance

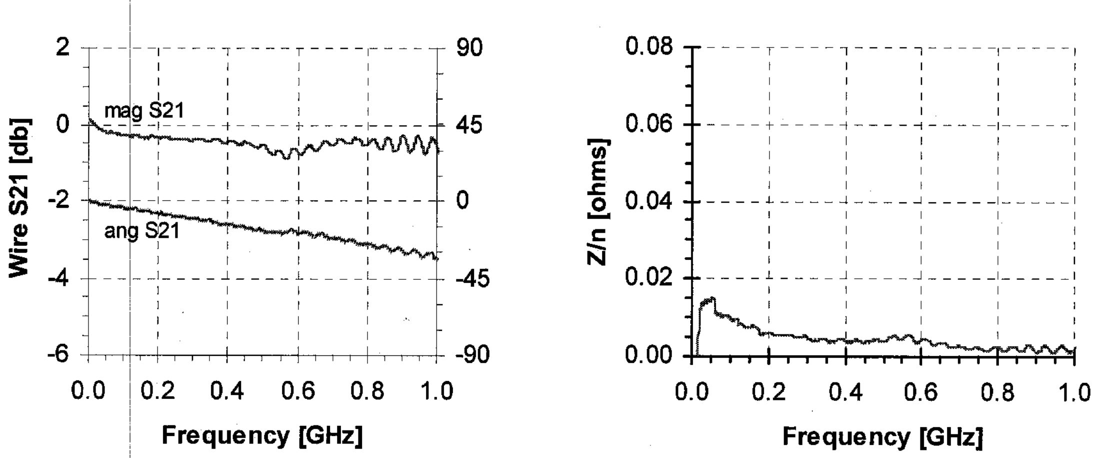


Figure 1. Measured attenuation and longitudinal impedance of one Tevatron C0 and one F0 lambertston.

For a round pipe, the transverse impedance can be calculated from the longitudinal impedance, figure 2. This model, or Panofsky equation, is often used to estimate transverse impedance for more complex devices.

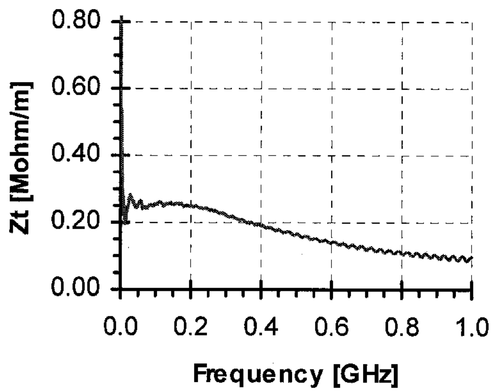
$$Z_T \approx \frac{2c}{\omega b^2} Z_L \qquad Z_L \textit{ measured at the center of radius } b$$

$$= \frac{2c}{\omega_0 b^2} \frac{Z_L}{n}$$

C0 lambertson estimated transverse impedance

$$\frac{2c}{\omega_0 b^2} = 7.94x10^6$$

$$for\ b = 0.625", \quad \frac{\omega_0}{2\pi} = 47713Hz$$



F0 lambertson estimated transverse impedance

$$\frac{2c}{\omega_0 b^2} = 1.99x10^6$$

$$for\ b = 1.25", \quad \frac{\omega_0}{2\pi} = 47713Hz$$

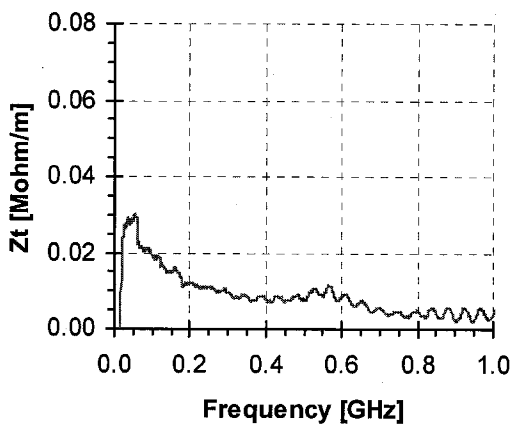


Figure 2. Tevatron C0 and F0 lambertston transverse impedance estimated from the longitudinal measurement. The impedance shown is for one magnet.

Measured Transverse Impedance

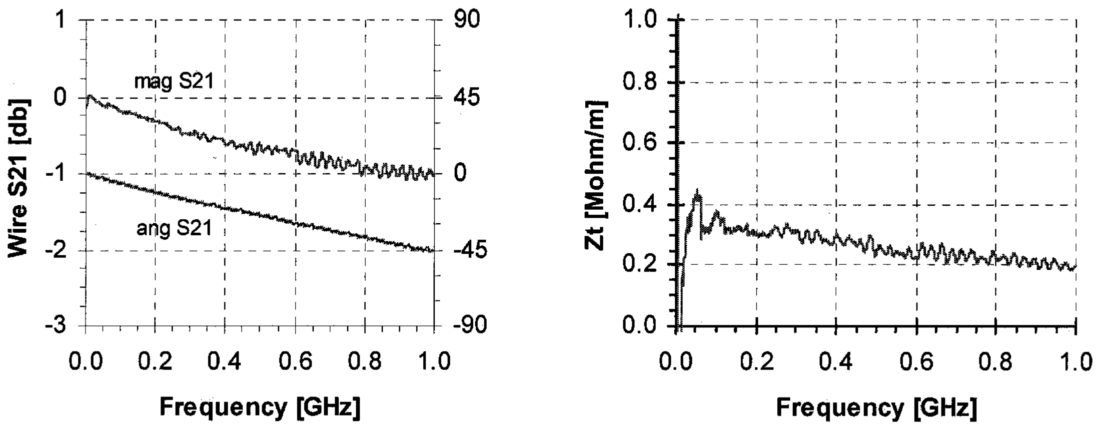
The transverse beam impedance can be determined more directly from the attenuation measured along two parallel wires driven differentially. Tin plated copper wires 0.010" in diameter were used to form a TEM balanced transmission line. A 5mm wire spacing was used in the smaller C0 lambertson (Zo=441Ω) and a 10mm spacing was used in the F0 lambertson (Zo=518Ω). Each end was matched to 100Ω with resistive L pads and combined/driven with a 100Ω broad band (5MHz to 1GHz) 180° hybrid splitter (Anzac H-1-4). A network analyzer was used to measure the transmission (S21) through the device. The measurement is normalized to a reference pipe, 1.250" ID for the C0 and 2.375" ID for the F0 lambertsons.

The vertical transverse impedance is measured with wires in a vertical plane to simulate vertical beam motion.

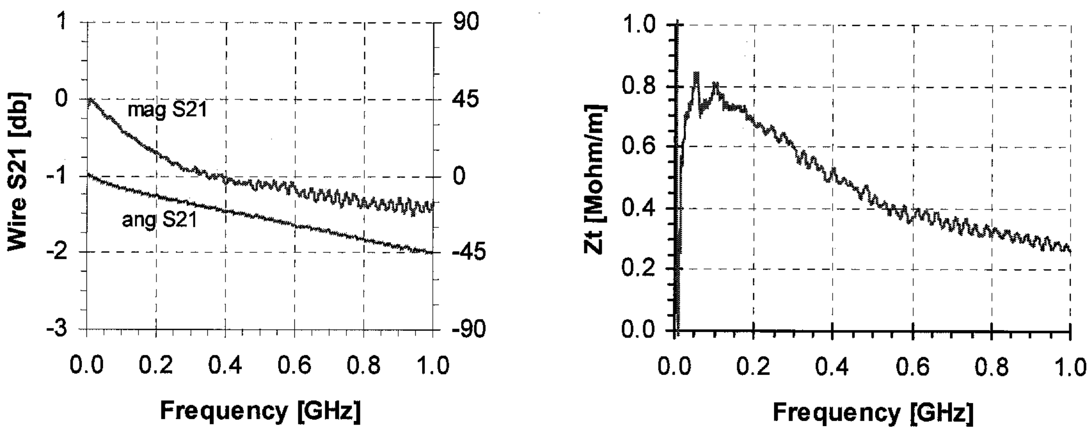
The transverse impedance is determined from the attenuation or S21 measurement, figures 3, 4.

$$Z_T = -\frac{c}{\omega\Delta^2} 2Z_0 \ln(S_{21}) = \frac{c}{\omega\Delta^2} 2Z_0 \frac{S_{21}[db]}{20\log(e)} \qquad \Delta \textit{ is the wire separation}$$

Tevatron C0 lambertson <u>horizontal</u> transverse impedance

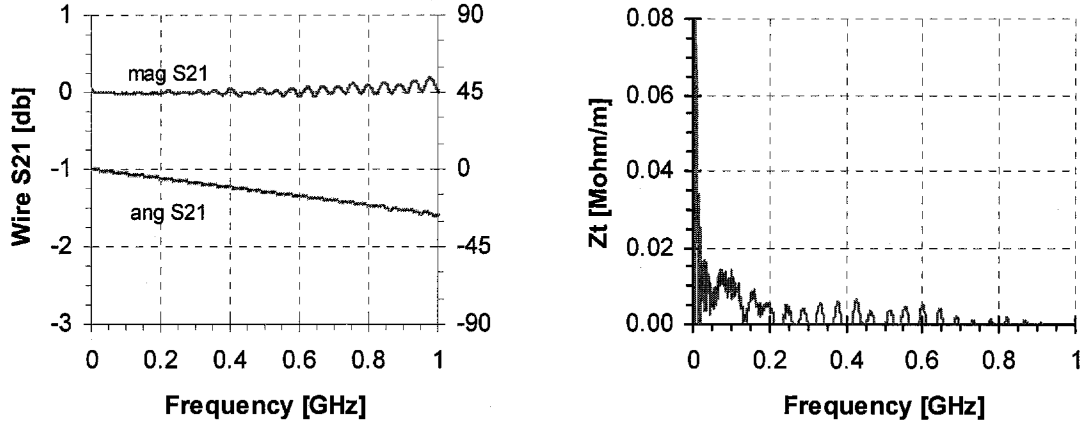


Tevatron C0 lambertson <u>vertical</u> transverse impedance

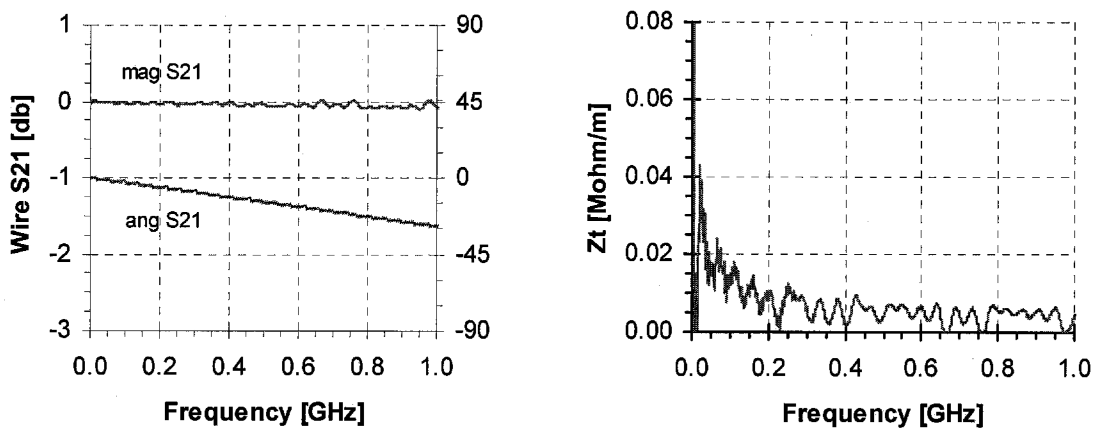


Figrure 3. Measured Transverse impedance of a Tevatron C0 lambertson. A 5mm wire spacing was used. The impedance shown is for one magnet.

Tevatron F0 lambertson horizontal transverse impedance



Tevatron F0 lambertson vertical transverse impedance



Figrure 4. Measured Transverse impedance of a Tevatron F0 lambertson. A 10 mm wire spacing was used. The impedance shown is for one magnet.

Position Dependance of Transverse Impedance

The C0 lambertson was measured with the balanced line at 5 positions. Both the horizontal and vertical impedances were smaller near the center of the aperture and increased as the balanced line was moved toward a wall. The impedance at the center was about half of that measured 2 cm toward the point.

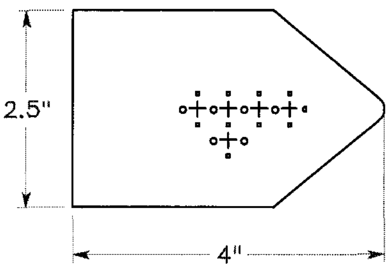


Figure 5. The transverse impedance was measured at 5 positions in the F0 lambertson. The conductors of the balanced line were spaced by 1cm. The positions are Center, 1 and 2 cm toward the point, 1cm down, and 1cm away from the point.

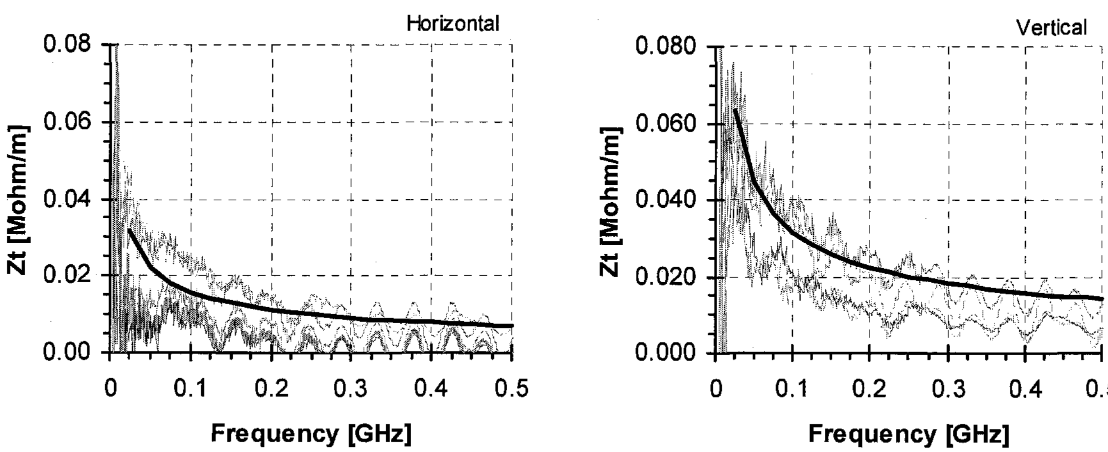


Figure 6. Horizontal and Vertical impedance at 5 different positions for one of the F0 lambertsons. The smooth black line indicates one over square root frequency dependence. The lowest impedance was found near the center.

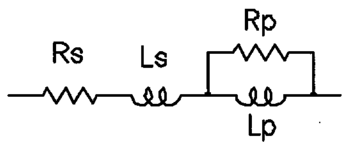
Comparison of the Magnet Lead Impedance

The purpose of the laminations is to reduce or prevent eddy currents that would stop fast changing magnetic fields from reaching the beam. The magnet laminations are typically welded together at the outside edge. If the laminations were shorted to each other near the beam, the beam impedance would be greatly reduced. Shorted laminations would increase losses in the laminations and should be apparent in the magnet impedance seen by the driving power supply.

The lead impedance of an F0 and C0 lambertson was measured. A good electrical model for a laminated magnet is shown below. The model includes a parallel R/L component to represent the lamination associated loss and inductance. The series R/L component represents copper loss in the windings and leakage flux. The F0 magnet had a resonance in the winding at 86.4 KHz. The first resonance in the C0 magnet was at 297 KHz. Figure 4 displays the measured values and the fit. No indication of shorted laminations is evident. Assuming the reluctance is dominated by the field region gap, the magnet inductance can be estimated as shown below. The estimated inductance is nearly double the measured value. A is magnet length times gap width and h is the gap height.

$$L = N\frac{\phi}{I} \qquad \phi = \frac{MMF}{R} \qquad MMF = NI \qquad \frac{1}{R} \approx \mu_o \frac{A}{h}$$

$$Lp \approx N^2 \mu_o \frac{A}{h}$$



	F0	C0
Rs	0.50Ω	.23Ω
Ls	850uH	50uH
Rp	3.8Ω	0.8Ω
Lp	10mH	1.0mH
$N^2 \mu_o \frac{A}{h}$	20mH	1.7mH
N	24	6
gap width	17"	7"
gap height	2"	1"
magnet length	129"	216"
self resonance	86.4 KHz	297 KHz

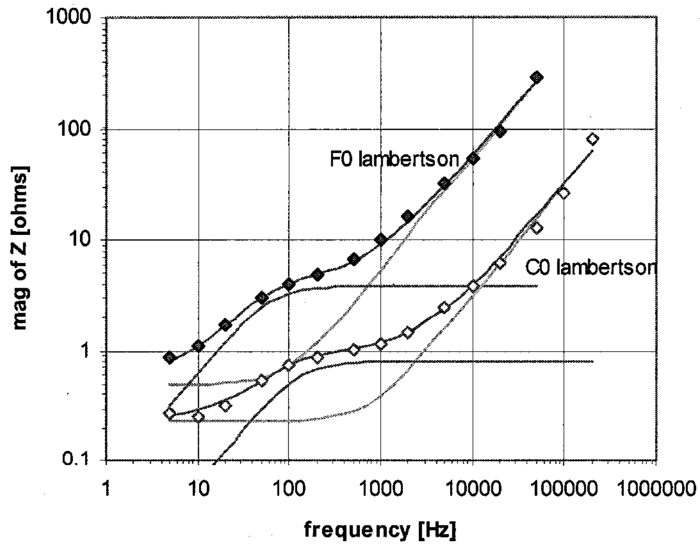


Figure 7. Measured coil impedance of the F0 and C0 lambertsons. The points are actual measurements and the solid lines are fits. The series and parallel components are shown separately.

Discussion

Although larger wire spacing provides greater sensitivity to beam impedance, it also makes the characteristic impedance change more with position which complicates the measurement. A reasonable compromise was to use a 5mm separation in the C0 lambertson and 10 mm in the F0 lambertson. Attenuation is small in the two wire measurement making reference measurements critical.

The Panofsky equation provides the transverse impedance from the longitudinal impedance measured at the center of a round pipe. The relationship is frequently employed to estimate the transverse impedance in more complicated geometries. For a 6.28 km circular beam pipe with resistivity ρ, radius b, and beam displacement x_o; the longitudinal and transverse impedances are shown below.

$$\frac{Z_L}{m} = \frac{\rho}{\delta 2\pi b}\left(1+2\frac{x_0^2}{b^2}\right) \qquad \delta = skin\ depth = \sqrt{\frac{2\rho}{\omega\mu}}$$

$$\frac{Z_T}{m} = \frac{2c}{\omega b^2}\frac{Z_L}{m} = \frac{2c}{\omega b^2}\frac{\rho}{\delta 2\pi b}\left(1+2\frac{x_0^2}{b^2}\right)$$

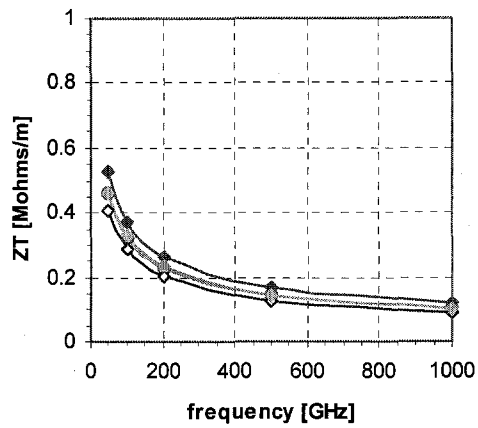


Figure 8. Transverse impedance of 6.28 km of 3" diameter round stainless pipe at room temperature and at 3°K (ρ= 90 and 70e-8Ω-m). Data is also shown for beam at center and offset by 1cm. At 100MHz the impedance ranges from 0.29 to 0.37 MΩ/m.

The Panofsky equation suggests the transverse impedance scales as one over the diameter cubed and may explain the large difference in impedance between the C0 and F0 lambertsons.

$$\left(\frac{d_{F0}}{d_{C0}}\right)^3 = 8.0 \qquad \frac{Zt_{C0}}{l_{C0}}\frac{l_{F0}}{Zt_{F0}} = 11.6$$

	C0	F0
Zt/magnet (long. est.)	0.3 MΩ/m	0.03 MΩ/m
Ver Zt/magnet	0.8 MΩ/m	0.04 MΩ/m
Hor Zt/magnet	0.4 MΩ/m	0.02 MΩ/m
lamination length	218"	126"
ID	1.25"	2.5"

Conclusion

At 100MHz, the transverse impedance was about 2.4MΩ/m for the three C0 lambertsons (recently removed) and 0.2MΩ/m for the four F0 lambertsons remaining in the Tevatron. The impedance is dominated by skin effect losses on the lamination surface and should decrease like one over the square root of frequency. In comparison, a rough estimate of 6 km of stainless beam pipe is 0.3MΩ/m.

References:

1) Sacherer, F., Nassibian, G.; "Methods for Measuring Transverse Coupling Impedances in Circular Accelerators"; Nuc. Inst. And Meth., Vol. 159, pgs 21-27, 1979.
2) Panofsky, W.K.H., Wenzel, W.A.; "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", Rev. Sci. Instrum., Vol 27, pg. 967, 1956.
3) Crisp, J., Fellenz, B.; "Measured Longitudinal Beam Impedance of a Tevatron Separator"; Fermilab TM-2194; November 25, 2002.
4) Chao, A., Tigner, M. ; "Handbook of Accelerator Physics and Engineering"; World Scientific Publishing; 1999; pg. 570.
5) "Reference Data for Radio Engineers"; sixth edition; Howard W. Sams; 1984.
6) M. Harrison, F.N. Rad; "Symmetric and Non-Symmetric Lambertson Magnets"; FN-398; February 1984.

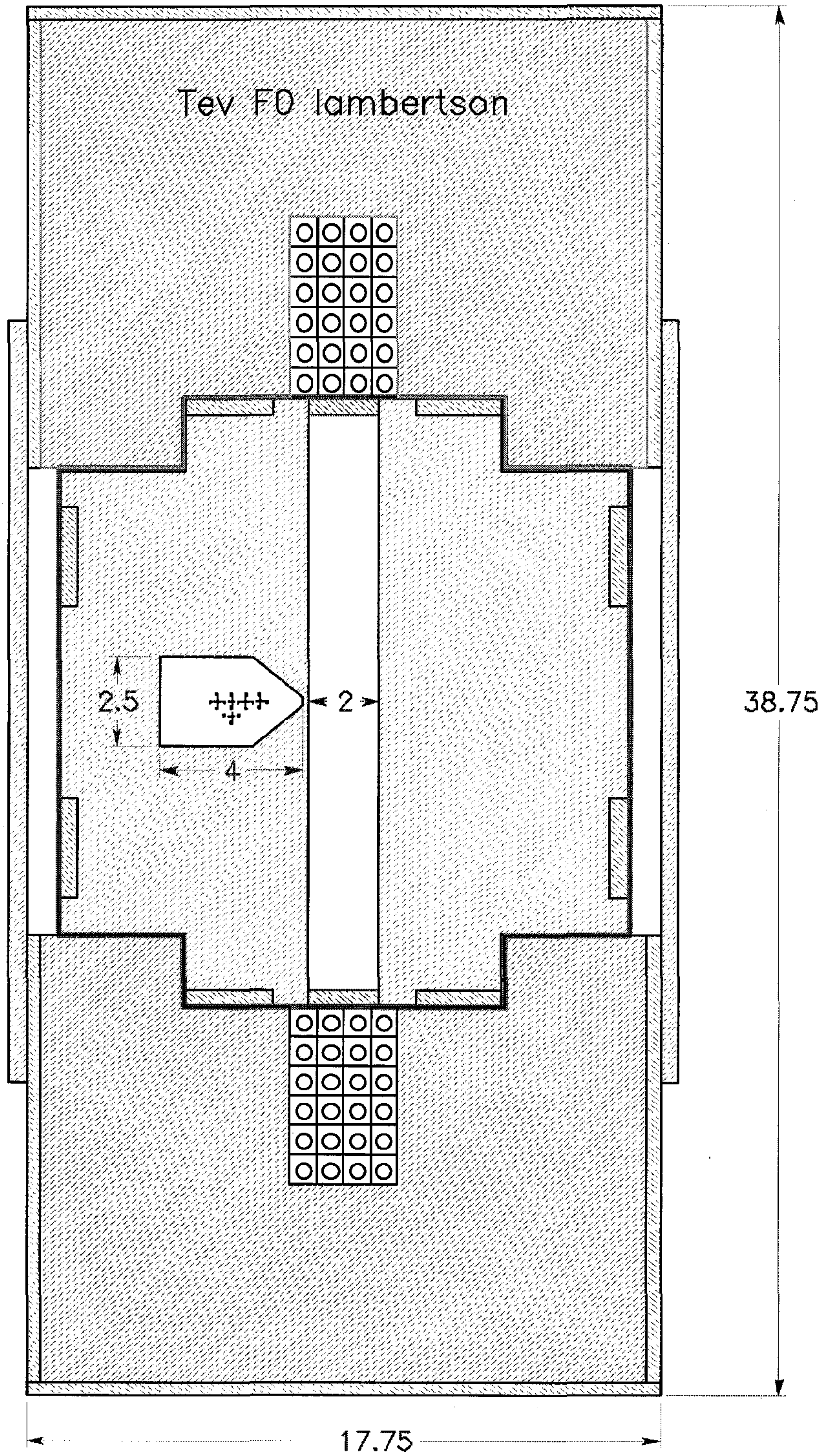
Wire Data:

wire	tin plated copper	(30 gauge)	
wire diameter	0.010"		
wire resistance <200MHz	10.4	'Ω/m@1GHz'	(Cu - 1.74e-8 Ω-m)
wire resistance >200MHz	26.6	'Ω/m@1GHz'	(Sn – 11.4e-8 Ω-m)
skin depth in tin @200MHz	.00047"		

F0 lambertson:

lamination thickness	0.0375"		
number of laminations	3360"		
lamination length	~126"	(128.25" with end blocks)	
wire length	132"		
aperture	2.5x4"		
reference pipe ID	2.375"	(2.5" stainless pipe, Zo=328Ω)	
reference pipe length	130.75		
Single wire impedance	331	2.5/.01	60 ln(D/d)
loss	10.6	'db/100ft@1GHz'	
Lpad	310/58.7Ω	-27.6db	
Balanced line spacing	10mm		
impedance	518Ω	.394/.01	~120 ln(2h/d)
loss	13.6	'db/100ft@1GHz'	
Lpad	205/127Ω	-18.4db	

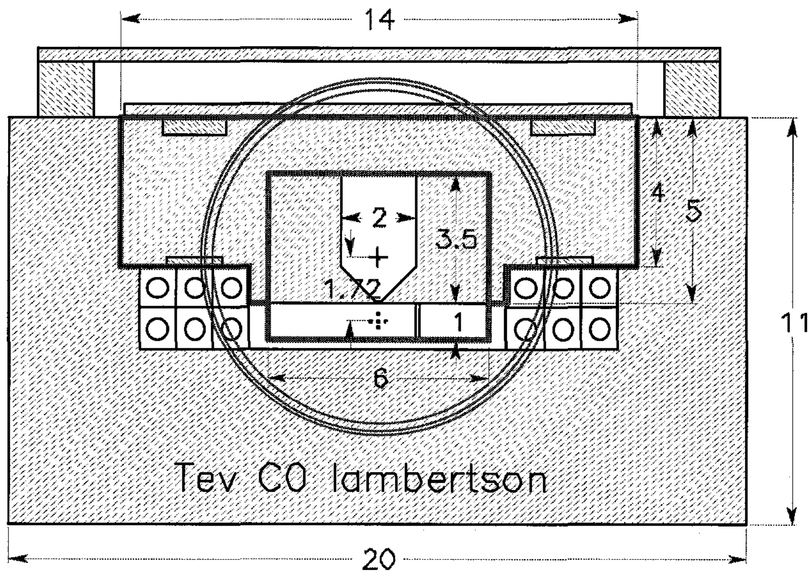
Sketch of Tevatron F0 lambertson: (scale 1:5)



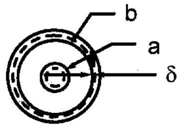
C0 lambertson:

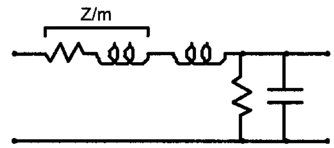
lamination thickness	0.0375”	
number of laminations	5812	
lamination length	218”	
wire length	228”	
aperture height	1”	(vertical 1/8” stainless bar 1” from center)
reference pipe ID	1.25”	(1.375” copper pipe, Zo=289Ω)
reference pipe length	228”	
Single wire impedance	291Ω	1/.01” 60 ln(4h/πd)
loss	12.1	‘db/100ft@1GHz’
Lpad	255/58.7Ω	-26.0db
Balanced line spacing	5mm	
impedance	437Ω	.197/.01” ~120 ln(2h/d)
loss	16.1	‘db/100ft@1GHz’
Lpad	162/137Ω	-16.4db

Sketch of Tevatron C0 lambertson: (scale 1:5)



Appendix 1: *(Skin effect losses in a stretched wire)*





$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

high frequency approximation $R \ll \omega L$ *and* $G \ll \omega C$

$$Z_0 \approx \sqrt{\frac{L}{C}} \qquad \alpha = \frac{R}{2Z_0} + \frac{G}{2} Z_0 \approx \frac{R}{2Z_0} \qquad \beta = \frac{\omega}{\upsilon_p} \approx \omega\sqrt{LC}$$

The inductive impedance equals the real impedance in a good conductor.

$$Z/_{m} \approx \left(\frac{\rho_a}{a\pi\delta_a} + \frac{\rho_b}{b\pi\delta_b} \right)(1 + j) = R/_{m}(1 + j)$$

'a' is the diameter of the outer conductor and 'b' is that of the center conductor. (the impedance is generally dominated by the losses on the small center conductor)

skin depth in good conductor: $\qquad \delta = \sqrt{\frac{2\rho}{\mu\omega}}$

or $\qquad R/_{m} \approx \alpha 2 Z_0 \sqrt{f/_{f_0}} \qquad$ *for* α *measured at* f_0

A good approximation is to calculate the attenuation at 1GHz and assume the actual attenuation is proportional to the square root of the frequency.

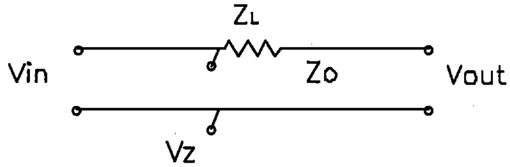
$$\frac{Vout}{Vin} = e^{-\gamma d} \qquad mag = e^{-\alpha d} \qquad ang = \left(\frac{\omega}{\upsilon_p} - \beta \right) d$$

It was interesting to see a knee in the attenuation versus frequency using 0.010" (30 gauge) tin plated copper wire. Below 200MHz the attenuation corresponds to the resistivity of copper (1.74e-8Ω-m) at higher frequencies it matches the value using the resistivity of tin (11.4e-8Ω-m). The skin depth at 200MHz in tin is .00047", about the plating thickness one would expect.

Appendix 2: *(useful equations for beam impedance measurements)*

Longitudinal Beam Impedance

lumped impedance:



$$Vz = Vinc + Vref = Vin\left(1 + \frac{(Z_L + Z_0) - Z_0}{(Z_L + Z_0) + Z_0}\right)$$

$$Vout = Vz\frac{Z_0}{Z_L + Z_0} = Vin\frac{Z_0}{Z_L + Z_0}\frac{2(Z_L + Z_0)}{Z_L + 2Z_0}$$

$$\frac{Vout}{Vin} = S_{21} = \frac{2Z_0}{Z_L + 2Z_0}$$

$$Z_L = 2Z_0\frac{1 - S_{21}}{S_{21}}$$

high frequency approximation:

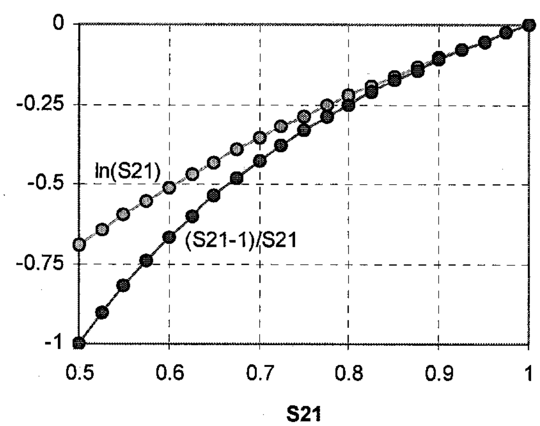
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

at high frequency $R \ll \omega L$ *and* $G \ll \omega C$

$$Z_0 \approx \sqrt{\frac{L}{C}} \qquad \alpha = \frac{R}{2Z_0} + \frac{G}{2}Z_0 \approx \frac{R}{2Z_0} \qquad \beta = \frac{\omega}{\upsilon_p} \approx \omega\sqrt{LC}$$

$$\frac{Vout}{Vin} = S_{21} = e^{-\alpha l}$$

$$R = -2Z_0 \ln S_{21} = -2Z_0\frac{S_{21}[db]}{20\log(e)}$$



Transverse Impedance

Panofsky equation: (estimate transverse impedance from longitudinal measurement)

$$Z_T = \frac{2c}{\omega b^2} Z_L \qquad \textit{for radius } b \textit{ and } Z_L \textit{ measured at the center}$$

$$Z_T = \frac{2c}{\omega b^2} 2Z_0 \frac{S_{21}[db]}{20\log(e)} \qquad \textit{for distributed impedance} \quad (\textit{from longitudinal } S_{21})$$

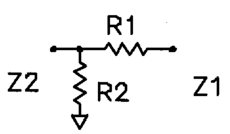
Transverse impedance: (using a balanced or differential transmission line)

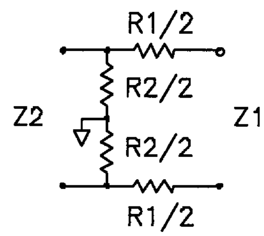
$$Z_T = \frac{c}{\omega \Delta^2} 2Z_0 \left(\frac{1 - S_{21}}{S_{21}} \right) \qquad \textit{for lumped impedance} \quad (\Delta = \textit{wire spacing})$$

$$= \frac{c}{\omega \Delta^2} 2Z_0 \ln S_{21} \qquad \textit{for distributed impedance}$$

$$Z_T = \frac{c}{\omega \Delta^2} 2Z_0 \frac{S_{21}[db]}{20\log(e)}$$

Impedance Matching





$$R1 = \sqrt{Z1\,(Z1 - Z2)}$$

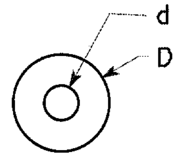
$$R2 = \frac{Z1\,Z2}{R1}$$

ideal resistor values:

	Z1	Z2	R1	R2	db 0Hz	db/100' 1GHz	d	D	h
F0 coaxial	331	50	305	54.3	27.76	10.6	.01"	2.375"	
F0 balanced	518	200	406	255	18.32	13.6	.01"		10mm
C0 coaxial	291	50	265	54.9	26.55	12.1	.01"	1"	
C0 balanced	437	200	326	272	16.46	16.1	.01"		5mm

Characteristic Impedance

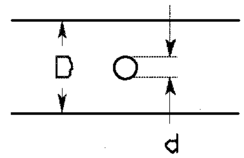
coaxial line:



$$Zo = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{D}{d} = 60\ln\frac{D}{d}$$

Zo=331Ω For 0.010" diameter wire in a 2.5" pipe:

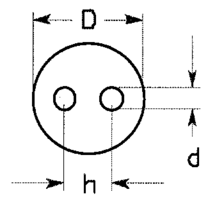
parallel plate line:



$$Zo = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{4D}{\pi d} = 60\ln\frac{4D}{\pi d}$$

Zo=291Ω For 0.010" wire between 1" plates:

balanced line:

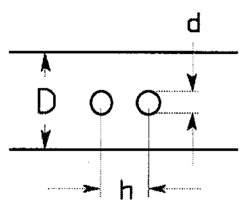


$$Zdiff = \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{2h}{d}\frac{1-\left(\frac{h}{D}\right)^2}{1+\left(\frac{h}{D}\right)^2}\right) \approx 120\ln\frac{2h}{d} \qquad (d << h, D)$$

$$Zcom = \frac{1}{4\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{h}{2d}\frac{1-\left(\frac{h}{D}\right)^4}{\left(\frac{h}{D}\right)^2}\right) \approx 30\ln\frac{D^2}{2hd}$$

Zo=518Ω For 0.010" diameter wires in a 2.5" pipe:

balanced line between parallel plates:



$$Zdiff = \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{4D}{\pi d}TanH\frac{\pi h}{2D}\right) \approx 120\ln\frac{2h}{d} \qquad (d << h, D)$$

Zo=437Ω For 0.010" wires between 1" plates: