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Momentum Errors in an RF Separated Beam

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Abstract

The purity of an RF separated beam is affected by the difference in mass of the particle types and the momentum bite of the beam. The resulting time-of-flight difference between different types allows separation to occur; the finite momentum bite results in chromatic aberation. Both these features also give rise to a particle type dependent velocity bite, which must also be taken into account. This memo demonstrates a generalizable method for calculating the effect.

1 Introduction.

While modeling the purity of the proposed RF separated beamline for Fermilab E921/CKM[1 2], one of us (Coleman) noted that the spectrum of transmitted protons looked "funny" — only off-momentum particles were transmitted (see Figure 1). Further investigation showed that, additionally, protons were the dominant contaminant; and the purity decreased as a function of RF kick (see Figure 2). The purity was re-calculated, accounting for the obvious affects of pion decay and collimation due to apertures, but the effects remained. This puzzled us.

RF separation works because of the difference in time-of-flight for wanted and unwanted particles. For a given species, off-momentum particles will be affected differently than on-momentum particles (due to their different speed). However, two different species with the same momentum offset will have different velocity offsets due to their differing mass. The optics for the beamline allow one to select a given momentum range; however, this same momentum range becomes different velocity ranges depending on the particle type. This can cause off-momentum particles to pass around the absorber. Additionally, the optics between the RF stations are not achromatic.

The following analysis will be specific to the CKM beamline, but the techniques are easily generalized to any RF separated beamline. Also, all references to the CKM beamline should be understood to mean the model of the CKM beamline (as of the time of writing, the beamline has not been built).

[1] "Charged Kaons at the Main Injector", Fermilab-Proposal-CKM

[2] J. Doornbos, NIM Physis Research A 455 (2000) 253-270, Possibilities for a 15-25 GeV/c RF-separated charged kaon beam

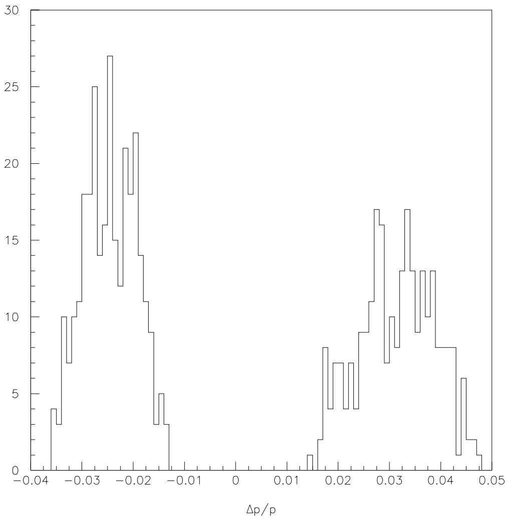

Figure 1: Spectrum of protons transmitted past beam-plug in CKM beamline.

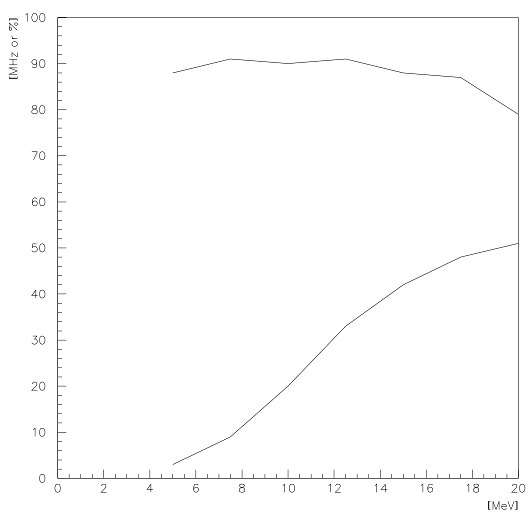

Figure 2: Hardonic purity [%] and rate [MHz] (upper and lower curves, respectively) as functions of RF kick [MeV]. Note that the kaon purity decreases as the transverse kick increases.

species	$\Delta p/p$	β_{lower}	β_{upper}	ΔRF_{lower}	ΔRF_{upper}
$\pi+$	$\pm 2\%$	0.999979	0.999981	0.32°	-0.34°
$K+$	$\pm 2\%$	0.999738	0.999758	3.96°	-4.20°
$p+$	$\pm 2\%$	0.999054	0.999127	14.29°	-15.17°

Table 1: Difference in velocity spread and RF phase error for same momentum-bite of pions, kaon, and protons, assuming a 22 GeV/c central momentum. Because the protons are more massive, they have a larger velocity spread, which results in a larger RF phase error.

2 The CKM Beamline

CKM requires a high rate, high purity, small momentum-bite, K^+ beam. To achieve this, RF separation is employed.

Two RF stations are used. The distance between them is chosen such that the unwanted pions and protons slip one RF cycle with respect to each other (Kaons slip by about 93° with respect to the pions). The second RF station is run 180° out of phase (or, equivalently, in phase but opposite polarity), with respect to the first station. A +1 F0D0 is placed between the stations. The result, for on-momentum particles, is that the pions and protons receive no net kick, while the kaons see a sinusoidally varying kick. After the second RF station, a 90° cell, with some magnification, transports the particles to a beam-plug. The pions and protons are absorbed in the plug while the Kaons pass around it.

In the CKM beamline, each RF station gives a total transverse kick of 15 MeV. The RF cavities are run at 3.9 GHz. The two stations are separated by 86 m. The current configuration gives a 42 MHz, 90% pure, kaon beam for a 5×10^{12} protons-per-second 120 GeV/c primary beam. The momentum spread is about $\pm 2\%$.

3 Explanations

3.1 Proton Excess

Although the protons, pions, and kaons all have the same momentum, they are traveling at different velocities — this is why RF separation works. Additionally, although all three species have the same momentum bite, the velocity spread is different (refer to Table 1). Thus, the same momentum-bite for pions and protons results in a much larger velocity-bite (and therefore RF phase error) for protons. (Note, also, that a symmetric $\Delta p/p$ results in an asymmetric $\Delta v/v$.) The result is that the separators are more effective for pions than protons.

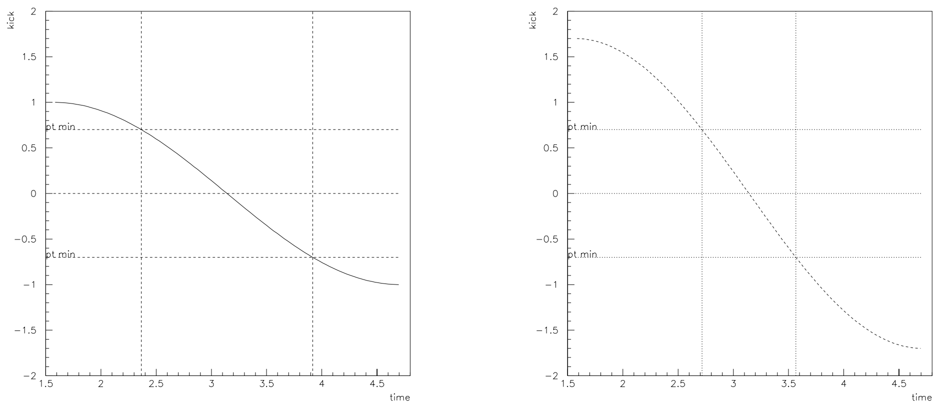

Figure 3: Kaon kick as a function of time for two different peak amplitudes. In these plots, the kick is given in arbitrary, but constant units. The horizontal lines labeled "pt_min" are drawn at an arbitrary, but constant, value. Note that as the amplitude increases, the amount of time where the magnitude of the kick is less than "pt_min" derceases.

3.2 Purity Decrease.

The purity decreasing with increasing kick may be understood with a simple model. The kaon beam sees a time dependent transverse kick, and some minimum kick is required to deflect the kaons around the beam plug. Considering only on-momentum particles, the pions, kaons, and protons will have the same spot-size. Thus, the beam plug may be sized such that the majority of the pions/protons are absorbed. At this point, if p_t is increase, the amount of time the kaon beam is deflected around the beam plug increases, so the purity increases due to an increase in kaon flux (refer to Figure 3).

The situation becomes a bit more complicated when a finite momentum spread is introduced. We observed that a momentum offset is equivalent to arriving at the second station with a phase offset. Concentrating only on the protons, the net kick may be expressed as:

$$A = sin(\theta) + sin(\theta + \delta + 180^\circ)$$

where θ is the time-dependent field in the RF station, δ is the momentum-dependent phase offset and 180° indicates the constant phase difference between the two RF stations.

For the sake of simplicity, we will assume that the off-momentum proton enters the first station when $\theta = 0$, thus reducing the amplitude function to a simple function of the momentum offset, δ. The momentum offset is related to

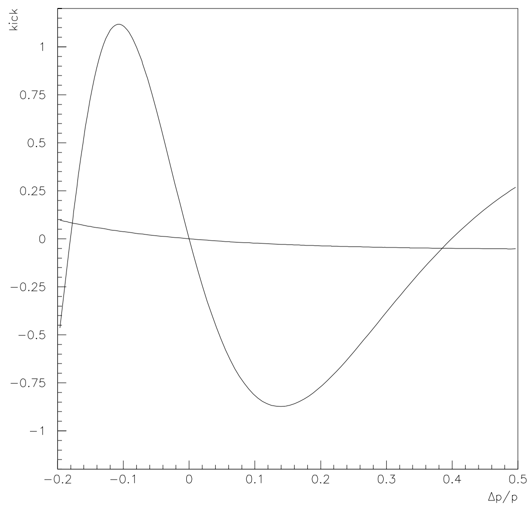

Figure 4: Kick as a function of phase offset for off-momentum protons and pions. Vertical scale is such that an on-momentum Kaon (not shown) would receive a maximum unit kick. The curve for the pion is much more constant than that of the proton.

the phase offset by:

$$\delta = 2\pi \frac{d}{c} \left(\frac{1}{\beta} - \frac{1}{\beta_0} \right)$$

where d is the separation between RF stations.

Finally, the dependence of the transverse kick on the particle momentum must be included. The magnetic field in the RF cavity is responsible for the change in transverse momentum of the particle. The radius of curvature of the particle's trajectory will be proportional to the particle's momentum (we assume that the phase velocity of the RF cavity and the velocity of the particle are the same). Thus, $\Delta\theta/\theta = -\Delta p/p$.

Figure 4 shows the kicks given to a proton and pion, each entering the first station at $\theta = 0$, as a function of $\Delta p/p$. The vertical scale is arbitrary units, in which an on-momentum Kaon, initially in time with the pion and proton, would receive a 1 unit kick (precisely, a $sin(93°) = 0.9986$ unit kick). Note that the effect for the pion is very small compared to the proton.

4 An Example.

As an example of this effect, consider the CKM beamline. After the second RF station there is a 90° cell with ×10 magnification which focuses the beam on a 6 mm half-width beam plug. Thus, for a zero emittance beam, any particle which receives greater than a 600 μradian kick will go around the plug. The RF station is expected to give p_t = 15 MeV/c, so the maximum deflection is 682 μradian. This means that any particle on our graph with a kick greater than 600/682 = 0.88 is deflected around the beam plug. Drawing a horizontal line on Figure 4 at 0.88, this line intersect the curve at $\Delta p/p = -0.08$ and -0.14. So any proton with $\Delta p/p$ between -8% and -14% will not be absorbed.

Unfortunately, this does not explain our results. We see protons with $\Delta p/p = \pm 2\%$ "sneaking" around the absorber.

5 Chromatic Aberration

As previously stated, the CKM beamline employs a +1 transfer between the two RF stations and a $\times 10$ 90° cell between the last RF station and the beam plug. However, these values occur only at the central momentum—the beamline suffers from chromatic aberration.

Chromatic aberation will not be covered in any detail in this paper—interested people can refer to the standard literature[3]. Only results will be quoted.

For the CKM beamline, at $\Delta p/p$ =-2%, the vertical transfer matrix of the unitary F0D0 changes to:

$$\mathbf{M} = \begin{pmatrix} 0.874 & -2.085 \\ 0.018 & 1.102 \end{pmatrix}$$

The $\times 10$ 90° cell between the second RF station and the beam-plug becomes:

$$\mathbf{M} = \begin{pmatrix} 0.026 & 10.194 \\ -0.098 & 0.024 \end{pmatrix}$$

6 An Example Revisited.

Assume a proton with $\Delta p/p = -0.02$ enters first station at the peak field. Because the proton is below nominal momentum it will receive a kick of 1.020 units. The proton leaves the first station, goes through the F0D0, and enters the second station. Note that the transfer matrices is no longer a unit matrix (due to chromatic aberation) — the angular kick is magnified by 1.102 (the m_{22} term of the transfer matrix). Additionally, the proton is out of phase by $-14.29°$. The net kick (after traversing the second station) is:

$$\frac{1.102}{0.98} - \frac{sin(-14.29°)}{0.98} = 0.8726$$

The proton then passes through the 90° cell, which now magnifies by 10.194 (again, due to chromatic aberation), resulting in a value of 8.895. Referring back to Section 4, we see that any particle with a value greater that 8.8 will clear the beam plug. Thus, due to the combined effects of phase lag and chromatic aberation, the proton clears the beam plug.

[3] See, for example, "The Optics of Charge Particle Beams" by David C. Carey.

7 Summary.

In an RF separated beam, two effects degrade the purity: phase-offset of off-momentum particles, and chromatic aberation. Chromatic aberation can be minimized, or even eliminated, using standard techniques. However, the phase error is dependent on the particle type and cannot be eliminated completely. The purity of the beam is ultimately set by the velocity spread of the unwanted particles.

Acknowledgments.

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