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Tevatron Bunch Length Studies at CDF

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1 Introduction

A luminous interaction region can be described by the distribution in (x, y, z) over which $p\overline{p}$ interactions are observed in a detector. The spatial size of this distribution can be written in terms of expressions that involve only combinations of the proton and anti-proton bunch sizes. Hence, it is not possible to determine the sizes of the proton and anti-proton bunches independently by fitting the distribution of (x, y, z) from the recorded events.

If, in addition to the coordinates (x, y, z) at which $p\overline{p}$ interactions occur, the times at which the interactions took place are also measured, then it becomes possible to measure the lengths of the proton and anti-proton bunches separately. This sensitivity is due to a correlation between z and t that arises from the fact that the proton and anti-proton bunches travel in opposite directions.

The derivation presented in section 2 quantifies this correlation, resulting in an expression for the probability density as a function of z and t for $p\overline{p}$ interactions. By fitting the distributions observed at CDF using this model, we measure the lengths of the proton and anti-proton bunches at times throughout several Tevatron stores. From this analysis the evolution of the bunch lengths can be studied. We attempt to correlate these with other measures of the bunch length obtained using different experimental techniques.

2 Bunch Shape Models

The model we considerer assumes that proton and anti-proton bunches have a Gaussian charge density and travel at the speed of light in opposite directions along the z-axis. These are expressed, for proton and anti-proton bunches, by

$$\rho(x,y,z,t) = \frac{N}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z} e^{-x^2/2\sigma_x^2} e^{-y^2/2\sigma_y^2} e^{-(z\mp ct)^2/2\sigma_z^2} \tag{1}$$

where N is the number of particles in the bunch and σ_x, σ_y and σ_z are the Gaussian bunch widths. The expression $z-ct$ or $z+ct$ in the exponent corresponds to proton and anti-proton bunches, respectively.

The horizontal and vertical bunch widths are functions of z and can be expressed in terms of the transverse emittance and the transverse β-function, evaluated at z:

$$\sigma_{x,y}^2(z) = \epsilon_{x,y}\beta_{x,y}(z) \tag{2}$$

where

$$\beta_{x,y}(z) = \beta_{x,y}^* + \frac{z^2}{\beta_{x,y}^*} \tag{3}$$

We will assume that the vertical and horizontal values of the β-function at $z=0$ are the same and refer to this value as β^*.

The luminosity is related to the integral of the overlap of the proton and anti-proton densities:

$$\mathcal{L} = 2cf_{\text{rev}} \int \rho_p(x,y,z,t)\rho_{\overline{p}}(x,y,z,t)dxdydzdt. \tag{4}$$

Performing the integral over x and y gives

$$\mathcal{L} = \frac{2cf_{\text{rev}}N_pN_{\overline{p}}}{(2\pi)^2} \int \frac{1}{\sigma_x(z)\sigma_y(z)\sigma_p\sigma_{\overline{p}}} e^{-(z^2+c^2t^2)(1/2\sigma_p^2+1/2\sigma_{\overline{p}}^2)-zct(1/\sigma_{\overline{p}}^2-1/\sigma_p^2)}dzdt \tag{5}$$

$$= \frac{cf_{\text{rev}}N_pN_{\overline{p}}}{2\pi^2\beta^*(\epsilon_p+\epsilon_{\overline{p}})\sigma_p\sigma_{\overline{p}}} \int \frac{1}{(1+z^2/\beta^{*2})} e^{-(z^2+c^2t^2)(1/2\sigma_p^2+1/2\sigma_{\overline{p}}^2)-zct(1/\sigma_{\overline{p}}^2-1/\sigma_p^2)}dzdt \tag{6}$$

where $\sigma_x^2(z) = \sigma_{px}^2(z) + \sigma_{\overline{p}x}^2(z)$, $\sigma_y^2(z) = \sigma_{py}^2(z) + \sigma_{\overline{p}y}^2(z)$ and where σ_p and $\sigma_{\overline{p}}$ are the lengths of the proton and anti-proton bunches, respectively.

The integrand of this expression must be normalized in order to construct a probability density in z and t. Thus, we integrate first over t, which yields the z-distribution of the luminous region, and then over z.

Integrating equation 6 over time gives

$$\mathcal{L} = \frac{f_{\rm rev} N_p N_{\overline{p}}}{(2\pi)^{3/2}\beta^*(\epsilon_p + \epsilon_{\overline{p}})\sigma_z} \int \frac{e^{-z^2/2\sigma_z^2}}{(1 + z^2/\beta^{*2})} dz \tag{7}$$

where

$$\sigma_z = \frac{\sqrt{\sigma_p^2 + \sigma_{\overline{p}}^2}}{2} \tag{8}$$

Integrating this expression over z yields

$$\mathcal{L} = \frac{f_{\rm rev} N_p N_{\overline{p}}}{2\sqrt{2\pi}(\epsilon_p + \epsilon_{\overline{p}})\sigma_z} e^{\beta^{*2}/2\sigma_z^2} {\rm erfc}(\beta^*/\sigma_z\sqrt{2}) \tag{9}$$

The probability density for the distribution of z and t for primary interactions is then

$$F_{p\overline{p}}(z,t) = \left(\frac{\pi c \beta^* e^{\beta^{*2}/2\sigma_z^2} {\rm erfc}(\beta^*/\sigma_z\sqrt{2})}{\sqrt{1/\sigma_p^2 + 1/\sigma_{\overline{p}}^2}} \right)^{-1} \frac{e^{-(z^2+c^2t^2)(1/2\sigma_p^2 + 1/2\sigma_{\overline{p}}^2) - zct(1/\sigma_{\overline{p}}^2 - 1/\sigma_p^2)}}{(1 + z^2/\beta^{*2})}. \tag{10}$$

These models are used to describe the observed distribution of z and t for $p\overline{p}$ collisions recorded at CDF.

3 Primary Vertex Fitting

In the CDF experiment, the position, z, of a primary interaction can be measured by clustering the longitudinal impact parameters of tracks found in the central tracking chamber. The time, t, at which the interaction occurred can be measured with a precision of < 100 ps using CDF's time-of-flight detector. This section describes some of the details of the algorithm used to measure z and t using these subdetectors.

Primary vertices are found by first selecting sets of tracks with longitudinal impact parameters that cluster in z. The tracks used in the analysis were found using only the Central Outer Tracker (COT) and do not make use any information from the silicon detectors in CDF. They are required to pass within approximately 2.5 mm of the average beam position which is determined on a run-by-run basis using tracks from the COT[1].

For each track, the arc length measured from the origin to the point of closest approach to the average beam position in the r-ϕ plane is calculated. The longitudinal

[1] Short runs are combined so that there is sufficient statistics for a precise beam position measurement.

impact parameter, z_0, and its uncertainty, σ_{z_0}, are then calculated at this arc length. Primary vertices are identified as local maxima in the function:

$$g(z) = \sum_i \frac{1}{\sqrt{2\pi}\sigma_{z_0}^{(i)}} e^{-((z-z_0^{(i)})/\sigma_{z_0}^{(i)})^2/2}. \tag{11}$$

The Time-of-Flight (TOF) detector is used to measure the time at which clusters of tracks originating from a common region in z were produced. The algorithm used is essentially the one used to analyze data from the prototype TOF system[1]. For each track, the distance, d, from the production point to the TOF scintillator is calculated and the time-of-flight is given by

$$T_j = \frac{d}{c}\sqrt{1 + \frac{m_j^2}{p^2}} \tag{12}$$

where p is the track's momentum and m_j $(j = \pi, K, p)$ is the hypothesized mass of the particle. A likelihood function is constructed from the measured times $t^{(i)}$, with uncertainty $\sigma_t^{(i)}$, at which tracks hit the scintillator.

$$\mathcal{L}(t) = \prod_i \left(\sum_j f_j \frac{1}{\sqrt{2\pi}\sigma_t^{(i)}} e^{-((t-t^{(i)}+T_j)/\sigma_t^{(i)})^2/2} \right). \tag{13}$$

where f_j are the pion, kaon and proton particle fractions in the event. These are assumed to be $f_\pi = 0.8$ and $f_K = f_p = 0.1$. The production time of the event is then the value of t that maximizes $\log \mathcal{L}(t)$.

4 Analysis of Tevatron Stores 1137-1253

Table 1 lists the initial luminosity, initial proton and initial anti-proton currents for Tevatron stores 1137-1253. An unbinned likelihood fit to the distribution given by equation 10 was used to determine the parameters σ_p and $\sigma_{\overline{p}}$ for sets of events recorded throughout these stores. In addition, the parameter β^* was determined in the fit. Section 4.5 describes a cross check of the analysis in which the fitted values are compared with the nominal value of $\beta^* = 35$ cm.

4.1 Correlation with initial luminosity

Figure 1 shows the normalized initial luminosity, defined as the initial luminosity divided by the initial proton and anti-proton currents, plotted versus the initial value of σ_z. No significant correlation between these quantities is observed. However, there are other quantities, for example, the transverse emittance, that also affect significantly the initial luminosity. Until these additional effects are be removed it the comparison, it is perhaps not surprising that the correlation is not statistically significant.

store	$\mathcal{L}$ (cm^{-2}s^{-1})	$i(p)$	$i(\overline{p})$	ℓ (cm^{-2}s^{-1})
1126	0.942×10^{31}	4759×10^{9}	278×10^{9}	0.712×10^{7}
1137	0.922	5223	269	0.656
1140	1.227	5340	305	0.753
1142	1.218	6113	291	0.685
1144	1.376	6955	298	0.664
1150	1.052	6581	261	0.613
1152	1.085	6966	265	0.588
1154	0.829	5502	295	0.511
1172	1.372	6543	293	0.716
1174	1.468	6692	336	0.653
1176	1.224	5815	329	0.640
1178	1.322	6213	283	0.752
1240	1.584	6432	348	0.707
1242	1.261	5904	286	0.747
1243	1.327	6185	297	0.722
1253	1.716	6133	374	0.749

Table 1: Initial luminosities and beam currents for the stores analyzed. The initial values are measured at the beginning of the first CDF run recorded in each store. The last column is the normalized initial luminosity, defined as the initial luminosity divided by the beam currents.

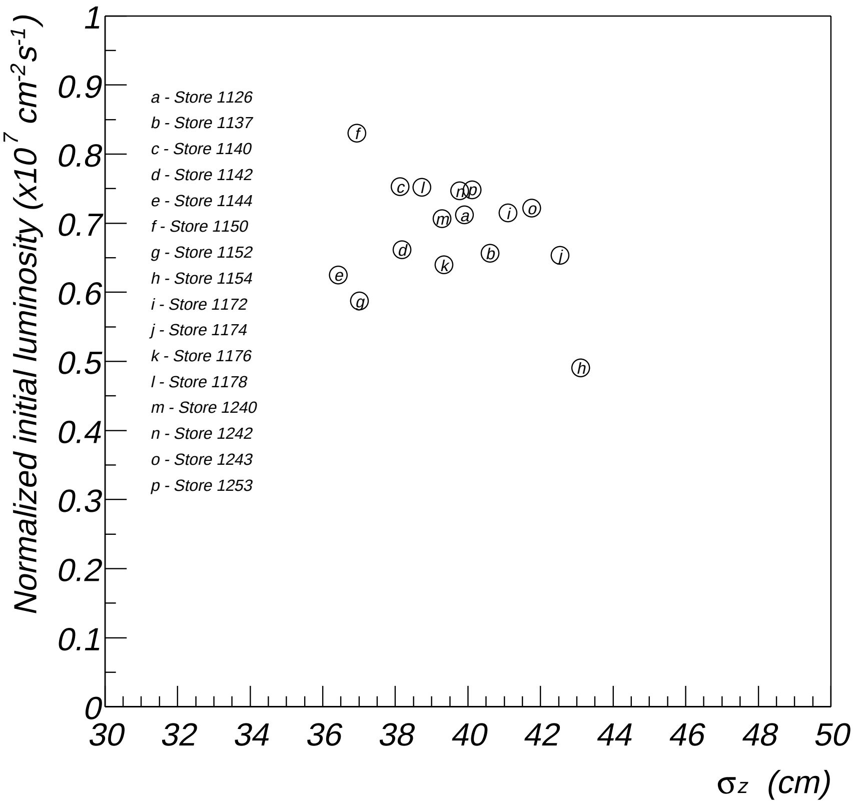

Figure 1: Normalized initial luminosity (initial luminosity divided by the beam currents) plotted against the $\sigma_z = \frac{1}{2}\sqrt{\sigma_p^2 + \sigma_{\bar{p}}^2}$, measured using data collected during the first 10^8 turns in each store.

4.2 Evolution of bunch lengths

The evolution of the bunch lengths can be examined throughout a store by fitting the distribution of z and t for primary interactions in chronologically ordered sets of events. Figure 2 shows the evolution of the proton and anti-proton bunch lengths measured at various times since the beginning of each store. Each point is obtained from a fit to 10^4 events. Bunch lengths are also measured from the waveform of the image current measured across a resistor that jumpers two electrically isolated sections of the Tevatron beam pipe. From these waveforms the RMS bunch lengths are determined and are logged as the ACnet parameter T:SBDMS. For comparison with the proton bunch lengths measured at CDF, the value of this parameter throughout the store is also plotted in figure 2. Although there is a clear correlation between the measured proton bunch length and T:SBDMS, there are also systematic differences.

4.3 Lengths of individual bunches

It is also possible to measure the lengths of the each of the 36 proton and anti-proton bunches separately. To obtain sufficient statistics, data recorded throughout the entire store was used and hence, any increase in the bunch length over time will be folded into these estimates.

Figure 3 shows the comparison between the average proton bunch lengths measured at CDF and as obtained from the SBD readings. The comparison is made for each of the 36 bunches for stores 1240, 1243 and 1253. It is possible that the worse correlation seen for store 1240 is due to the non-uniform weighting in time of the bunch lengths measured at CDF, which can be seen in figure 2. In general, it is apparent that the bunch lengths measured by the SPD signal are longer than those measured at CDF by approximately 20%.

4.4 z-distribution of luminous region

Although timing information allows one to measure the lengths of colliding bunches, there is still considerable interest in the shape of the z-distribution of $p\overline{p}$ interactions. Tails or asymmetries in this distribution could provide significant information about the interaction region. Figure 4 shows an example of the distribution of the z-coordinate of $p\overline{p}$ interactions for events recorded during the initial part of store 1126. The distribution was fit to the functional form of the integrand in equation 7. In general, this function describes the distribution well, but there are deviations observed at large z and the χ^2 per degree of freedom indicates that systematic differences are significant. However, without additional detailed studies of the trigger acceptance, it is difficult to attribute these effects entirely to the shape of the luminous region.

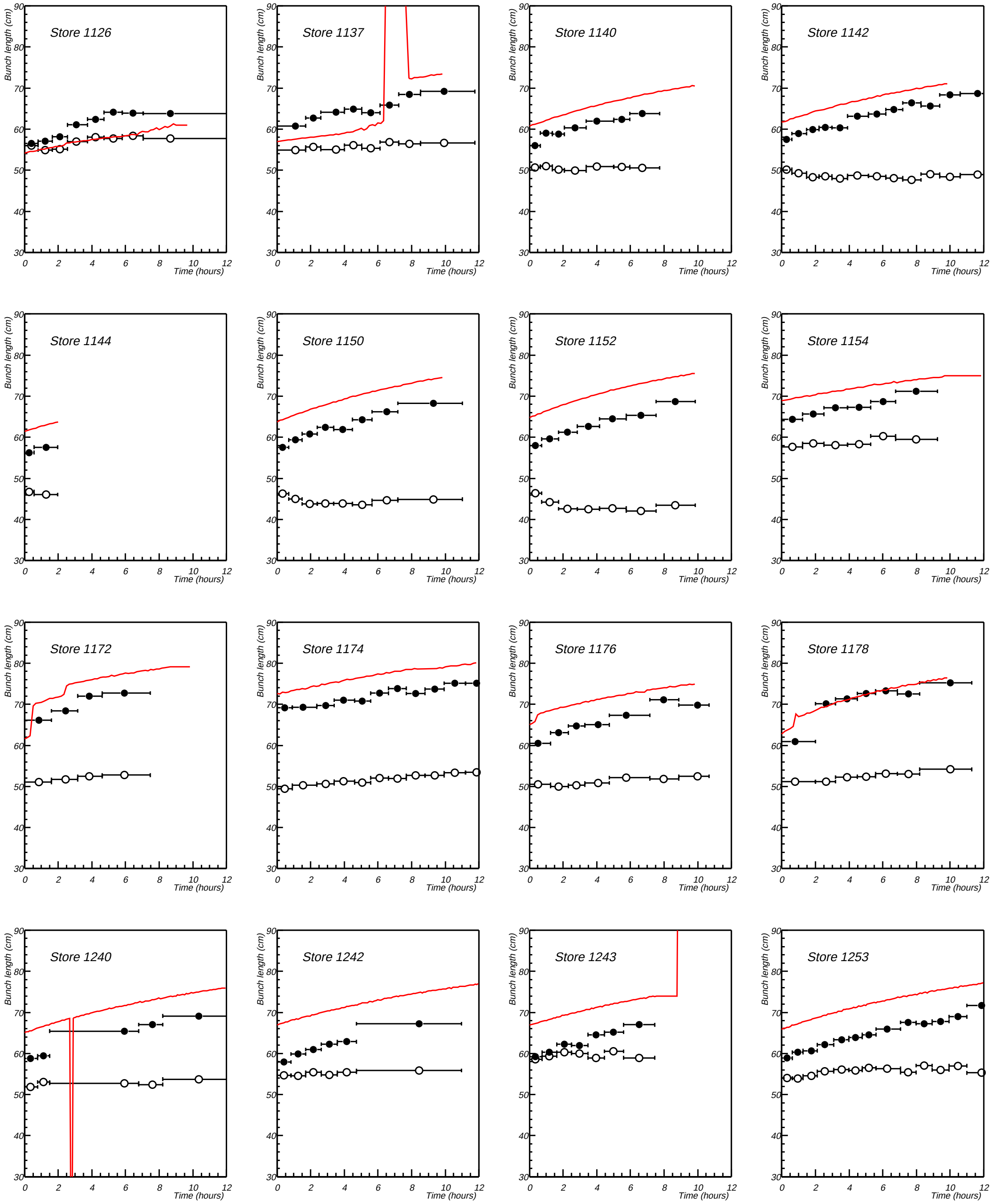

Figure 2: Evolution of proton bunch lengths. The solid and open symbols are the proton and anti-proton bunches, respectively. The solid line is the value of the ACnet parameter T:SBDMS.

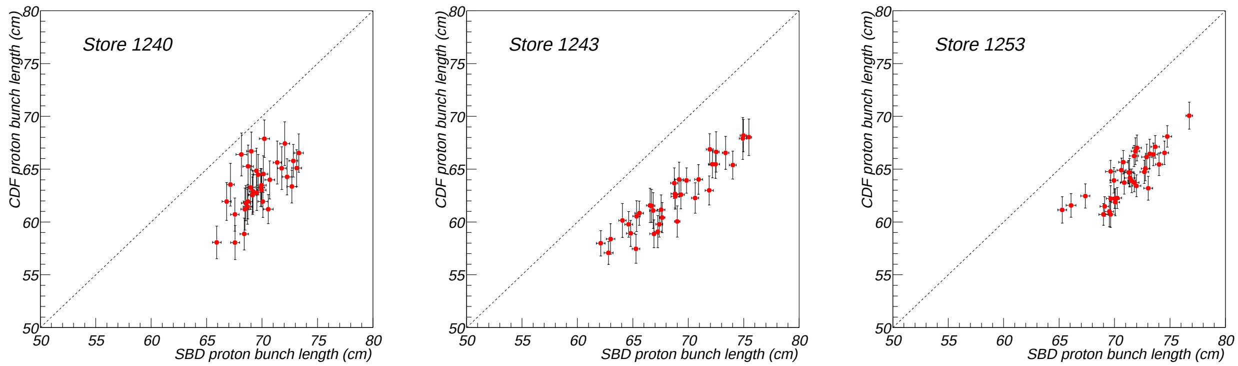

Figure 3: A comparison of the lengths of individual proton bunches in stores 1240, 1243 and 1253, measured at CDF and with the the SBD signal.

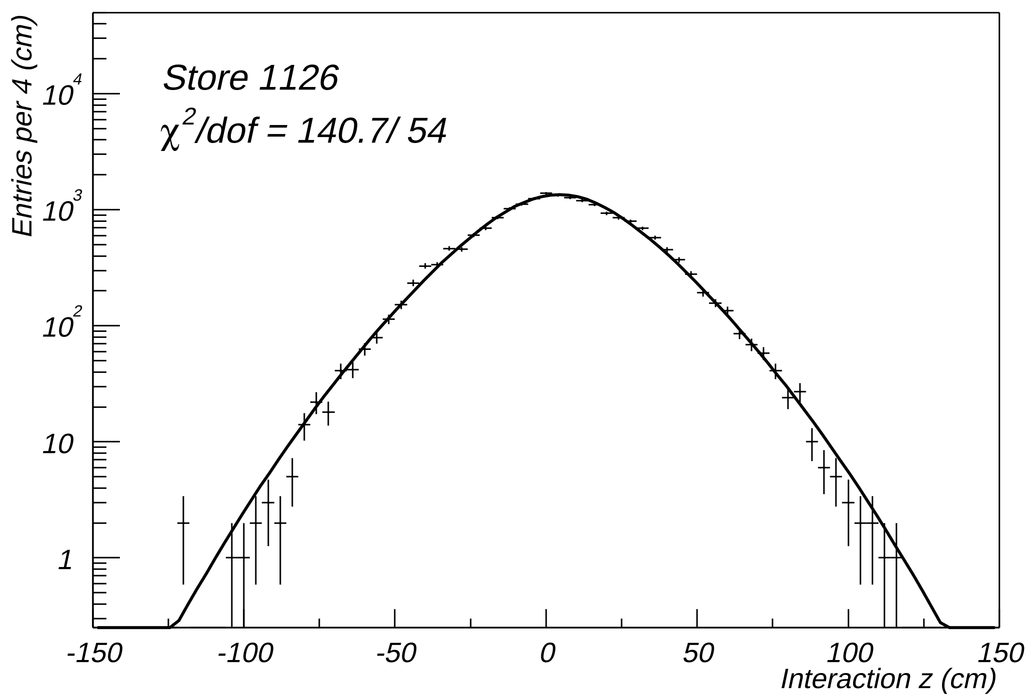

Figure 4: Distribution of the z-coodinates of $p\overline{p}$ interactions recorded at the start of store 1126.

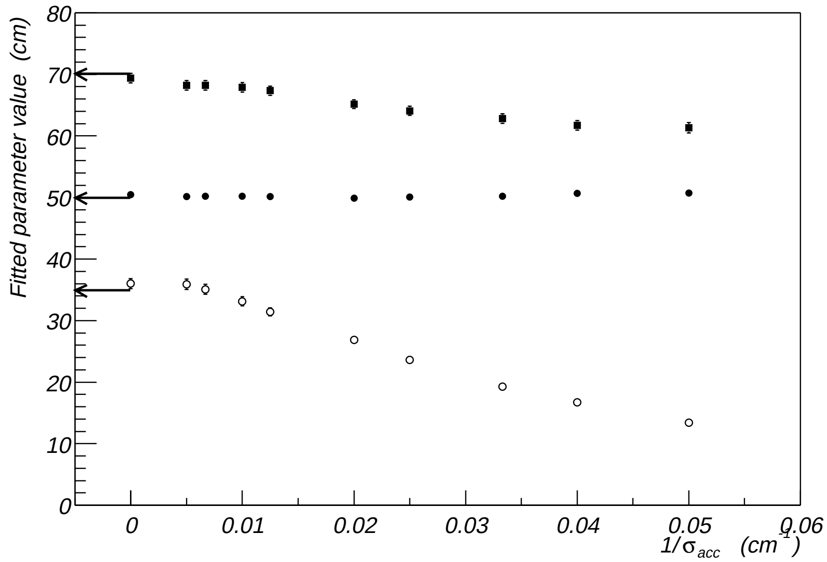

Figure 5: Fitted parameters as a function of $1/\sigma_{\rm acc}$, indicating the level of bias introduced by a non-uniform acceptance of the trigger in z. The arrows indicate the generated values $\beta^* = 35$ cm, $\sigma_{\overline{p}} = 50$ cm and $\sigma_p = 70$ cm.

4.5 Cross checks

In addition to the bunch lengths, the parameter β^* was also determined from the fit. The fitted values of β^* were compared with the nominal value of 35 cm, providing a cross check of the analysis. This check is important since we have not corrected for any non-uniformities of the trigger acceptance or efficiency in z and t. These might be expected to reduce the number of events reconstructed at large values of $|z|$ and would result in a fitted value of β^* that is systematically low.

To study this effect, a toy Monte Carlo study was performed in which the distribution of z and t was generated according to equation 10 but with an additional z-dependent efficiency, parameterized by the function

$$\epsilon(z) = \begin{cases} e^{-z^2/2\sigma_{\rm acc}^2} & |z| < 150 \text{ cm} \\ 0 & |z| > 150 \text{ cm.} \end{cases} \tag{14}$$

The resulting distribution of z and t was then fitted, assuming a flat acceptance function. Figure 5 shows the fitted values of σ_p, $\sigma_{\overline{p}}$ and β^* as a function of $1/\sigma_{\rm acc}$. The fitted values of σ_p and $\sigma_{\overline{p}}$ are seen to have a weaker dependence on any non-uniformity in the trigger acceptance in z than does the value of β^*. In principle, if this model for trigger acceptance were correct, the fitted value of β^* could be used to determine the bias in the measured proton and anti-proton bunch lengths.

The distribution of the fitted values from all sections of the data analyzed is shown in figure 6. Although the mean is close to the nominal value of $\beta^* = 35$ cm,

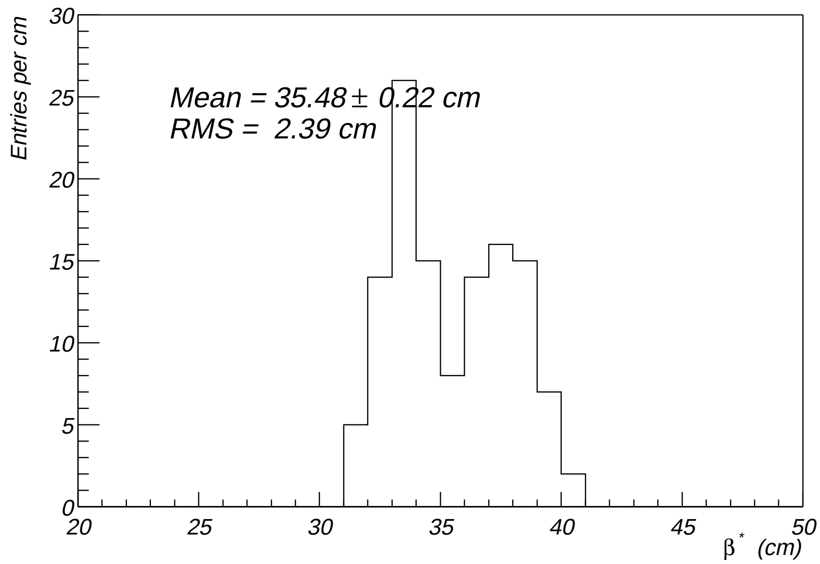

Figure 6: Distribution of the fitted values for β^* in the stores analyzed.

the distribution is clearly bi-modal. This might be due to changes in the CDF trigger table and could result in systematic biases in the measured bunch lengths that depend on which sets of data were analyzed. We intend to investigate this hypothesis but for now rely on this toy Monte Carlo study described above to argue that the bias in the measured bunch lengths might be of the order of a few centimeters.

An additional cross-check was performed in which the quantity $\frac{1}{2}\sqrt{\sigma_p^2 + \sigma_{\overline{p}}^2}$ was compared with the value of σ_z, obtained by fitting the only the observed distribution of the z-coordinate of $p\overline{p}$ interactions to the function

$$F_{p\overline{p}}(z) = N \frac{e^{-z^2/2\sigma_z^2}}{1 + z^2/\beta^{*2}}. \qquad (15)$$

Figure 7 shows this comparison performed on the individual bunches in store 1174.

5 Comparison with Beam Loss Measurements

The CDF Beam Shower Counters (BSC) consist of rings of scintillator that are divided into four quadrants and surround the beam pipe. One such counter arrangement is located on each side of the detector about 20 ns away from the interaction point. Figures 8 and 9 show the location and construction of the BSC's at CDF. The LOSTP signal is defined as the coincidence between any of the west counters, delayed by about 41 ns, with any of the east counters.

The time at which a LOSTP signal arrives with respect to the bunch 0 marker signal (B0) determines with which bunch the lost proton is associated and also the

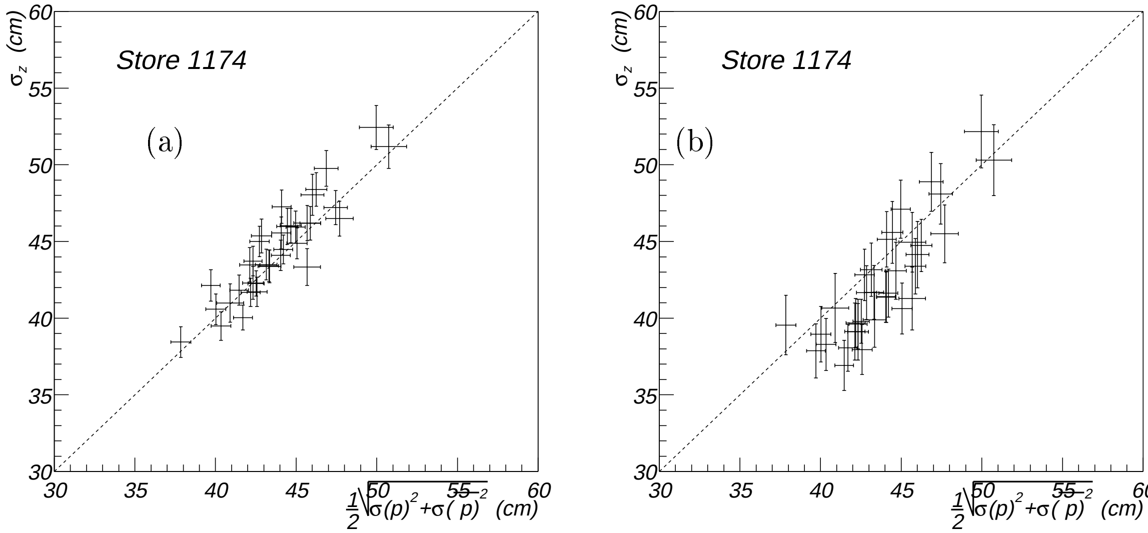

Figure 7: A comparison of the width of the luminous region, σ_z, obtained by simply fitting the z-distribution of primary interactions and calculated from the fitted proton and anti-proton bunch lengths. In (a), the value of β^* was fixed at 35 cm in the fit to the z-distribution while in (b) it was allowed to vary.

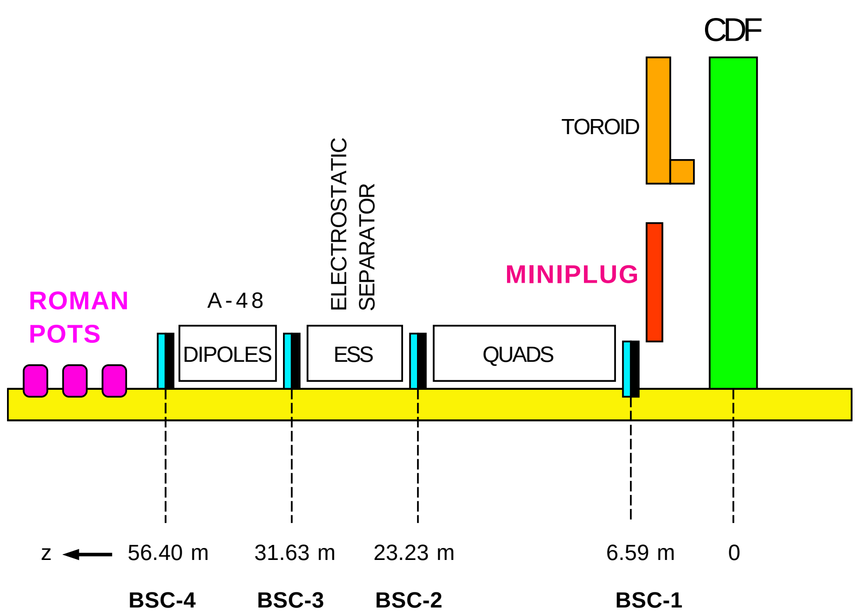

Figure 8: Locations of the beam shower counters at CDF. The BSC-2 detectors are used to define the LOSTP signal.

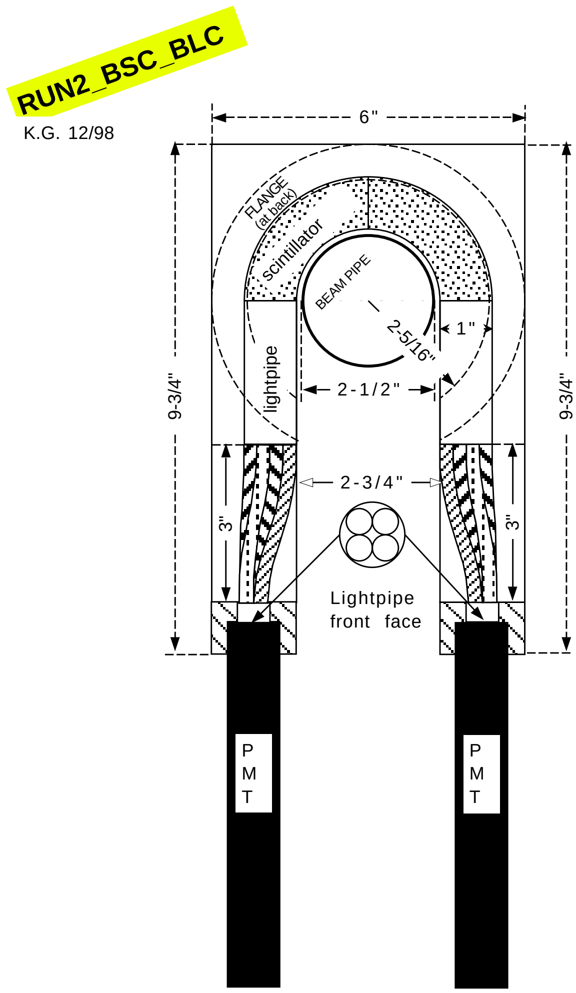

Figure 9: Construction of the BSC-2 beam shower counters at CDF.

position along the length of the bunch from which it was lost. The LOSTP signal was used as input to a discriminator and the resulting output signal was clipped to give a pulse that was approximately Gaussian with a width of 0.94 ns. This clipped signal was used to trigger a digital oscilloscope. For each trigger, the time between the LOSTP and the next B0 marker signal was measured. The average waveform from many triggers represents the time structure of the losses, convoluted with a Gaussian of this width.

Figure 10 shows the measured distribution of LOSTP times recorded during store 1126. Figure 11 shows the widths of the individual bunches in store 1126. These were determined by fitting Gaussian functions to the parts of the average waveform corresponding to a particular bunches with the resolution of 0.94 ns subtracted in quadrature from the resulting widths.

It is clear that the lengths of the bunches measured in this way using the LOSTP signal are significantly larger than the widths measured using reconstructed $p\overline{p}$ interactions. One explanation that has been proposed is that the LOSTP signal is sensitive to the length of the part of the bunch that is being lost, whereas the length measured using $p\overline{p}$ events is sensitive to the length of the part of the bunch that is not.

6 Conclusion

A primary motivation for this analysis was to establish whether the lengths of the proton and anti-proton bunches measured at CDF could be interpreted in a meaningful way. Although no corrections for systematic effects have been applied to the results, the size of these effects is believed to be of order a few centimeters. This is significantly smaller than the variation observed in the bunch lengths from store to store, or even among the individual bunches themselves. The correlation of the measured proton bunch lengths with the SBDMS measurement and with the SBD measurements of the lengths of each bunch support the suggestion that this method could provide an accurate measure of the bunch lengths at CDF.

In its current state, this analysis does not provide evidence for a strong correlation between the instantaneous luminosity and the proton or anti-proton bunch lengths. Although one expects such a correlation to be present, it is recognized that several additional effects may be present that have not been accounted for. Further studies are required to determine the extent to which the bunch lengths are currently influencing the instantaneous luminosity at CDF.

For most stores, the bunch lengths measured using the technique described here are significantly smaller than those obtained with the SBD signals or by monitoring beam loses with the beam shower counters. Nevertheless, they are significantly longer than the early estimate of 37 cm for Run-II[2].

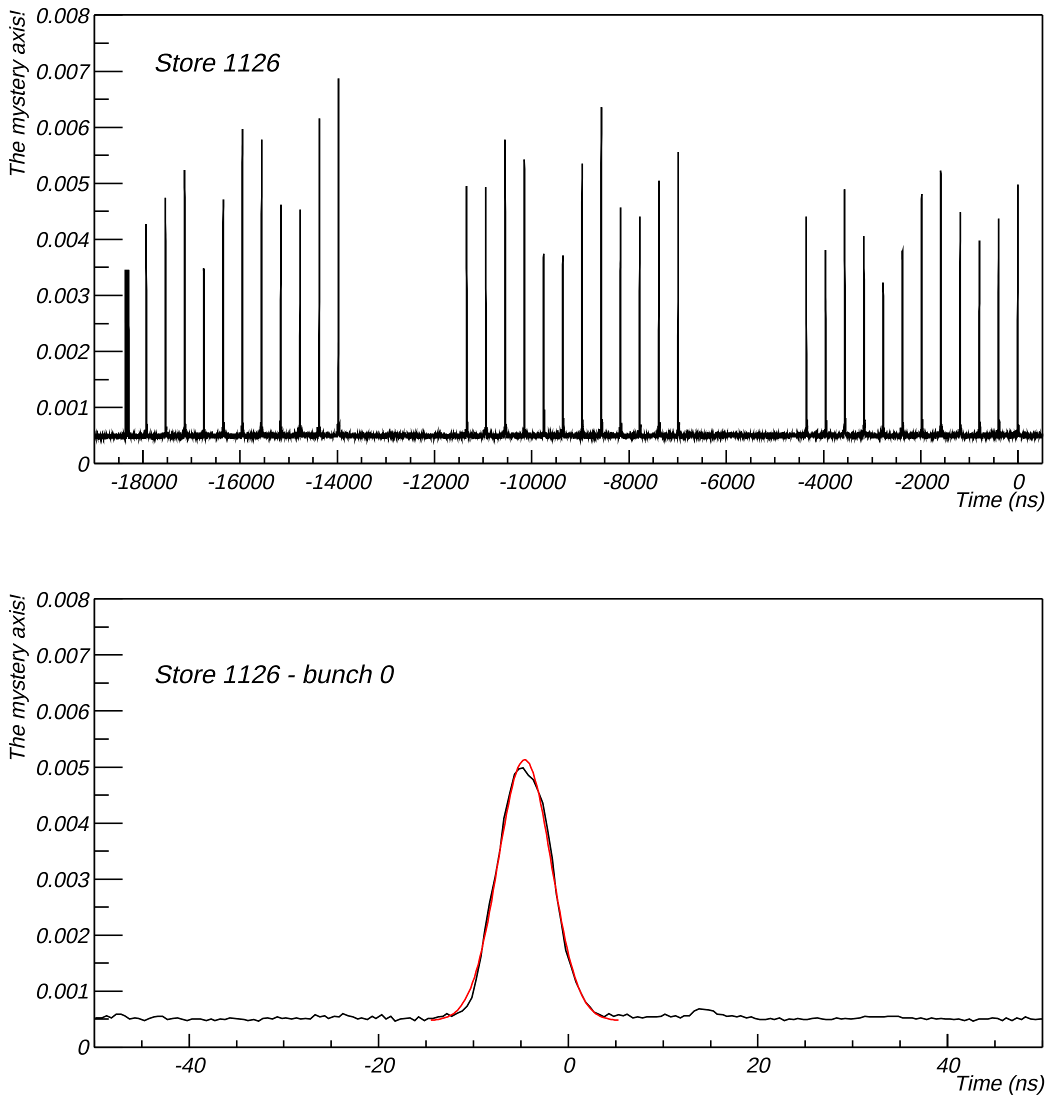

Figure 10: The upper plot shows the distribution of LOSTP signal times with respect to the B0 marker signal. The lower plot shows only the region of the upper distribution corresponding to the first bunch. The Gaussian function fitted to the time distribution for this bunch is superimposed.

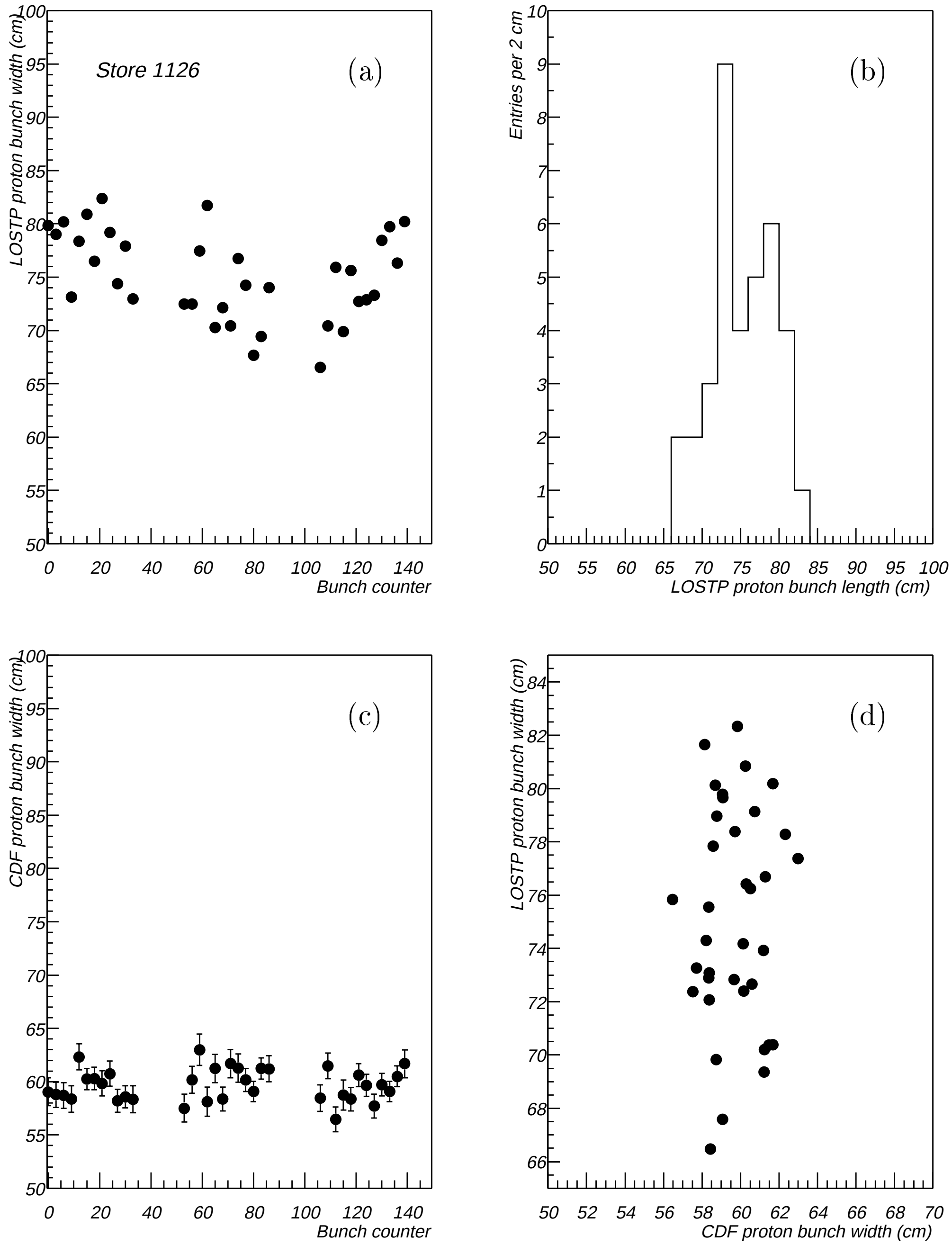

Figure 11: (a) Individual proton bunch lengths measured from the widths of the LOSTP signal times. (b) Histogram of individual bunch lengths measured with LOSTP signal times. (c) Individual bunch lengths measured with collisions at CDF. (d) A comparison of the bunch lengths measured with the two techniques.

References

[1] CDF Note 3662, “Time-of-Flight Test System Performance”, J.G. Heinrich, *et.al.*, April 22, 1996.

[2] “Luminosity Distribution During Collider Run II”, Mike Martens and Peter Bagley, January 21, 2000.