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Effects of Inflow on NuMI Groundwater Concentrations

J. Donald Cossairt

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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Introduction

Recent discussions of the NuMI groundwater problem have been concerned with the effect of inflow of water into the tunnel on the overall groundwater concentration. The purpose of this note is to document calculations of these effects using simple mathematical models. These results can, then, be compared with the results obtained using more elaborate methods such as computer modeling techniques.

At Fermilab, a concentration model has been developed to address groundwater activation concerns[1]. While this model has evolved to some degree over time, the main features have remained stable. A principal result of this work, and latter modifications[2], is the derivation of the following expression for the concentration, C_i, of radionuclide, i, in the medium as a function of irradiation time, t,

$$C_i = \frac{N_p < S > K_i L_i}{1.17 \times 10^6 \rho w_i}[1 - \exp(-\lambda_i t)] \text{ (pCi cm}^{-3}\text{)}, \quad (1)$$

where N_p is the number of protons incident annually, $<S>$ is the star density (stars cm^{-3}) averaged over some volume of interest, K_i is the production yield of radionuclide i (atoms star^{-1}), L_i is the fraction of radionuclide i that is leachable by water, ρ is the density (g cm^{-3}) of the medium, w_i, is the ratio of the weight of the water to the weight of soil that corresponds to the leaching fraction for the radionuclide i, and λ_i is the activity constant (inverse mean life) for radionuclide i. The numerical constant in the denominator includes the appropriate unit conversion factors.

In this equation, the product ρw_i in the demoninator of the right-hand side merits further examination. Originally, the definition of the quantity w_i was developed for unconsolidated media such as soil. For these materials, while not completely free of arbitrariness, the selection of certain specific values of w_i is supported by several measurements.

1 A. J. Malensek, A. A. Wehmann, A. J. Elwyn, K. J. Moss, and P. M. Kesich, "Groundwater Migration of Radionuclides at Fermilab", Fermilab Report TM-1851, August 1993.
2 J. D. Cossairt, A. J. Elwyn, P. Kesich, A. Malensek, N. Mokhov, and A. Wehmann, "The Concentration Model Revisited", Fermilab Environmental Protection Note No. 17, June 24, 1999.

For consolidated materials such as the dolomite in which portions of the NuMI Decay Tunnel is sited, one should consider the porosity, *p*, in terms of densities and volumes, *V*, of rock and water,

$$p = \frac{V_{water}}{V_{rock}} = \frac{w_{water}}{\rho_{water}} \frac{\rho_{rock}}{w_{rock}} = \rho_{rock} \frac{w_{water}}{w_{rock}} = \rho_{rock} w_i \, , \qquad (2)$$

since $\rho_{water} = 1$ g cm^{-3}. Thus, it may be beneficial to make this substitution into Eq, (1) since the porosity is, in principle, a measurable quantity.

The STS borings of 1997[3] have improved our knowledge of the values of the density, ρ, and the porosity, *p*, for the various geologic strata in which the NuMI tunnel will be placed. These values and their averages and standard deviations are provided in Table 1.

Table 1 Values of Density and Porosity for Major Strata Encountered by the NuMI Decay Tunnel

Strata	***<ρ> (g cm^{-3})***	***<p> (%)***
Silurian Dolomite	2.78	19
Scales Formation (upper dolomite)	2.65	16.6
Scales Formation (lower-shaley)	2.84	22
Average Values	**2.757 ± 0.097**	**19.2 ± 2.7**

The Fluor-Daniel report of May 30, 1997[4] gave an average estimate of the minimal rate of inflow for the NuMI tunnel of 50 gal min^{-1} mile^{-1}. This "minimal" value, while subject to considerable uncertainty, is taken as a benchmark value in the remainder of this discussion. Performing a conversion to units that are more useful for the present calculation, one observes that an inflow, *I*, of one gal min^{-1} mile^{-1} (gpm/mile) represents the following inflow rate:

$$I = \frac{1\,\text{gal}}{\text{minute mile}} \times \frac{3875\,\text{cm}^3}{\text{gal}} \times \frac{1\,\text{min}}{60\,\text{sec}} \times \frac{1\,\text{mile}}{5280\,\text{ft}} \times \frac{1\,\text{ft}}{30.48\,\text{cm}} = 4.013\times10^{-4} \frac{\text{cm}^3\,\text{sec}^{-1}}{\text{cm of tunnel}} \, . \qquad (3)$$

Thus, the rate at which water enters the tunnel per cm of length of tunnel is,

$$\frac{dV}{dt} = 4.013\times10^{-4}\, I\, (\text{cm}^3\ \text{s}^{-1}) = 1.264\times10^{4}\, I\, (\text{cm}^3\ \text{yr}^{-1}) \, , \qquad (4)$$

where the engineering units of *I* in gpm/mile has been explicitly retained.

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[3] STS Consultants, Ltd.,"Hydrogeological Evaluation Report for Neutrino Main Injector", April 2, 1997.
[4] Fluor-Daniel and Harza Engineering, "Review of the STS Hydrogeological Evaluation Report for the NuMI Project dated April 2, 1997 and Initial Discussion of Expected Inflows into the NuMI Tunnel", May 30, 1997.

The NuMI Decay Tunnel has an approximate radius $r_o \approx 3.3$ meters = 330 cm. Given the tunnel size along with the formula for the surface area of a cylinder of one cm length, we can determine the average velocity, v_o, of the inflowing water at the tunnel boundary to be,

$$v_o = \frac{dV/dt}{2\pi r}\left(\frac{\mathrm{cm}^3\mathrm{s}^{-1}}{\mathrm{cm}^2}\right) = \frac{4.013\times10^{-4} I}{2\pi r} = 6.387\times10^{-5}\,\frac{I}{r_o}\,(\mathrm{cm\,s}^{-1}). \tag{5}$$

A particular result is an inflowing velocity $v = 9.677 \times 10^{-6}$ cm sec^{-1} for the "minimal" value of $I = 50$ gpm/mile and the approximate value of $r_o = 330$ cm.

At larger radii, assuming the inflow into the tunnel dominates over other water movement mechanisms over a reasonable region of concern, for $r > r_o$,

$$v(r) = v_o r_o / r. \tag{6}$$

This is a vector quantity that is naturally directed <u>inward</u>. The inflow is implicity assumed to be uniform over azimuthal angle around the cylindrical tunnel. Two methods will now be employed to calculate the effect of inflow on concentrations.

Method 1: Mixing Within the Averaging Volume of TM-2009[5]

The authors of TM 2009 averaged the production of radionuclides over the region $r_o < r < r_o + 150$ cm. This region, per centimeter of tunnel length, contains a total rock volume, V_{rock} given by,

$$V_{rock} = \pi(r_o + 150)^2 - \pi r_o^2 = 3.817 \times 10^5\ \mathrm{cm}^3. \tag{7}$$

Using the average value of porosity from Table 1, this rock contains a volume of water, V_{water} given by,

$$V_{water} = pV_{rock} = 7.329 \times 10^4\ \mathrm{cm}^3. \tag{8}$$

Taking the inflow as calculated above, the average mean residence time of water in this particular space, τ_R, is,

$$\tau_R = \frac{V_{water}}{dV/dt} = \frac{7.329\times10^4\,\mathrm{cm}^3}{4.013\times10^{-4}\, I\ \mathrm{cm}^3\,\mathrm{sec}^{-1}} = \frac{1.826\times10^8\ \mathrm{sec}}{I}. \tag{9}$$

[5] A. Wehmann, W. Smart, S. Menary, J. Hylen, and S. Childress, "Groundwater Protection for the NuMI Project", Fermilab Report TM-2009 and NuMI Note B-279, October 10, 1997.

The additional time for the water to flow down the tunnel to the sump pit is, by comparison, very small and thus is ignored. For the "minimal" value of I = 50 gpm/mile, τ_R = 3.652 x 10^6 = 0.116 yr. From this it is straightforward to calculate a "rinsing" *removal* constant,

$$\lambda_R = 1/\tau_R = 5.476 \times 10^{-9}\, I\ (\mathrm{sec}^{-1}) = 0.172\, I\ (\mathrm{yr}^{-1}). \tag{10}$$

For the "minimal" value of I = 50 gpm/mile, λ_R = 8.62 yr^{-1}.

If one merges all the constants associated with production in Eq. (1) as a factor denoted by Γ_i and denotes the average number of *removable*[6] atoms cm^{-3} of rock of radionuclide i as a function of time t as $n_i(t)$, one has the differential equation,

$$\frac{dn_i(t)}{dt} = -\lambda_{p,i} n_i(t) - \lambda_R n_i(t) + \Gamma_i\ , \tag{11}$$

where the $\lambda_{p,i}$ denotes the *physical* decay constant for radionuclide i. For the two radionuclides of principal concern in groundwater activation analysis,

$$\lambda_p(^3\mathrm{H}) = 0.0562\ \mathrm{yr}^{-1} \text{ and}$$

$$\lambda_p(^{22}\mathrm{Na}) = 0.266\ \mathrm{yr}^{-1}.$$

If one starts at $t = 0$ with no activity having been produced, the solution is,

$$n_i(t) = \frac{\Gamma_i}{\lambda_{pi} + \lambda_R}\left[1 - \exp\left\{-(\lambda_{pi} + \lambda_R)t\right\}\right]. \tag{12}$$

One actually desires activity concentration, $a_i(t)$, rather than atomic concentration $n_i(t)$, so

$$a_i(t) = \lambda_{pi} n_i(t). \tag{13}$$

Thus,

$$a_i(t) = \frac{\lambda_{pi}\Gamma_i}{\lambda_{pi} + \lambda_R}\left[1 - \exp\left\{-(\lambda_{pi} + \lambda_R)t\right\}\right]. \tag{14}$$

The result is that one can calculate the ratio of the concentration including inflow to the static concentration after a period of operation, t,

$$R_{\mathrm{Meth\,1}}(t) = \frac{\lambda_{pi}}{\lambda_{pi} + \lambda_R}\,\frac{\left[1 - \exp\left\{-(\lambda_{pi} + \lambda_R)t\right\}\right]}{\left[1 - \exp\left\{-\lambda_{pi} t\right\}\right]}. \tag{15}$$

It is clear that as λ_R -> 0, $a_i(t)$ approaches the value realized for static conditions, as it should.

[6] Here the term *removable* atoms of a particular radionuclide takes into account the fraction that might be leachable or, for ^{3}H, the total which can flow through the rock and is not trapped in closed pores.

Method 2: Use of the Continuity Equation[7]

At a given value of *r*, one can write down the following continuity equation which can be used to obtain the atomic density (cm^{-3}), $n_i(r,t)$, as a function of both time and position, *r*, in the rock[8],

$$\frac{\partial n_i(r,t)}{\partial t} + v(r)\frac{\partial n_i(r,t)}{\partial r} + \lambda_{pi} n_i(r,t) = P_i S(r,t). \tag{16}$$

In this equation, the left hand side includes the effects of transport (second term, here characterized by the inflow velocity $v(r)$) and radioactive decay (the third term). The transport term of Eq. (16) is the only nonzero part of the scalar product, $\vec{v}(\vec{r}) \cdot \bullet \nabla n_i(\vec{r},t)$ that appears in generalized continuity equations of this type. In the cylindrical coordinate system (r, z, ϕ) used in the present discussion,

$$\nabla n_i(r,t) = \hat{r}\frac{\partial}{\partial r} n_i(r,t) \tag{17}$$

since, in the present approximation, $\dfrac{\partial n_i(r,t)}{\partial \phi} = \dfrac{\partial n_i(r,t)}{\partial z} = 0.$

$P_i\, S(r,t)$ is a production term closely related to the quantity denoted by Γ_i that was used in Method 1. For the NuMI decay tunnel, as reported elsewhere[9], $S(r,t)$, the star density at any given location and time, is well-described by,

$$S(r,t) = S_{max}\exp\{-\mu(r - r_o)\} \text{ with } \mu = 0.0307 \text{ cm}^{-1}. \tag{18}$$

For the NuMI tunnel, *r* is rather large and $v(r)$ varies rather slowly with *r* compared with the radial exponential attenuation of the production of the radionuclides in the rock. In view of this observation, $v(r)$ will be taken to be a constant, *v*, in developing an approximate solution to Eq. (16). The solution of this equation, again assuming $n_i(r,0) = 0$ is,

$$\begin{aligned} n_i(r,t) &= P_i \int_0^t dt' S(r - vt', t') \exp[-\lambda_{pi} t'] = P_i S_{\max} \int_o^t dt' \exp[-\mu(r - r_o - vt')] \exp[-\lambda_{pi} t'] \\ &= P_i S_{\max} \exp[-\mu(r - r_o)] \int_o^t dt' \exp[(\mu v - \lambda_{pi}) t'], \end{aligned} \tag{19}$$

[7] This equation and its solution is due to N. Mokhov (1997). It ignores effects of diffusion.
[8] An example of a similar continuity equation is found in K. R. Symon, *Mechanics*, 2nd Edition (Addison Wesley, Palo Alto, 1960).
[9] A. Wehmann, et al., *op cit*.

$$n_i(r,t) = P_i S_{\max} \exp[-\mu(r-r_o)] \left[\frac{\exp\{(\mu v - \lambda_{pi})t'\}}{\mu v - \lambda_{pi}} \right]_o^t$$
$$= \frac{P_i S_{\max} \exp[-\mu(r-r_o)]}{\mu v - \lambda_{pi}} \left[\exp\{(\mu v - \lambda_{pi})t\} - 1 \right]. \quad (20)$$

Converting to specific activity rather than the atomic concentration,

$$a_i(r,t) = \frac{\lambda_{pi} P_i S_{\max} \exp[-\mu(r-r_o)]}{\mu v - \lambda_{pi}} \left[\exp\{(\mu v - \lambda_{pi})t\} - 1 \right]. \quad (21)$$

In this situation, $v < 0$ as it is *inflow* rather than *outflow*. Removing the awkward negative sign so that we can simply use the magnitude of v, we have,

$$a_i(r,t) = \frac{\lambda_{pi} P_i S_{\max} \exp[-\mu(r-r_o)]}{\mu v + \lambda_{pi}} \left[1 - \exp\{-(\mu v + \lambda_{pi})t\} \right]. \quad (22)$$

As $v \to 0$, one can see that, again with this method, the static values are approached. The functional form is identical to the result of Method 1 expressed in Eq. (14).

Looking at the "minimal" inflow values, at $r = r_o$, $v = v(r_o) = 9.677 \times 10^{-6}$ cm s^{-1} so that $\mu v = 2.97 \times 10^{-7}$ s^{-1} = 9.36 yr^{-1}. This is very nearly equal to the value obtained in Method 1 for $\lambda_R = 8.62$ yr^{-1}.

For other values of the inflow parameter, I, for $r > r_o$,

$$\mu v(r) = \mu \left[\frac{6.387 \times 10^{-5} I}{r_o} \frac{r_o}{r} \right] = \frac{6.387 \times 10^{-5} I \mu}{r} \text{sec}^{-1}. \quad (23)$$

For the expected value of $\mu = 0.0307$ cm^{-1},

$$\mu v(r) = \frac{1.96 \times 10^{-6} I}{r} \text{sec}^{-1} = 61.71 \frac{I}{r} \text{yr}^{-1} \text{ for } r > 330 \text{ cm}. \quad (24)$$

Since previous groundwater calculations[10] have taken the average star density to be approximately 0.19 S_{max}, it is a sensible approximation to simply take v to be a constant at the value of r that corresponds to 0.19 S_{max}, determined by solving

$$0.19 = \exp[-\mu(r' - r_o)] \,. \quad (25)$$

With $\mu = 0.0307$ cm^{-1}, $r' = 384$ cm.

[10] A. Wehmann, et al., *op cit*.

One can thus calculate the ratio of the concentration including inflow to the static concentration after a period of time of operation, *t*, due to this method.

$$R_{\text{Meth 2}}(t) = \frac{\lambda_{pi}}{\lambda_{pi} + \mu v} \frac{\left[1 - \exp\left\{-(\lambda_{pi} + \mu v)t\right\}\right]}{\left[1 - \exp\left\{-\lambda_{pi} t\right\}\right]}. \tag{26}$$

Results and Discussion

A plot of the relative concentrations that would exist after a ten year run at constant intensity is give in Figure 1 for both methods derived. Figure 2 expands this plot for the region of small inflows. The results of the two methods agree quite well with each other.

In summary, any significant amount of inflow represents a rather dramatic reduction in radionuclide concentrations in the water found in the vicinity of the NuMI decay tunnel. More sophisticated analyses are presently underway to better take into account the details of the hydrogeological conditions. The results reported here should be used with some degree of caution as they represent an *average* condition. There would likely be significant local variations of the inflow rates due to the known spatial variations of the rock formations. However, the drawing of water in a well also averages over a considerable volume. The author would like to acknowledge the helpful comments of Kamran Vaziri, Alex Elwyn, Nancy Grossman, Paul Kesich, and Peter Lucas in various stages of preparing this paper.

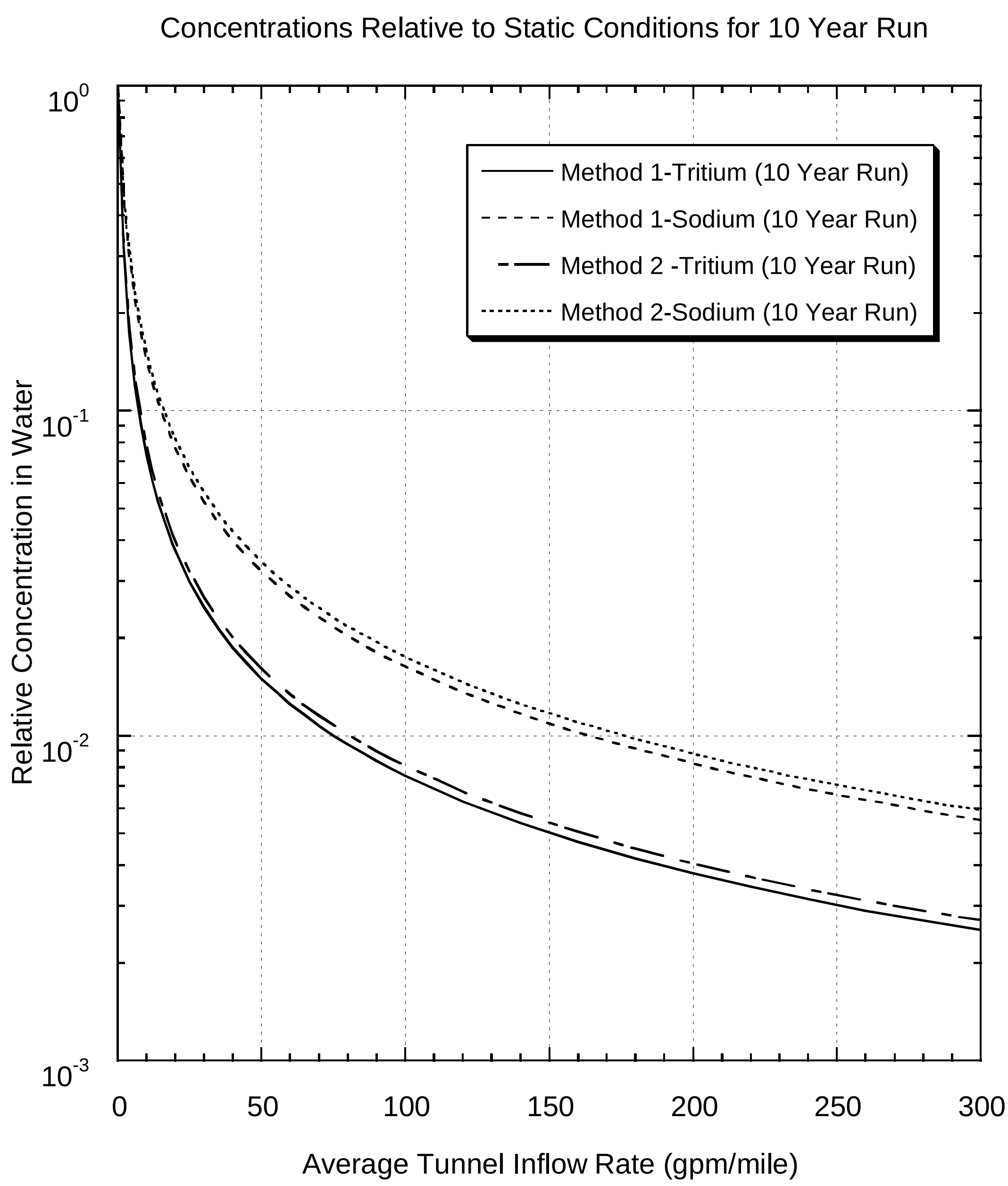

Figure 1 Concentrations of ^{3}H ("Tritium") and ^{22}Na ("Sodium") in water flowing into the NuMI tunnel relative to those which might be obtained with no inflow plotted as a function of the tunnel inflow rate, *I*. The results plotted are based upon the two different methods of calculation presented in the text. A ten year long period of operations is presumed.

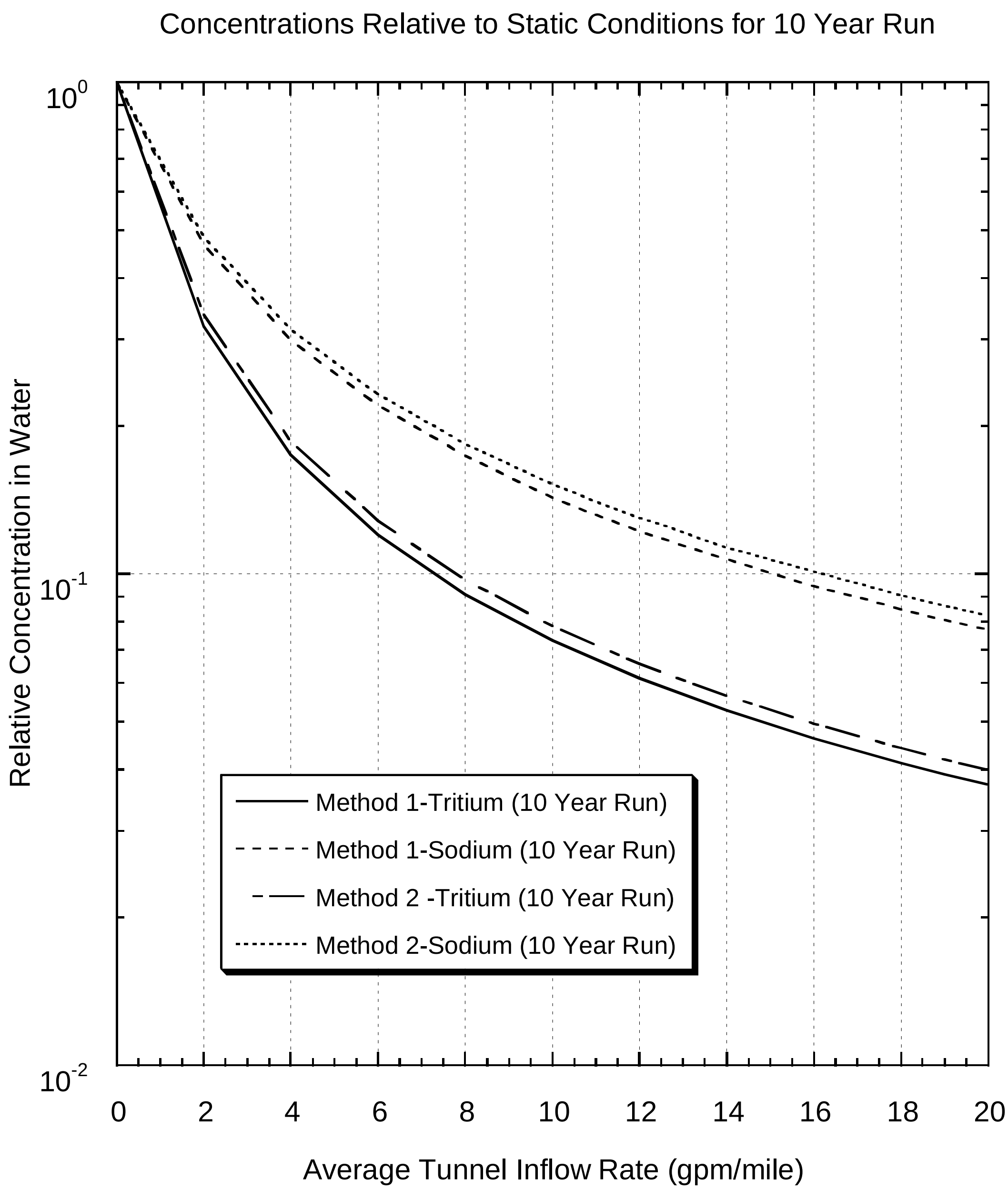

Figure 2 The same quantities plotted in Figure 1 are displayed over a smaller domain of low inflow values.