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**Abstract**

Beam crossing schemes in the LHC interaction regions impose non-zero vertical closed orbit in the low-$\beta$ triplets, which excite a perturbative periodic dispersion ; the phenomenon is described and quantified in detail. It is shown that this dispersion reaches values at the limit of tolerances in the nominal optics of Version 5.0 of the LHC ring, and prohibitively large values in particular in the low-$\beta$ quadrupoles and interaction regions in the foreseen extreme $\beta$-squeeze case ($\beta^* = 0.25$ m). Such behaviour justifies including a local correction in the LHC design, in order to damp the effect and confine it as much as possible in the vicinity of the excitation sources (the low-$\beta$ triplets). An optical compensation scheme based on the use of skew quadrupoles is described in detail, as well as the entailed residual dispersion.

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# 1 Introduction

Crossing angle and orbit off-centering schemes at interaction points (IP) in LHC ring in collision optics mode are foreseen [1, 2], for full separation of the beams during energy ramping phase, or early separation of the beams beyond the IP during collision, in order to reduce as much as possible harmful effects related to beam-beam interactions in that region where they share a common vacuum pipe. Both planes may be affected by crossing or off-centering, e.g., in the 45 deg. inclined crossing plane scheme. In terms of orbit design this means non-zero closed orbit (c.o.) angle (crossing) or non-zero c.o. off-centering (separation) at the IP of concern, and in consequence in the low-$\beta$ triplets, which has a sensible effect on the dispersion function in collision optics when betatron functions reach very large values. In terms of the equations of motion, the non-zero c.o. induces dispersive terms of first order in momentum deviation, with corresponding particular closed solution.

This phenomenon has been subject to detailed investigation in Ref. [3] in the frame of the LHC Version 4.2. We now address the recently designed Version 5.0 [5] of the ring. The main aspects of dispersion excitation are recalled ; it is shown that in the nominal optical conditions ($Q_x/Q_y = 63.32/59.31$, $\pm 0.1$ mrad vertical crossing, $\beta_y^* = 0.5$ m) and under propitious betatron phase relations between IP's, the so induced vertical dispersion may reach the limit of tolerances in the case of a single crossing and even exceed it in the case of several crossings. It is also shown that prohibitively large figures are attained in the extreme $\beta$-squeeze conditions ($\pm 0.2$ to $\pm 0.4$ mrad vertical crossing, $\beta_y^* = 0.25$ m), therefore justifying foreseeing a local correction scheme.

Correction strategies for horizontal crossing induced dispersion have already been investigated in detail and are now part of the LHC design [4]. Vertical crossing induced dispersion and correction principles for its compensation have also been investigated in Ref. [3], however a practical correction scheme for LHC V5.0 still remained to be defined, which is done here. The device is based on the use of skew quadrupoles located as close as possible to the low-$\beta$ triplets at the neighbouring arc ends. Corrector strengths are derived analytically and allow quantifying the needs for Version 5.0.

The report is organized as follows. In Section 2 the differential equation for the vertical crossing induced dispersion is established and its effects are derived and quantified. Section 3 describes the proposed correction optics. Numerical applications and simulations undertaken in the report are based on the Version 5.0 of the LHC optics [5]. MAD [6] simulations are performed wherever necessary, with the regular LHC lattice files [7] and preliminary $\beta^* = 0.25$ m optics [9]. The present work largely leans on Ref. [3] which may in particular be referred to for comparison with prior similar study involving Version 4.2 of the optics.

# 2 Vertical crossing/off-centering induced dispersion

## 2.1 Perturbative periodic dispersion ; scaling

Vertical dispersive effects related to c.o. geometry derive from the equation of motion

$$d^2y_r/ds^2 + K(s)y_r = -(1-\delta)\Delta B_y(s)/B\rho + K(s)y_r\delta \tag{1}$$

in which $y_r$ is the transverse excursion w.r.t. machine axis, $B\rho$ is the particle rigidity, K(s) the quadrupole strength and $\delta$ the momentum deviation. The field term $-\Delta B_y(s)/B\rho$ is due to the c.o. dipoles and its factor $(1-\delta)$ accounts for their first order chromatic effect. The second order dispersive term $K(s)y_r\delta$ is due to quadrupoles. Taking $y_r = y + y_{co}$ ($y_{co}$ = c.o. excursion, y = particle excursion w.r.t. c.o.) leads to the differential equation

$$d^2D_y/ds^2 + K(s)D_y = \Delta B_y(s)/B\rho + K(s)y_{co} \tag{2}$$

for the vertical dispersion $D_y = y/\delta$. The elementary kick approximation $K(s)y_{co}(s) = \int K(s)y_{co}(s)\delta(s-s_q)ds_q$ [$\delta(s-s_q)$ = Dirac impulse at azimuth $s_q$], yields the solution

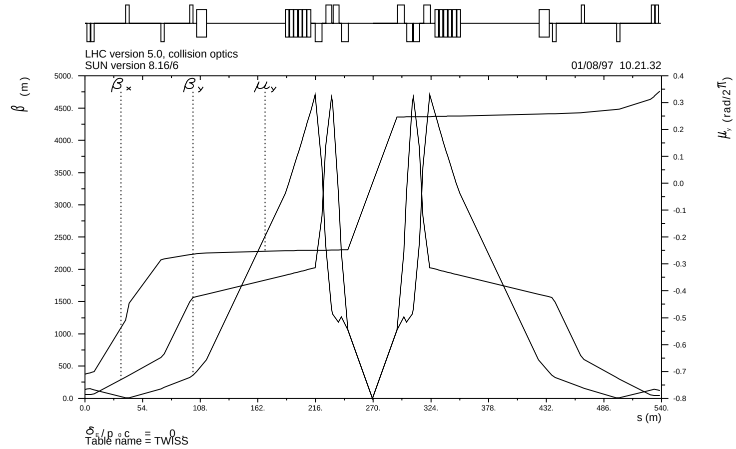


Figure 1: Betatron functions and vertical betatron phase in IR5, Version 5.0, $\beta^* = 0.5$ m.

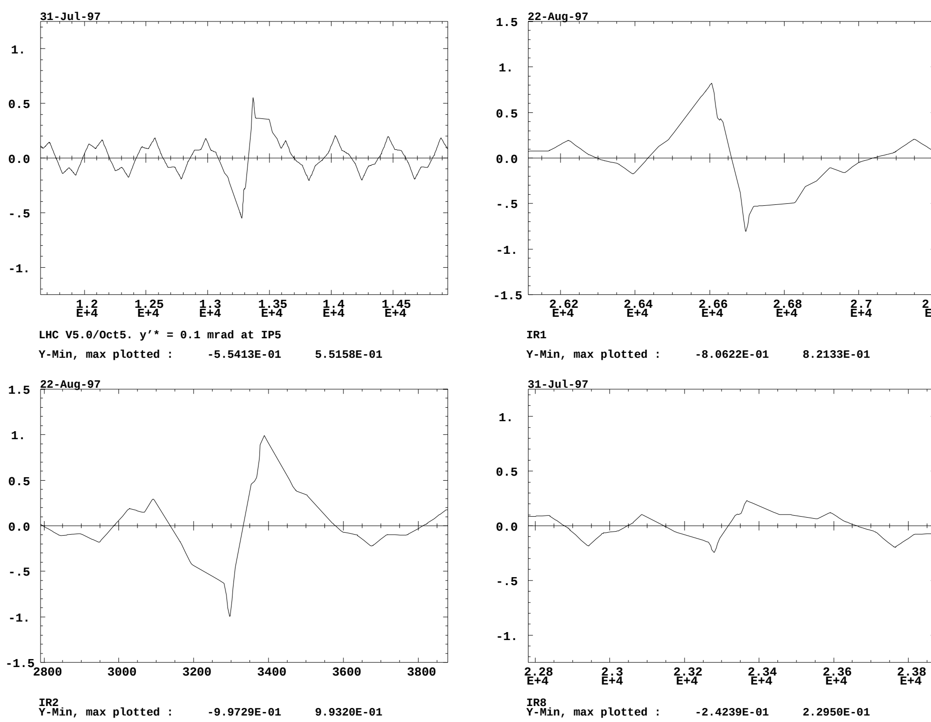


Figure 2: Perturbative $D_y$ (Eq. 3) induced by $y'^* = 10^{-4}$ rad c.o. angle at IP5 with $\beta^* = 0.5$ m, as observed at Octant 5 and in IR1, IR2 and IR8 regions. With $\beta_y^* = 0.25$ m and $y'^* = 0.4$ mrad (assuming identical phase at IP) the vertical scales increase by a factor of $4\sqrt{2}$.

Table 1: Values of the sums relevant with the calculation of the perturbative dispersion. ($\beta_y^* = 0.5$ m, $y'^* = 0.1$ mrad).

| IP | $\sum_q (KL)_q \beta_y(s_q)$ Left/Right | $\sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)}$ Left/Right ($10^{-2}$) |
|---|---|---|
| IP1/5 | -160 / -209 | -1.13 / 1.47 |
| IP2/8 | -210 / -156 | -1.44 / 1.10 |
| | scaling $\approx \frac{1}{\beta_y^*}$ | scaling $\approx \frac{1}{\sqrt{\beta_y^*}}$ |

$$D_y(s) = -y_{co}(s) + \sqrt{\beta_y(s)}/(2\sin\pi Q_y)\sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)}\cos Q_y[\pi - |\phi(s) - \phi(s_q)|] \quad (3)$$

where $Q_y$ = vertical tune, $\phi(s) = 1/Q_y \int ds/\beta_y$ is the normalized vertical betatron phase, $\phi(s_q)$ = normalized phase at the kick, $\beta_y$ = betatron function. The summation $\sum_q$ extends over those quadrupoles located within the c.o. bump. Plots of $D_y(s)$ obtained in the collision optics conditions schemed in Fig. 1 and c.o. geometry discussed in Appendix A ($y'^* = 0.1$ mrad c.o. angle at IP5, Fig. (6-left)) are given in Fig. 2 ; it is shown below that 1 mm c.o. displacement (Fig. 6-right) causes less than 8% the effect of 0.1 mrad c.o. angle.

*Calculation of $D_y(s)$ from the nominal lattice*

In Version V5.0 of LHC optics the c.o. bump encompasses quadrupoles in the range Q4/Q6 (App. A). However the quadrupoles Q5, Q6 have very weak effect (e.g., less than 6% contribution for $\beta_y^* = 0.5$ m with the c.o. geometry used here) and it is good enough approximation to assume that all the c.o. bump dipoles are located beyond the quadrupoles (namely, Q1-Q3) sources of the dispersion. This allows to introduce the c.o. in terms of unperturbed first order optics by its transport from IP,

$$y_{co}(s_q) = y^*\sqrt{[\beta_y(s_q)/\beta_y^*}\cos Q_y[\phi(s_q) - \phi^*] + y'^*\sqrt{\beta_y(s_q)\beta_y^*}\sin Q_y[\phi(s_q) - \phi^*] \quad (4)$$

in which '*' denotes quantities taken at the IP and beam divergence $\alpha^* = 0$ is assumed. Note that, due to the product $\sqrt{\beta_y(s_q)\beta_y^*}$ being preserved $\forall \beta_y^*$ the closed orbit $y_{co}(s)$ in the low-$\beta$ quadrupoles region does almost not depend on $\beta_y^*$ and becomes simply proportional to the c.o. angle $y'^*$ ; this feature will be useful in determining the maximum necessary correction strength. Reporting Eq. (4) in Eq. (3) yields

$$\begin{aligned} D_y(s) &= -y_{co}(s) \\ &+ y^*\sqrt{\beta_y(s)/\beta_y^*}/(2\sin\pi Q_y)\textstyle\sum_q (KL)_q\beta_y(s_q)\cos Q_y[\phi(s_q)-\phi^*]\cos Q_y[\pi - |\phi(s)-\phi(s_q)|] \\ &+ y'^*\sqrt{\beta_y(s)\beta_y^*}/(2\sin\pi Q_y)\textstyle\sum_q (KL)_q\beta_y(s_q)\sin Q_y[\phi(s_q)-\phi^*]\cos Q_y[\pi - |\phi(s)-\phi(s_q)|] \end{aligned} \quad (5)$$

This conveniently allows calculation of the perturbative dispersion from the bare optics, regardless of c.o. bump excursion beyond the IP.

*Characteristics of the perturbation*

Figure 1 shows that the betatron phase is about constant over the all low-$\beta$ triplet, which permits factorizing the sine and cosine terms out of the summations in Eq. 5 ; this is convenient since it allows quantifying the effect of the perturbation by simply evaluating $\sum_q (KL)_q\beta_y(s_q)$. This is detailed in

Table 1 for $\beta_y^* = 0.5$ m, at all four IP's. By comparison one had quasi-identical values of that sum with Version 4.2 optics [3, App. B]. From this we see that the two Versions are about equivalent as to crossing angle and c.o. displacement induced dispersion, and also that all IP's in Version 5.0 generate similar perturbation. Another consequence is that all IP's need identical sets of correctors.

On the other hand, given that the low-$\beta$ quadrupole strengths stay unchanged whatever $\beta_y^*$ [5], it can be shown that the amplitude and shape of the function $\beta_y^*\beta_y(s_q)$ across the low-$\beta$ triplets does not significantly change whatever $\beta_y^* < few\ meters$ ; besides, the closed orbit is also independent of $\beta_y^*$. It results that in the 0.25 m to about 5 m $\beta_y^*$ range the quantity $D_y(s)/\sqrt{\beta_y(s)}$ scales as $1/\sqrt{\beta_y^*}$ (cf. Eq. 7) (its behaviour for $\beta_y^*$ values above 5 m is of no concern since the dispersion then does not exceed $\approx 0.1$ m, $\forall s$ as shown below).

## 2.2 Upper limits of the perturbation

It can be shown that the perturbative dispersion amplitude is locally bound by [3]

$$
\begin{aligned}
D_{y,extr}/\sqrt{\beta_y(s)} \quad &= \quad \{\, [\textstyle\sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)} \cos Q_y(\pi + \epsilon\phi(s_q))]^2 \\
&+ \quad [\textstyle\sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)} \sin Q_y(\pi + \epsilon\phi(s_q))]^2 \,\}^{1/2} / (2\sin\pi Q_y)
\end{aligned}
\tag{6}
$$

with $\epsilon = \pm 1$ *for resp.* $\phi(s) \gtrless \phi(s_q), \forall q$. Calculation of the cosine and sine squared sums above from the first order optical functions in collision optics yields

$$
D_{y,extr}/\sqrt{\beta_y(s)}|_{y^*=0} \approx 109\, y'^*/\sqrt{\beta_y^*}, \qquad D_{y,extr}/\sqrt{\beta_y(s)}|_{y'^*=0} \approx -0.84\, y^*/\sqrt{\beta_y^*} \tag{7}
$$

Eq. (7) confirms that $10^{-4}$ rad c.o. angle has about 13 times the effect of $10^{-3}$ m c.o. off-centering at IP. Under crossing, typical upper limits to $D_y(s)$ for $\beta_y^* = 0.5$ m are, 0.21 m in the arcs ($\beta_{y,max} \approx$ 180 m/rad) ; 1.06 m in the odd-type low-$\beta$ triplets ($\beta_{y,max} = 4700$ m/rad) ; 1.02 m in the even-type low-$\beta$ triplets ($\beta_{y,max} = 4420$ m/rad). These extrema increase by a factor of $4\sqrt{2}$ in the $\beta_y^* = 0.25$ m, $y'^* = 0.4$ mrad optics (leading to 1.19 m peak in the arcs and 6 m in the low-$\beta$ triplets), and by an additional factor of four in the worst configuration of uncorrected 4-crossing optics (see section 2.4) ; they are attained *iff* adequate betatron phase is reached at $\beta_{y,max}$ (contrary to what happens with 63.31/59.32 tuning, see Fig. 2), however the correction should allow for such possibility (e.g., with non-split tunes $Q_x/Q_y = 63.28/63.31$ in LHC Version 4.2) as well as for the $y'^*/\sqrt{\beta_y^*}$ scaling.

## 2.3 Dispersion at IP's

Dispersion at IP's under c.o. angle $y'^*$ is given with good precision by

$$
D_y(IP^*) = \frac{y'^*\beta_y^*}{2}\,\Sigma^-, \quad D_y(IP \neq IP^*) = \pm\frac{y'^*\sqrt{\beta_y^*}\sqrt{\beta_y(IP)}}{2\sin(\pi Q_y)} sinQ_y[\pi - |\phi(IP) - \phi^*|]\,\Sigma^+, \tag{8}
$$

with $\pm$ sign in $D_y(IP \neq IP^*)$ for resp. $\phi(IP) \gtrless \phi^*$, and $\Sigma^\pm = \sum_{Q1/Q6}(KL)_q\beta_y(s_q)|_{Left} \pm \sum_{Q1/Q6}(KL)_q\beta_y(s_q)|_{Right}$. The left sum in Eq. (8) holds for the IP where the crossing occurs, the right one for the other IP's. Taking $y'^* = 0.1$ mrad with $\beta_y^* = \beta_y(IP) = 0.5$ m and (Version V5.0, $Q_x/Q_y = 63.32/59.31$) phases $\phi(IP1, 2, 5, 8) = 0,\ 7.601,\ 29.769,\ 51.715\ (\times 2\pi/Q_y)$ respectively, and with $\Sigma^+ \approx -369$ and $\Sigma^- \approx 50$ (Table 1, col. 2), one gets

$$
D_y(IP1, 2, 5, 8) = 6.93\ mm, \quad -0.56 mm, \quad 1.22\ mm, \quad -10.6\ mm, \tag{9}
$$

(MAD simulations give 6.87, -0.51, 1.21 and -10.6 mm). These values scale as $y'^*/\sqrt{\beta_y^*}$.

As to the impact on beam size at IP it amounts to up to $\sigma_d = D_y\,\delta p/p \approx 1$ $\mu$m at $\sigma_{\delta p/p} = 10^{-4}$, that is, about 7%. In the extreme optics $\beta^* = 0.25$ m, $y'^* = 0.4$ mrad the effect on beam size at IP

Table 2: Peak $D_y$ values in the arcs and in the low-$\beta$ triplets in the nominal Version 5.0 of LHC ($\beta_y^* = 0.5$ m at IP1, 2, 5 and 8), due to $y'^* = 0.1$ mrad vertical crossing set at a single IP, 2 IP's or 4 IP's, in the absence of local compensation (the + and - signs in the first column stand for the respective crossing signs at the various IP's of concern). Following Eq. (7) these values scale as $y'^*/\sqrt{\beta_y^*}$ and thus, considering identical betatron phase behaviour, need be multiplied by $4\sqrt{2}$ in the case of the extreme squeeze optics ($\beta_y^* = 0.25$ m and $y'^* = 0.4$ mrad).

| Crossing | Peak vertical dispersion (m) In the arcs | In the low-$\beta$ triplets |
|---|---|---|
| IP1 | 0.20 | 0.95 (at IR2, 8) |
| IP1 and 5 | | |
| + + | 0.33 | 0.71 (at IR8) |
| + - | 0.40 | 1.96 (at IR2) |
| IP1, 2, 5 and 8 | | |
| + + + + | 0.50 | 2.21 (at IR1) |
| - + + + | 0.73 | 3.45 (at IR1) |
| + - + + | 0.45 | 1.15 (at IR5, 8) |
| - - + + | 0.50 | 2.40 (at IR2) |
| - - - + | 0.51 | 0.82 (at IR6) |
| - - + - | 0.32 | 0.58 (at IR1) |

is about 5.7 times larger leading to $D_y \approx 60\ 10^{-3}$ m and $D_y\ \delta p/p \approx 6\ \mu$m, while the betatron size squeezes down to about 11$\mu$m, which entails a prohibitive effect of more than 50%. This is in the case of a single, uncompensated crossing ; things are liable to worsen by an additional factor of four with four crossings (see below).

### 2.4 Interferences

Interference arises when crossings are set at several IP's. They may be either destructive or constructive, depending on the local phase, on the phase difference between IP's of concern and on the signs of the crossings. Note that the issue of phase shift from IP to IP is still pending ; indeed, recent studies [8] tend to show that special relations ought to be preserved in order to improve the dynamic aperture.

For instance interference between IP1 and IP5 with $y'^* = 10^{-4}$ rad vertical c.o. angle and $\beta_y^* = 0.5$ m give the following extremum in the IP5 low-$\beta$ triplets (with $\epsilon = y'^*_{IP1}/y'^*_{IP5}$) [3]

$$\begin{array}{ll} D_{y,extr} = -y'^* \sqrt{\beta_{y,max}\beta_y^*}/(2\sin\pi Q_y)(1+\epsilon\cos\pi Q_y)\sum_q (KL)_q\beta_y(s_q) & (\epsilon=\pm 1) \\ yielding, \quad -y'^* \sqrt{\beta_{y,max}\beta_y^*}/(2\tan\pi Q_y/2)\sum_q (KL)_q\beta_y(s_q) \approx 0.46\,m \quad if \quad \epsilon=+1 & \\ and \quad\quad -y'^* \sqrt{\beta_{y,max}\beta_y^*}/2\tan\pi Q_y/2\sum_q (KL)_q\beta_y(s_q) \approx 1.64\,m \quad if \quad \epsilon=-1 & \end{array} \tag{10}$$

liable to be reached with close to $\pi$ (normalized) phase shift between IP1 and IP5. In the particular case of the nominal V5.0 tuning 63.32/59.31, these extrema are not attained, actual peaks are, 0.25 m if $\epsilon = +1$ (identical signs) and 1.36 m if $\epsilon = -1$ (opposite signs). This shows the possible enhancement of harmful effects under multiple crossing or separation schemes in the absence of local correction. The situation can be worse and the dispersion peak can reach twice the values above with 4-IP interference, and about 50% more with $\beta_y^* = 0.25$ m leading for instance to $D_y$ extremum of more than 20 m in low-$\beta$ triplets. Given $\sigma_{\delta p/p} \approx 10^{-4}$ this entails a non-negligible contribution of $\approx$2 mm to the transverse aperture in the - costly - bore of the low-$\beta$ quadrupoles. More details obtained from MAD simulations can be found in Table 2.

In terms of beam size at IP, contributions of various crossings are liable to add up and entail significant beam cross section increase, e.g., from MAD simulations, $D_y(IP8) = -19$ mm with

$y'^* = 0.1$ mrad at all IPs', $D_y(IP5) = -18$ mm with $y'^* = 0.1$ mrad at IP1,2 and -0.1 mrad at IP5,8.

# 3 Correction of the dispersion

The c.o. angle/displacement induced vertical dispersion can be compensated by arrangement of skew quadrupole correctors located at the neighbouring arc ends, whose effect is to locally close independently the Left and Right chromatic c.o. bumps excited by the off-centering in the low-$\beta$ quadrupoles[1].

## 3.1 Correction strength

Adding skew quadrupole correctors of strength R(s) to the structure results in coupled motion whose dispersion function verifies [3]

$$d^2D_y/ds^2 + K(s)D_y = R(s)D_x \tag{11}$$

with solution (index SQ denotes skew correctors)

$$D_y(s) = \sqrt{\beta_y(s)}/(2\sin\pi Q_y)\sum_{SQ}(RL)_{SQ}D_x(s_{SQ})\sqrt{\beta_y(s_{SQ})}\cos Q_y[\pi - |\phi(s) - \phi(s_{SQ})|] \tag{12}$$

where $\phi$ is the normalized vertical betatron phase. This is to be added to the solution (Eq. 3) of Eq. (2), hence the condition for mutual compensation beyond the chromatic bump writes (index q designates the low-$\beta$ triplet quadrupoles)

$$\begin{array}{c}\sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)}\cos Q_y[\pi - |\phi(s) - \phi(s_q)|] \\ -\sum_{SQ}(RL)_{SQ}D_x(s_{SQ})\sqrt{\beta_y(s_{SQ})}\cos Q_y[\pi - |\phi(s) - \phi(s_{SQ})|] = 0\end{array} \tag{13}$$

Now, on the one hand the correction strength $(RL)_{SQ}$ is minimized if $D_x(s_{SQ})\sqrt{\beta_y(s_{SQ})}$ is maximized, on the other hand better horizontal/vertical decoupling is insured if correctors are located at maximum $\beta_y/\beta_x$, which dictates that they be placed at defocusing quadrupoles. The phase difference from the correctors to the source of the defect may significantly differ from $\pi/2Q_y$ $[\pi/2Q_y]$[2,3] which imposes the use of two sets SQ1 and SQ2 of correction lenses (one set can for instance be a pair of skew quadrupoles, as below) to compensate the excitation source $(KL)_q$. It can be shown (App. B) that the strength $[(RL)_{SQ1}, (RL)_{SQ2}]$ of the correctors (SQ1,SQ2) is given by

$$(RL)_{SQ1} = -\sin(\epsilon)\frac{\Sigma_q}{D_x\sqrt{\beta_y}}, \quad (RL)_{SQ2} = \cos(\epsilon)\frac{\Sigma_q}{D_x\sqrt{\beta_y}} \tag{14}$$

with radius $\Sigma_q = \sum_q (KL)_q y_{co}(s_q)\sqrt{\beta_y(s_q)}$ and $\epsilon = Q_y(\phi^* - \phi(SQ)) - \pi/2$ $[\pi/2]$.

As an illustration the correction strengths in the LHC Version 5.0 optics with split tunes 63.31/59.32 can be calculated, as follows. From MAD simulations reported on col. 4 of Tables 3 we get $\epsilon_{Left} \approx 0.10 \cdot 2\pi$ on the left hand side of IP5, $\epsilon_{Right} \approx 0.15 \cdot 2\pi$ on the right hand side, and $D_x\sqrt{\beta_y} \approx 14$ at the corrector locations ; from col. 3 of Table 1 we get $\Sigma_{q,Left} \approx -1.13\ 10^{-2}$

[1] A compensation by an opposing bump in the arcs can be thought of ; principle of such correction has been addressed in Ref. [3] as concerns the dispersion excited in the dispersive plane by the horizontal component of the crossing. As to the vertical plane we leave it to further investigations.

[2] Modulo $\pi/2Q_y$.

[3] Referring to works under completion at FNAL [11], local chromaticity correction might in the future lead to imposing $\pi/2Q_y$ $[\pi/2Q_y]$ IP to arc phase shift. In such case this remark is no longer true and a single set SQ will insure local $D_y$ compensation.

while $\Sigma_{q,Right} \approx 1.473\ 10^{-2}$. We also consider sets SQ made of a pair of quadrupoles placed $\pi$ apart from one another and with opposite strengths in order to minimize effects on the focusing, hence the strength in a single corrector is half the value $(RL)_{SQ}$ above which we note $(RL)_{\frac{1}{2}SQ}$, namely (Eq. 14)

$$\begin{array}{ll} (RL)_{\frac{1}{2}SQ1} \approx \pm 2.3\ 10^{-4}, \quad (RL)_{\frac{1}{2}SQ2} \approx \pm 3.3\ 10^{-4} & \textit{on the Left side of IP} \\ (RL)_{\frac{1}{2}SQ1} \approx \pm 4.2\ 10^{-4}, \quad (RL)_{\frac{1}{2}SQ2} \approx \pm 3.1\ 10^{-4} & \textit{on the Right side of IP} \end{array} \tag{15}$$

This is in agreement with MAD simulations reported in col. 5 of Tables 3[4].

### 3.2 Corrector characteristics

After Eq. (14) the maximum strength a corrector may be demanded is $(RL)_{\frac{1}{2}SQ} = \Sigma_q / 2D_x\sqrt{\beta_y}$. Considering that $\Sigma_q$ is proportional to $y'^*/\sqrt{\beta_y^*}$ (Eqs. 6,7) with a maximum value $\approx 1.5\ 10^{-2}$ (col. 3 of Table 1), one gets the maximum strength of a skew quadrupole in a pair

$$(RL)_{\frac{1}{2}SQ,max} = 3.8\, y'^*/\sqrt{\beta_y^*} \tag{16}$$

that is for instance, $5.37\ 10^{-4}$ m$^{-1}$ for 0.1 mrad c.o. angle at $\beta_y^* = 0.5$ m. Given $B\rho = 23352$ Tm rigidity (7 TeV) the gradient of a 0.32 m long skew quadrupole comes out to be about 39 T/m. About 10% strength has to be added to allow for simultaneous compensation of $y^* = 1$ mm c.o. orbit displacement (Section 2) for beam-beam separation during injection phase. Additional safety margin is to be foreseen to allow on the one hand for possible vertical beta and horizontal dispersion beatings that would affect correction efficiency at the location of the skew quadrupoles (scales as $D_x\sqrt{\beta_y}$), and on the other for beta beatings in the low-$\beta$ region that would enhance the strength of the excitation (scales as $\beta_y$). This leads to a safety margin of about 50% which allows the use of MQS type correctors whose nominal gradient is 86 T/m [10].

More demanding is the $\beta_y^* = 0.25$ m, $y'^* = 0.2$ mrad optics. $(RL)_{\frac{1}{2}SQ,max}$ reaches $15.2\ 10^{-4}$ m$^{-1}$ and the gradient about 110 T/m, which is close to the maximum value allowed by the MQS type correctors (120 T/m [10]). These may thus seem too weak ; however it can be foreseen to impose such betatron phase at IP as to have significant $\epsilon$ value (Eq. 14) so that the total strength is shared by the two sets SQ1 and SQ2[5]. Such is the case in the V5.0 optics where $(RL)_{\frac{1}{2}SQ}$ does not exceed about the two thirds of $(RL)_{\frac{1}{2}SQ,max}$. This would leave a safety margin of about 30% on the gradient w.r.t. 120 T/m.

However this margin would again be lost with $y'^* = 0.4$ mrad crossing which calls for integrated correction strength $(RL)_{SQ,max} \approx 40\ 10^{-4}$ m$^{-1}$ (about 140 T/m per skew quadrupole of a pair) ; in order to allow for these extreme working conditions, it may be foreseen installing quadruplets instead of doublets.

### 3.3 Short range correction scheme

We describe here a correction scheme which, apart from allowing for the various criteria above, provides the shortest chromatic c.o. bump. This is done by placing the skew quadrupoles as close as possible to the excitation source (the low-$\beta$ triplets) at such locations as specified in Table 3[6].

[4] Corrector strengths in the LHC Version 4.2 optics were calculated in Ref. [3] assuming the SQ1 pair to have very weak role, which was good enough since $\epsilon_{Left/Right} \approx 0.03/0.02 \cdot 2\pi$ ; however Eqs. (14) allow recovering all strengths with very good precision, namely taking $\Sigma_{q,Left/Right} \approx 1.5/1.12\ 10^{-2}$ and , $D_x\sqrt{\beta_y} \approx 14.2$ leads to $(RL)_{SQ1,Left/Right} \approx 1.0/0.50\,10^{-4}$ and $(RL)_{SQ2,Left/Right} \approx 5.19/3.91\,10^{-4}$, in excellent agreement with related MAD simulations [3, Table 9, col. 5].

[5] This is not possible though if fixed arc to IP phase shift is imposed by other considerations (see footnote 2 in Section 3 [11] ; in such case however the number of quadrupoles in the unique SQ set can be increased in order to allow for the desirable safety margin.

[6] This might impose moving the tune shift quadrupoles.

Table 3: Corrector strengths for $y'^* = 0.1$ mrad, $\beta_y^* = 0.5$ m (scale as $y'^*/\sqrt{\beta^*}$).

| INTERLEAVED CORRECTION SCHEME | | | | |
|---|---|---|---|---|
| Corrector name | Neighbouring quadrupole | $D_x\sqrt{\beta_y}$ ($m^{3/2}$) | Phase w.r.t. IP ($2\pi$) | $(RL)_{\frac{1}{2}SQ}$ ($10^{-4}m^{-1}$) |
| SQ2a/b.Left | QD13/17.Left | 14. | -1.351 /-1.838 | $^-/_+$ 3.28 |
| SQ1a/b.Left | QT.Q11/15.Left | 14. | -1.107 /-1.594 | $^-/_+$ 2.26 |
| SQ1a/b.Right | QD10/14.Right | 14. | 0.672 / 1.143 | $^-/_+$ 4.47 |
| SQ2a/b.Right | QD12/16.Right | 14. | 0.899 / 1.386 | $^-/_+$ 2.84 |

| NON-INTERLEAVED CORRECTION SCHEME | | | | |
|---|---|---|---|---|
| Corrector name | Neighbouring quadrupole | $D_x\sqrt{\beta_y}$ ($m^{3/2}$) | Phase w.r.t. IP ($2\pi$) | $(RL)_{\frac{1}{2}SQ}$ ($10^{-4}m^{-1}$) |
| SQ2a/b.Left | QD17/21.Left | 14. | -1.837 /-2.324 | $^-/_+$ 3.30 |
| SQ1a/b.Left | QT.Q11/15.Left | 14. | -1.106 /-1.594 | $^-/_+$ 2.00 |
| SQ1a/b.Right | QD10/14.Right | 14. | 0.673 / 1.143 | $^-/_+$ 4.22 |
| SQ2a/b.Right | QD16/20.Right | 14. | 1.387 / 1.874 | $^-/_+$ 2.90 |

Given the optical and geometrical similitude between all four IR's in LHC, identical corrector implementation can be used for all.

We investigate two implementations, interleaved (upper Table 3) and non-interleaved (lower Table 3) . The skew quadrupole correctors are placed next to the multipoles MSCV at $D_x\sqrt{\beta_y} \approx 14$ (Tables 3, col. 3). The integrated correcting strength is matched with constraints exclusively on DY = 0 and DY' = 0 at IP and Octant ends and indeed meets the results above (Eq. 15 and col. 5 of Tables 3). The strength is shared in comparable way by the SQ1a/b and SQ2a/b pairs on both the Left- and Right-hand sides of the IR.

Both interleaved and non-interleaved configurations have similar effects. The residual dispersion in the arcs is quasi-zero (Fig. 3). The absence of effect on the first order focusing is apparent in Tables 4 which display the ensuing values of the optical functions at IP1,2,5,8, as well as tunes and other parameters as obtained from a one-turn MAD TWISS procedure without any additional re-tuning of the IR ; it is clear that any induced mismatch is negligible. Horizontal and vertical $\beta$-beating in the arcs are also quasi-zero thanks to the $\pi$-distant opposite strength quadrupoles constituting the correction pairs, and the machine tunes are not affected. From these results the induced coupling appears negligible too.

## 3.4 Correction scheme for LHC V5.0 optics

The LHC V5.0 optics is particularized in that many corrector locations are already attributed. Our goal here is to insert necessary sets of skew quadrupole correctors proper to $D_y$ compensation within remaining locations. As to these, we refer to [10, 12]. All quadrupoles from Q11 to Q19 are occupied ; we don't consider placing correctors in the DS in order to avoid unbalancing of the pairs and to preserve a good $\beta_y/\beta_x$ ratio. The first available spots are from Q20 after moving the coupling skew quadrupoles and the octupoles towards the centre of the arc [12]. It does not make principle differences with the previous 'Short range correction scheme' (Section Sec3.3), the main effect will be an extension of the chromatic bump over a longer range.

Correctors pairs are interleaved at quadrupoles QD21-27 and QD20-26 on respectively the left- and right-hand side of the IR ; their strengths are as in the upper Table 3, col. 5. The resulting residual vertical dispersion is shown in Fig. 4.

Table 4: Optical functions after correction, with $y'^* = 0.1$ mrad, $\beta_y^* = 0.5$ m.

INTERLEAVED CORRECTION SCHEME

| pos. | | beta1 | alfa1 | mu1 | xco | pxco | Dx | Dpx |
|---|---|---|---|---|---|---|---|---|
| | | beta2 | alfa2 | mu2 | yco | pyco | Dy | Dpy |
| (m) | | (m) | | [2pi] | [mm] | [.001] | (m) | (rad) |
| 1 | IP1 | 0.500 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 599 | IP2 | 0.500 | -0.001 | 8.242 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.499 | 0.000 | 7.601 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2417 | IP5 | 0.500 | -0.001 | 31.838 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 29.770 | 0.000 | 0.100 | 0.000 | 0.000 |
| 4214 | IP8 | 0.500 | 0.000 | 55.378 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.004 | 51.715 | 0.000 | 0.000 | 0.000 | 0.000 |

| | |
|---|---|
| Q1 = 63.309890 | Q2 = 59.320094 |
| betax(max)= 4705.599893 | betay(max) = 4706.096404 |
| Dx(max) = 2.859208 | Dy(max) = 0.134132 |
| | $y_{co}$(max) = 4.850630 |

NON-INTERLEAVED CORRECTION SCHEME

| pos. | | beta1 | alfa1 | mu1 | xco | pxco | Dx | Dpx |
|---|---|---|---|---|---|---|---|---|
| | | beta2 | alfa2 | mu2 | yco | pyco | Dy | Dpy |
| (m) | | (m) | | [2pi] | [mm] | [.001] | (m) | (rad) |
| 1 | IP1 | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 599 | IP2 | 0.500 | 0.000 | 8.242 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 7.601 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2417 | IP5 | 0.500 | 0.000 | 31.838 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 29.769 | 0.000 | 0.100 | 0.000 | 0.000 |
| 4214 | IP8 | 0.500 | 0.000 | 55.378 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 51.715 | 0.000 | 0.000 | 0.000 | 0.000 |

| | |
|---|---|
| Q1 = 63.309969 | Q2 = 59.320013 |
| betax(max)= 4706..495006 | betay(max) = 4706.265351 |
| Dx(max) = 2.859727 | Dy(max) = 0.134131 |
| | $y_{co}$(max) = 4.850630 |

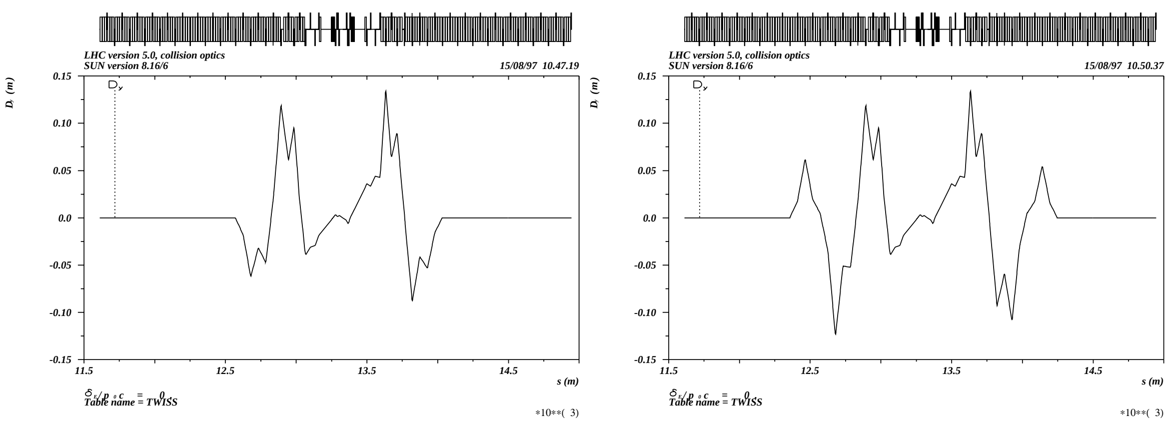


Figure 3: Residual vertical dispersion in Octant 5 after compensation of $y'^* = 10^{-4}$ rad c.o. angle at IP5 ($\beta_y^* = 0.5$ m) with four skew quadrupole pairs placed at the arcs ends ; the dispersion is zero in the rest of the machine. *Left plot* : interleaved scheme ; *Right plot* : non-interleaved scheme.

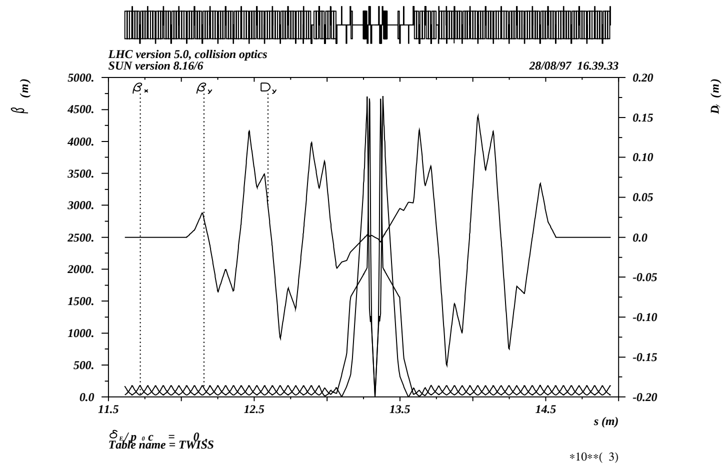


Figure 4: Residual vertical periodic dispersion in Octant 5 after compensation of $y'^* = 0.1$ mrad c.o. angle at IP5 ($\beta_y^* = 0.5$ m) with four skew quadrupole pairs placed at the quadrupoles QD21-27 and QD20-26 on respectively the left- and right-hand side of the IR. This correction differs from the 'Short range one' (Section 3.3) mostly by the increased extent of the closed chromatic bump.

Table 5: Corrector strengths for $y'^* = 0.2$ mrad, $\beta_y^* = 0.25$ m (to be doubled for $y'^* = 0.4$ mrad).

| INTERLEAVED CORRECTION SCHEME | | | | |
|---|---|---|---|---|
| Corrector name | Neighbouring quadrupole | $D_x\sqrt{\beta_y}$ ($m^{3/2}$) | Phase w.r.t. IP ($2\pi$) | $(RL)_{\frac{1}{2}SQ}$ ($10^{-4}m^{-1}$) |
| SQ2a/b.Left | QD13/17.Left | 14. | -1.392 /-1.879 | $^-/_+$ 7.25 |
| SQ1a/b.Left | QT.Q11/15.Left | 14. | -1.147 /-1.634 | $^-/_+$ 8.49 |
| SQ1a/b.Right | QD10/14.Right | 14. | 0.635 / 1.164 | $^-/_+$ 13.62 |
| SQ2a/b.Right | QD12/16.Right | 14. | 0.923 / 1.411 | $^-/_+$ 8.21 |

### 3.5 Ultimate squeeze

The LHC should eventually accommodate $\beta_y^* = 0.25$ m, while beam-beam effects impose $y'^* = 0.2$ mrad (and possibly even up to $y'^* = 0.4$ mrad [8]). Dispersion corrector characteristics in this case have been discussed in Section 3.2. We now show the behaviour of the interleaved scheme described in Section 3.4 [9]. Table 5 gives the corrector strengths ; it can be checked that their square sum is in a ratio of $y'^*\sqrt{\beta_y^*} \approx 2\sqrt{2}$ (Eq. 20) with the nominal optics (Table 3). Table 6 shows the residual dispersion function after correction ; the peak dispersion has been damped from 2.6 m (attained inside IR2 low-$\beta$ triplets) and in the arcs down to 0.35 m peak in IR5 region and practically zero beyond.

## 4 Conclusion

The vertical perturbative dispersion excited by crossing angle and orbit off-centering at IPs' in the Version 5.0 of the LHC ring in collision optics has been investigated and its most significant effects quantified.

It has been shown that in the nominal optical conditions ($Q_x/Q_y = 63.32/59.31$, $\pm 0.1$ mrad vertical crossing, $\beta_y^* = 0.5$ m) and under propitious betatron phase relations between IP's, the so induced vertical dispersion reaches values close to the limit of tolerances namely, $\pm 0.2$ m beats in the arcs, more than one meter in the low-$\beta$ triplets, and about 11 mm at the IP's in the case of a

Table 6: Optical functions after correction, with $y'^{*} = 0.2$ mrad, $\beta_y^{*} = 0.25$ m. Tunes, maximum beta, etc. are practically unchanged w.r.t. non perturbed case [9].

| INTERLEAVED CORRECTION SCHEME | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| pos.<br><br>(m) | | beta1<br>beta2<br>(m) | alfa1<br>alfa2 | mu1<br>mu2<br>[2pi] | xco<br>yco<br>[mm] | pxco<br>pyco<br>[.001] | Dx<br>Dy<br>(m) | Dpx<br>Dpy<br>(rad) |
| 1 | IP1 | 0.500 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 599 | IP2 | 0.500 | -0.001 | 8.242 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.499 | 0.000 | 7.601 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2417 | IP5 | 0.500 | -0.001 | 31.838 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.000 | 29.770 | 0.000 | 0.100 | 0.000 | 0.000 |
| 4214 | IP8 | 0.500 | 0.000 | 55.378 | 0.000 | 0.000 | 0.000 | 0.000 |
| | | 0.500 | 0.004 | 51.715 | 0.000 | 0.000 | 0.000 | 0.000 |
| Q1 = 63.326144 | | | | | Q2 = 59.383959 | | | |
| betax(max)= 9471.582314 | | | | | betay(max) = 9262.963729 | | | |
| Dx(max) = 2.932897 | | | | | Dy(max) =0.353314 | | | |
| | | | | | $y_{co}$(max) = 9.699951 | | | |

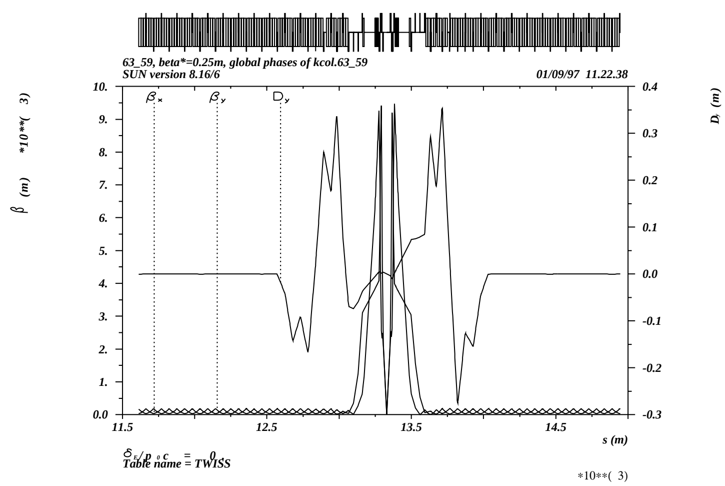


Figure 5: Residual vertical periodic dispersion in Octant 5 after compensation of $y'^{*} = 0.2$ mrad c.o. angle at IP5 with $\beta_y^{*} = 0.25$ m, by means of the interleaved correction scheme described in Section 3.4 (Optics after Ref. [9]) - the $D_y$ scale has to be doubled for $y'^{*} = 0.4$ mrad.

single crossing. These figures are four times larger in the case of four crossings, which is sufficient to justify local compensation. It has also been shown that these effects are prohibitively large in the extreme $\beta$-squeeze conditions ($\pm 0.2$ to $\pm 0.4$ mrad, $\beta_y^* = 0.25$ m), for instance more than 20 m dispersion in the low-$\beta$ triplets and 60 mm at IP's in the case of a single, uncompensated crossing.

The principle of a local compensation based on the use of skew quadrupoles is addressed and its major aspects are described. It is in particular shown that it allows killing the dispersion everywhere beyond a residual chromatic bump confined in the vicinity of the IR of concern. Correction schemes are shown in detail, as well as characteristics of the skew quadrupoles of concern. These can be of the MQS type already foreseen for coupling corrections, used by pairs down to $\beta^* = 0.25$ m as long as the projected crossing angle does not exceed $\pm 0.2$ mrad, or by quadruplets beyond.

# A Appendix : Beam crossing and off-centering schemes

Appendix A describes the crossing and/or off-centering c.o. bumps on which the numerical applications and other plots presented in this report are based.[7]

The steering is to include a combination of crossing or off-centering at IP in each plane (e.g., in order to achieve $\pm 45$ deg. inclined crossing plane). This imposes a pair of COD's per plane on each side of the IP (rather than a single one at $\pi/Q_y$ (normalized) phase-shift in crossing optics, for instance). Another argument in favour of early closing of the orbit bump is avoiding propagating non-zero c.o. in the dispersion suppressor region. This last criterion guided the present design in confining the c.o. bump within the range Q6.Left/Q6.Right : on each side of the IP, a vertical COD has been placed at Q6 and another one at D2.

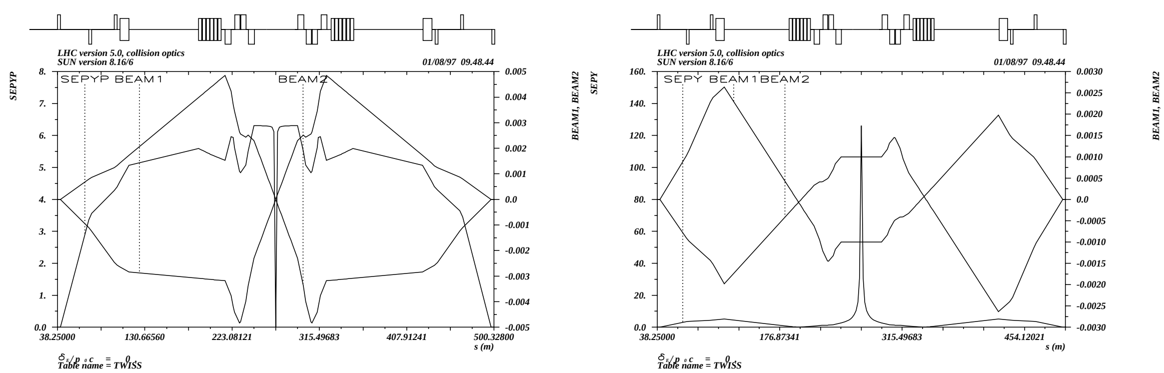


Figure 6: Vertical c.o. geometry and beam-beam separations (normalized to largest $\sigma$) in collision optics for $\beta_y^* = 0.5$ m, as used for numerical applications presented in the report. *Left plot* : $\pm 10^{-4}$ rad c.o. angle at IP5. *Right plot* : $\pm 10^{-3} m$ off-centering. The c.o. kicks do not exceed 6 $10^{-5}$ rad.

# B Appendix : Skew quadrupole strengths

The strengths of the skew quadrupoles to be located at arc ends for compensation of the vertical crossing angle induced dispersion are calculated. We start from Eq. (13) and considering that $\phi(s_q) = C^{ste} = \phi_q$ and $D_x\sqrt{\beta_y} \approx C^{ste}$ since the skew quadrupoles are placed close to arc quadrupoles, we rewrite it under the form

$$\cos Q_y[\pi - |\phi^* - \phi_q|]\Sigma_q = (D_x\sqrt{\beta_y})\{\sum_{SQ}(RL)_{SQ}\cos Q_y[\pi - |\phi^* - \phi_{SQ}|] + \sum_{SQ}(RL)_{SQ}\cos Q_y[\pi - |\phi^* - \phi_{SQ}|]\} \tag{17}$$

[7] c.o. implementation was still in discussion at CERN when this study was being performed [12].

For simplification we consider in a first step a single corrector SQ1 (respectively SQ2) located $\pi$ $[\pi]$ ($\pi/2$ $[\pi]$) away from the IP with strength $(RL)_{SQ1}$ (resp. $(RL)_{SQ1}$)and write, $Q_y(\phi^* - \phi_{SQ1}) = \pi + \epsilon_1$ $[\pi]$ ($Q_y(\phi^* - \phi_{SQ2}) = \pi/2 + \epsilon_2$ $[\pi]$). After some algebra Eq. 17 becomes

$$\sin(\pi Q_y)\frac{\Sigma_q}{D_x\sqrt{\beta_y}} = \cos(\pi Q_y - \epsilon_1)(RL)_{SQ1} - \sin(\pi Q_y - \epsilon_2)(RL)_{SQ2} \tag{18}$$

We now neglect the slight departure of the cell tune from $\pi/2$ (e.g., by up to about 2 deg in V5.0 optics) which entails $\epsilon_1 \approx \epsilon_2 \equiv \epsilon$ and, by identification between the left and right hand side terms in the equality above leads to

$$\left\{ \begin{array}{c} \cos(\epsilon)\,(RL)_{SQ1} + \sin(\epsilon)\,(RL)_{SQ2} = 0 \\ \sin(\epsilon)\,(RL)_{SQ1} + \cos(\epsilon)\,(RL)_{SQ2} = \frac{\Sigma_q}{D_x\sqrt{\beta_y}} \end{array} \right. \tag{19}$$

which leads to Eq. (14). If SQ1 (resp. SQ2) is a pair of skew quadrupoles $\pi/Q_y$ $[\pi/Q_y]$ distant both share the total strength with alternate signs. In particular Eq. (19) above leads to

$$\Big[\sum_{SQ1}|RL|_{SQ}\Big]^2 + \Big[\sum_{SQ2}|RL|_{SQ}\Big]^2 = \Big[\frac{\Sigma_q}{Dx\sqrt{\beta_y}}\Big]^2 \tag{20}$$

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