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Electron Optics in Hybrid Photodetectors in Magnetic Fields

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Introduction:

The CMS detector design has the hadronic calorimeter immersed in a 4 T magnetic field. The scintillator photon transducer must work reliably in this environment. The baseline phototransducer is the "hybrid photomultiplier", which consists of a standard photocathode (S20) followed by a high field acceleration onto the surface of a Si diode. Such a device has a linear response, 1 e out for every 3.6 eV of potential drop in excess of the threshold needed to penetrate the passivation layer of the diode [1]. A threshold voltage of ~ 2 kV is typical of these devices, leading to a gain of ~ 2000 at 10 kV applied voltage.

In the interest of reducing costs, the Si surface can be cut into pixels. However, the optics of the electron trajectories must be well understood so as to avoid crosstalk between pixels caused by misalignment of the accelerating electric field and the axis of the CMS magnetic field [2]. The depletion depth of the Si is quite standard, ~ 300 um. The source capacity is ~ 20 pF. The output pulse has a ~ 6 nsec risetime for > 60 V diode biasing. The device is expected to be highly immune to magnetic field effects due to the short spacing, ~ 3 mm, between photocathode and Si.

Equations of Motion:

The equations of motion are simply the 3-D form of the Lorentz force equation. The motion is nonrelativistic, which leads to simple expressions. We take as coordinates z along the electric field direction, the magnetic field in the (y,z) plane, inclined by an angle theta with respect to the z axis, and no field in the x direction. The cyclotron frequency is w, and the acceleration that an electron would experience in the absence of a magnetic field is a.

$$d2x/dt2 = w*[dy/dt*ct - dz/dt*st] \qquad (1)$$
$$d2y/dt2 = w*[\qquad - dx/dt*ct]$$
$$d2z/dt2 = w*[dx/dt*st \qquad] + a$$
$$st = \sin(theta),\ ct = \cos(theta)$$
$$w = eB/m$$
$$a = e(Vo/d)/m$$

For B = 4T, Vo = 10 kV, and d = 3 mm one finds that $w = 6.8 \times 10^{11}$ /sec, $a = 5.86 \times 10^{20}$ mm/sec^2.

The solution to this set of second order equations is straightforward and is given below. The "natural" length scale is a/w^2, and the "natural" angular scale is $w*t$, the rotation angle of the electron momentum in the absence of an electric field. As a simplification, knowing that the electron velocities after photoejection are small on the scale of kV, we adopt the approximation that both the initial positions and velocities are zero.

$$x = -(a/w^2)*st*(p - \sin(p)) \qquad (2)$$
$$y = (a/w^2)*st*ct*[(p^2)/2 + (\cos(p) - 1)]$$
$$z = (a/w^2)*[st^2*(1 - \cos(p) + ct^2*(p^2)/2]$$
$$p = w*t$$
$$x(0) = y(0) = z(0) = dx/dt(0) = dy/dt(0) = dz/dt(0) = 0.$$

The solutions are not particularly transparent. In order to build up some intuition, consider the case where there is no magnetic field. In that case, to is the time to fall to z = d, and the final velocity at z = d is vo. In the other limiting case without electric field, but with velocity vo, one has helical motion with a radius of curvature about the axis of the b field of ro.

$$
\begin{aligned}
&to^2 = (2*d/a) \qquad\qquad (3)\\
&(vo/c)^2 = (2*e*Vo)/m\\
&ro = (m*vo)/0.3*B
\end{aligned}
$$

For the case without magnetic field, for Vo = 10 kV, and d = 3 mm, one finds a final velocity of vo/c = 0.198, achieved at z = d at a time, to = 0.1 nsec. In the case of a 4 T magnetic field, an electron with velocity vo has a radius of curvature, ro, of 0.084 mm. In a 4T field, after a time to the momentum has rotated through an angle p = w*to = 68 radians, or about 11 full rotations. Given the numerical estimates, we expect nonrelativistic velocities, time scales in tenths of nsec, small radii of curvature on the scale of the electrode spacing, d, and tight helical orbits with many rotations about the direction of the magnetic field.

Results Relevant to CMS:

The numerical estimates given above lead one to expect that the slow e will be captured along the magnetic field direction, and rotate in tight helices. The time to get to z = d can be approximated by the time in the absence of the magnetic field (for small field inclination angles). The motion in x is expected to be small, since there is no magnetic or electric field in the x direction, by assumption. This expectation is borne out as shown in Fig. 1. The points plotted are those for all time , $0<t<to$, showing that taking the z motion to be solely due to the electric field is a sufficient approximation. The 11 rotations of the helix are seen, as is the smallness of the magnetic field effect. The maximum displacement is 0.016 mm.

The situation for motion along the y axis is rather different. A first view is shown in Fig. 2. Clearly, the main effect - for nonzero angles of magnetic field with respect to the electric field axis - is for the electron to be captured along the magnetic field direction. The linear relation, y = z*tan(theta) is a good representation of the trajectory in the (y,z) plane shown in Fig. 2. The "fine structure" in the (y,z) trajectory becomes evident when the main dependence is subtracted off, as shown in Fig. 3. The ~ 11 helical rotations are observed, with an amplitude ~ 0.0005 mm [3,4]. Clearly, the electrons are tightly captured to the magnetic field, with only minimal "jitter" due to the radius of curvature.

Shown in Fig. 4 are the (y,z) trajectories for 2 different angles of inclination of the magnetic field with respect to the electric field axis. The relationship, y = z *tan(theta) appears to represent the exact solution rather well, as the radius of curvature is almost unobservable in Fig. 4.

Thus the HPD electron optics are fairly simple. The transit time of the device can be roughly calculated using only the electric field and the total z distance between photocathode and Si diode. The transit time is not greatly effected by the magnetic field. The trajectory is, leading to an image shift by d*tan(theta). This shift must be minimized in pixel devices by aligning the magnetic field to the electric field axis. Displacements perpendicular to the field plane are minimal.

References:

1) Hybrid Photomultiplier Tubes, DEP Delft Instruments

2) Technical Proposal, CMS Collaboration, CERN/LHCC 94-38 (1994)

3) P. Cushman - private communication

4) A. Rhonzin - private communication

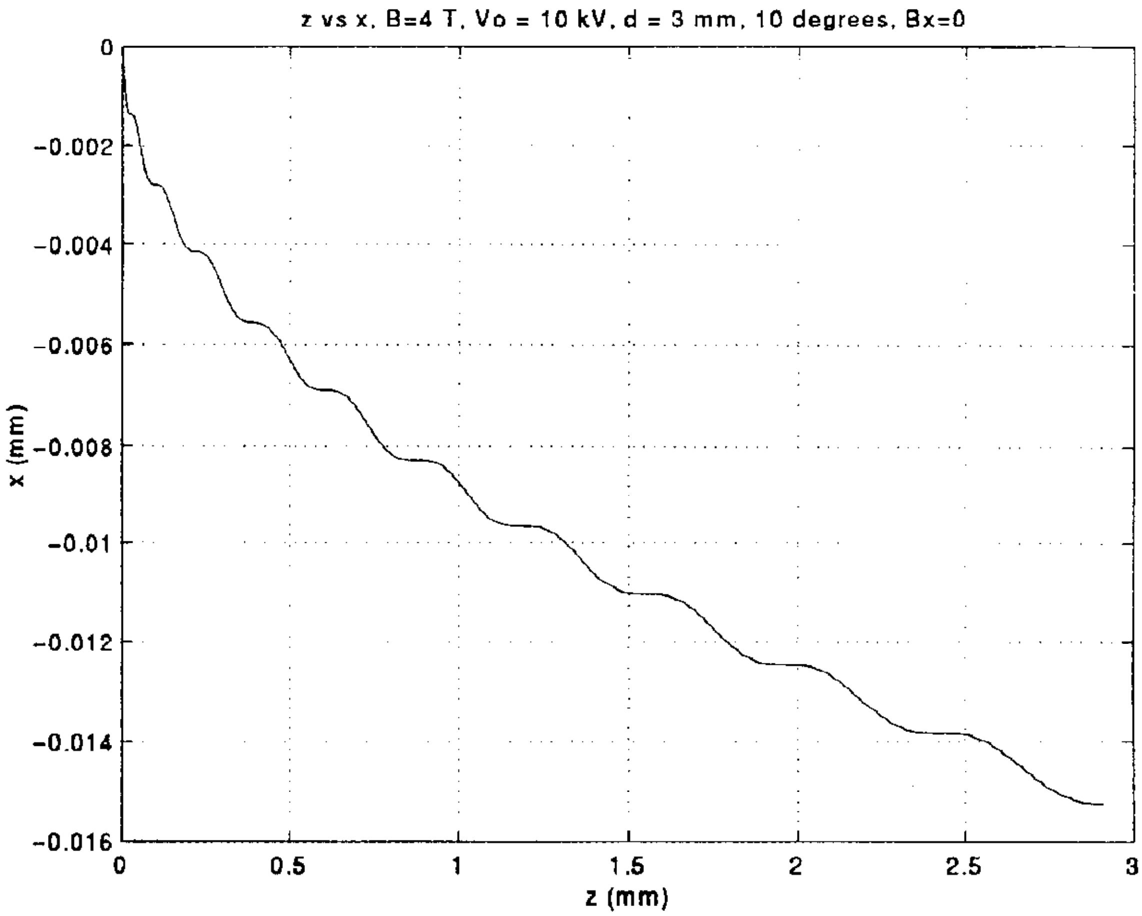

Figure 1: The trajectory z (electric field direction) vs x (Bx=0) for a 4T magnetic field in the (y,z) plane at angle of 10 degrees to the z axis. The field is Vo = 10 kV over d = 3 mm. The maximum deviation is 0.016 mm. The ~11 cyclotron rotations in the field modulating the trajectory are quite evident.

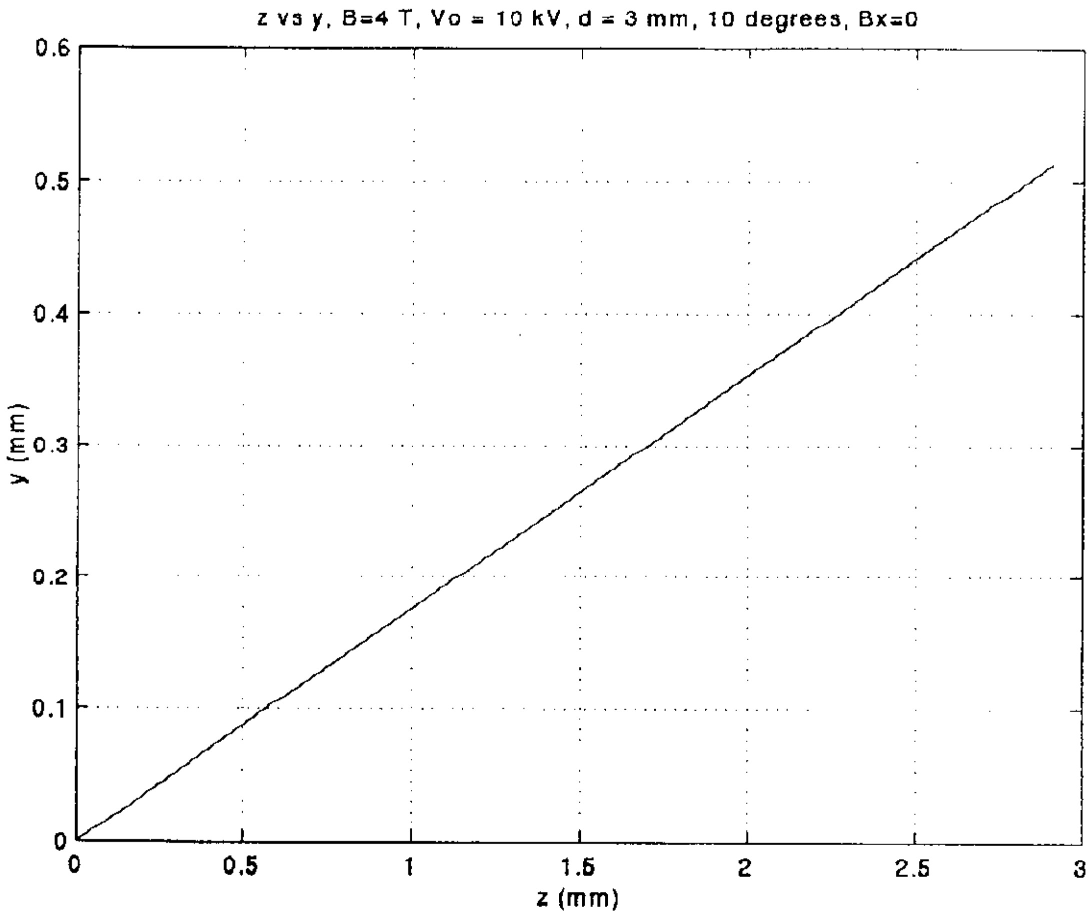

Figure 2: The trajectory for z vs y in the same situation as obtains in Fig.1. The maximum deviation is ~ 0.5 mm in y after the e drops through 10 kV in 3 mm in the z direction.

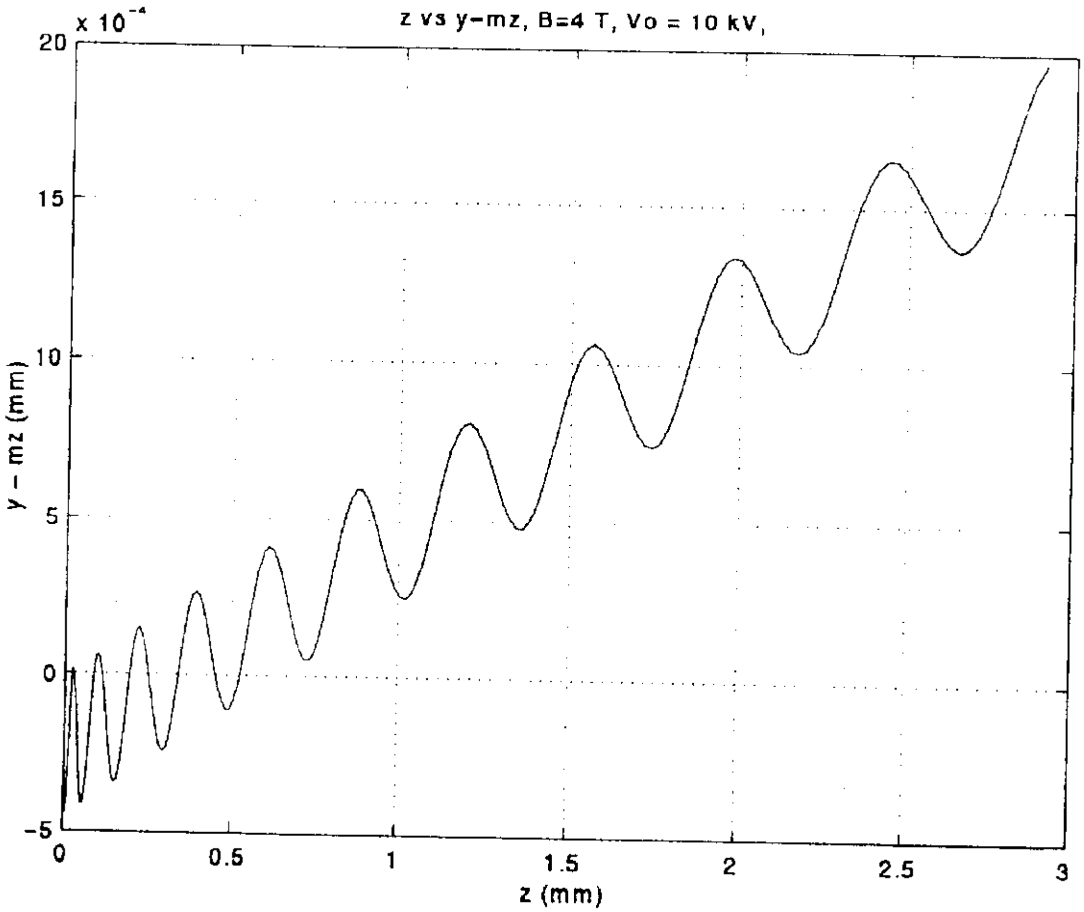

Figure 3: Same as Fig. 2 but with the relationship y = d*tan(theta) roughly subtracted off the y trajectory. The residual cyclotron rotation with amplitude ~ 0.0005 mm is evident.

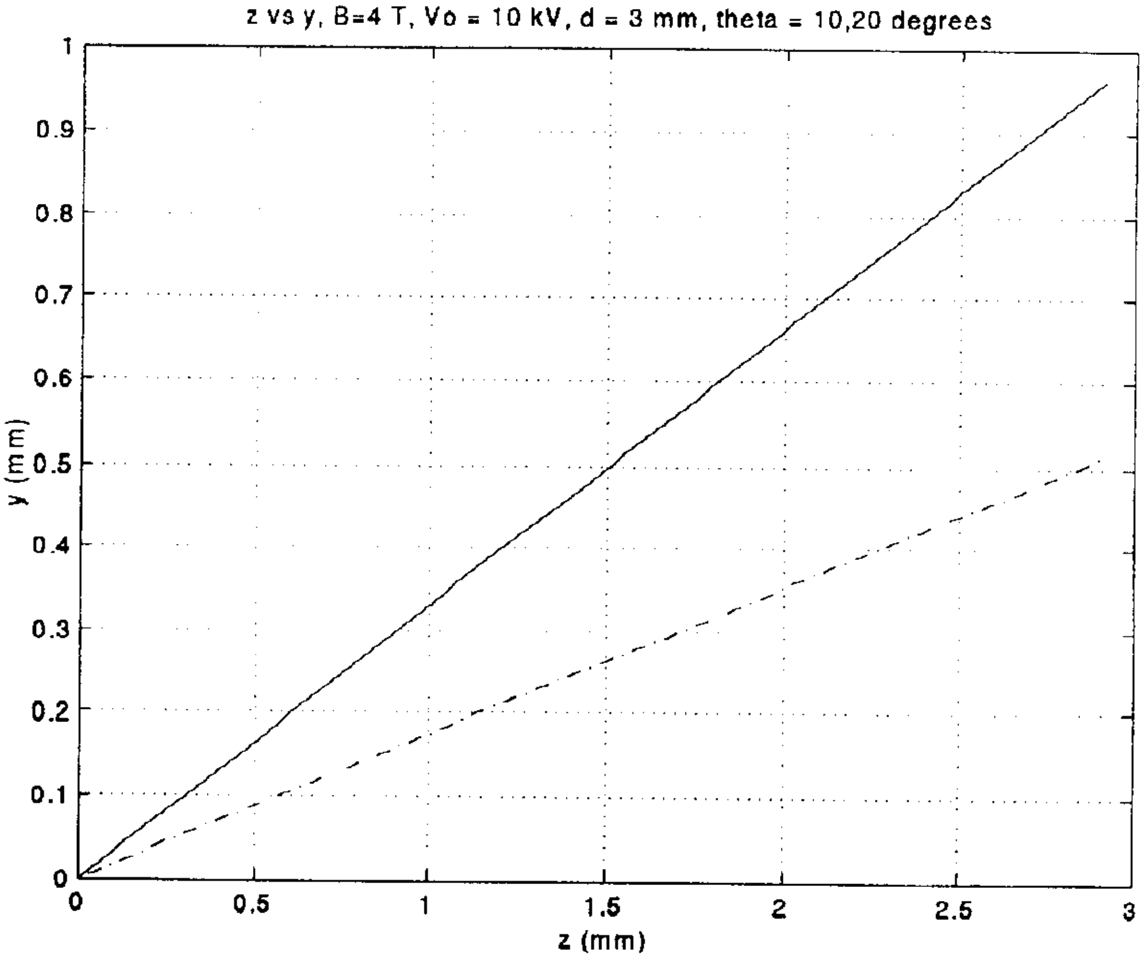

Figure 4: As in Fig. 2 except 2 different inclination angles are shown, 10 and 20 degrees. The expected relationship, y = d*tan(theta) is observed.