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Introduction

Recently, it has been suggested[1] that improved technology and reduced costs in remotely-drilled small-diameter tunnels, coupled with improvements in robotic technology, may make the original concept of the "desertron"[2] more realistic and affordable. In this concept, a long, small-diameter tunnel is drilled (<~1m diameter "sewer" pipe) and filled with long, low-cost magnets, which are installed and serviced robotically. To obtain high-energy then requires low cost magnets, which are iron-dominated "superferric" magnets (B~2 T). A large circumference is then required(~1000 km for ~100 TeV/beam). Table 1 shows parameters for a 200TeV proton-proton collider, based on the premise of a large low-cost ring with super-ferric magnets.

While outline designs for a low-cost ~2T dipole have been initiated, an accelerator requires beam stability, which means quadrupole fields for focusing, as well as sextupoles for chromatic correction, and further design tolerances and correctors to obtain sufficiently linear fields. Previously we have developed initial lattices and dynamic motion discussions for the earlier 40 TeV incarnation of the superferric supercollider.[3] In this note we apply those results to initiate discussions of the dynamic requirements of this 200 TeV collider.

Some comments on the parameter table are in order. To maintain a reasonable rate of point-like interactions, the luminosity should increase with the square of the interaction energy: $L \propto E^2$. With this scaling from the LHC/SSC baselines, the 200 TeV collider should have an ultimate design goal of $L \sim 10^{35}$ cm^{-2}s^{-1}, and actually more luminosity would be desired (up to 10^{36}). (The 10^{35} luminosity corresponds to a 10^{31} luminosity in the 2 TeV Tevatron.) The luminosity L is given by the usual formula:

$$L = \frac{f_0 n_B \gamma N^2}{4\pi\varepsilon_N \beta_\perp} ,$$

where the various parameters are defined in table 1. The collider parameters are constrained in the beam-beam tune shift $\Delta\nu$:

$$\Delta\nu = \frac{r_p N}{4\pi\varepsilon_N} .$$

where the (optimistic) constraint $\Delta\nu < 0.005$ is used. r_p is the classical proton radius, N is the number of protons per bunch, and ε_N is the normalized emittance. We have used the optimistic SSC value of $\varepsilon_N = 10^{-6}$, which is 4× smaller than current Tevatron values. (A conservative design should accommodate the larger values in setting aperture acceptances.) We have used 0.1m for the focussing parameter $\beta_\perp$, an extrapolation from SSC/LHC values of ~0.5m.

It is fairly straightforward to construct parameters for a high-luminosity megacollider. One parameter that becomes unavoidably large is the number of interactions per crossing: ~1000 from Table 1 (compared to ~10 for LHC). Also stored beam energy becomes large in this high-energy large-circumference ring: $2n_BNE$ or ~32GJ at the parameters of table 1.

Cells

Most of the lattice will be in alternating gradient (AG) FODO cells. As in previous accelerators a design choice between separated-function and combined-function machines must be made.

In a combined-function lattice, the quadrupole (and possibly first-order sextupole) functions are included in the bending dipoles, which means that these magnets must include a field gradient, which must alternate in sign. For ferric magnets, this means a hyperbolic tilt from rectangular in the pole tip orientation, which must alternate in sign from F to D dipoles. An additional curvature would add sextupole capabilities. For practicality we would like the gradient component of the magnets to be small compared to the bending dipole component. If we set a 10% limitation, and use 2T magnets with 1cm half gaps, this implies that we want gradients of < 20 T/m.

In a separated function lattice, the dipoles only include bending fields, and focussing and chromatic correction are provided by separate quadrupoles and sextupoles located in gaps between the dipoles, in a FODO cell configuration, where FODO refers to a configuration consisting of a focussing quad, long bending dipole(s) (O), and a defocussing quad followed by further dipoles (O). For ferric quads with 1cm radius apertures, gradients of 200T/m can be obtained. For practicality we require that the quad length/cell be small compared to the total dipole length (<~5%). This is naturally similar to the combined-function gradient limit. An important, and often decisive, advantage of separated function lattices is that they are insensitive to dipole alignment.

In tables 1—2 we have compiled FD and FODO cell parameters for various combined-function and separated-function lattices for rings for 100 TeV beams (1000 km circumference). Under the above limitations on focussing strengths, we require cell half-lengths of ~ 250 m for the 100 TeV ring (or longer) At these parameters maximum dispersions (η_{max}) are ~1—2m and maximum betatron functions are β_{max} ~ 700—900m. (S. Holmes has obtained similar lattice parameters.)

This dispersion is sufficiently small that even beam particles at $\Delta E/E \sim 10^{-3}$ have only ~1—2mm amplitude motions, and we expect beams of somewhat smaller $\Delta E/E$ in these rings. Similarly a beam with normalized emittance of $\varepsilon_N = 4\times10^{-6}$ (~existing Tevatron emittances) would have an rms beam size of $\sigma \sim 1$mm at 4 TeV (a possible injection energy). This is not so small when one considers that one needs beam acceptances of several σ, which implies that a good field aperture of at least several mm is required. Note that beam sizes do not decrease quickly with increasing beam energy. The relevant relationship is:

$$\sigma = \sqrt{\frac{\varepsilon_N \beta_{max}}{\gamma}}$$

In this expression β_{max} may be expected to increase as $E^{1/2}$, where E is the ring design energy. γ is simply proportional to E, so beam size decreases only following $\sigma \propto E^{-1/4}$. Beam size could decrease if emittance were decreased.

Closed orbits can be estimated from the equation:[4]

$$\hat{x} = 2.4\pi f(\nu)\sqrt{\frac{\beta_{max}\bar{\beta}}{N_D}}\frac{\Delta B_{rms}}{B}$$

where N_D is the number of dipoles($N_D \sim 4000$), β_{max}, β are the maximum and average betatron functions, ΔB_{rms} is the rms residual uncorrected error field, and $f(\nu)$ is a lattice-dependent factor of order unity. The factor of 2.4 sets the closed orbit amplitude limit x at the 98% statistical level. At $\Delta B/B \sim 10^{-5}$, this formula obtains ~1mm. This indicates that the minimal desired aperture should be ~1mm or more in the 200 TeV collider. Note that because of the dependence on the betatron functions β_{max}, β, the residual closed orbit amplitude would not be expected to decrease with increasing final beam energy.

IR Region

The other major optical feature of the megacollider is the IR region, where the beam is focussed to small spots. This region would be very similar to the SSC IR regions, and a triplet of quads could be used to obtain the IR focus. High-field quads should be used (up to ~10T or more). Asssuming equal-gradient quads the lengths of the IR quads would simply scale as $E_{final}^{1/2}$. Assuming maximum gradients of 333 T/m, the focusing triplets are ~100m long for the 200 TeV Collider and even after allotting a km or two for matching from the arcs, the IR regions are only a small fraction of the total circumference. The betatron functions will become very large in the IR quads, and these quads would be very sensitive to field errors. For typical low beta focuses of 0.1—1.0m, the maximum beta function would be 100,000—10,000m. These parameters do not seem particularly difficult.

Field Quality Requirements

Following similar estimates used for the SSC,[5] we require a linear aperture at ~0.5 cm amplitudes. The size scale for the linear aperture requirements is set at the amplitude scale for closed orbit distortions, which increases in comparison with the SSC. Intrinsic beam size would be less if we accept the SSC emittance, but becomes larger if we accommodate the Tevatron emittance. From comparisons, the 0.5cm at β_{max} (0.4cm$_{rms}$)appears a minimal choice for a linear aperture. The momentum aperture requirement has been reduced from the SSC value of 10^{-3} to 0.5×10^{-3}, and our lattice does have smaller dispersion (η) than the SSC. However, we do note that $\Delta E/E >\sim 10^{-4}$ is required to avoid the longitudinal microwave instability.

Setting a linear aperture by the tune shift criteria is rather approximate and may exaggerate the systematic multipole limits and underestimate the random multipole limits. However we do need long beam lifetime, and the expectation is that linear motion is needed for long beam lifetime. SSC tracking results (from Y. Yan) do demonstrate that indefinitely long beam stability is obtained within the linear aperture region given by the linear aperture criteria.[6]

Results of the linear aperture screening are displayed in table 4, for a representative ring with $L_{½cell}$ = 250m, μ=60° lattice. Note that systematic quad and sextupole errors can be corrected by adjusting quads and sextupoles, if they exist. Also, systematic b_3 and b_4 could be corrected by suitably placed correctors.[7] In general, avoidance or correction of systematic multipoles at the 10^{-5} level and of random multipoles at the 10^{-4} level appears desirable. This level is consistent with the closed orbit evaluations. This is ~one order of magnitude better than that readily achieved in small-aperture magnet design, but is also only one order of magnitude, and is plausibly within reach of design improvements. I do note that, unlike previous dipoles, the large-collider dipole symmetry, as presented in ref. 1, does allow systematic octupole (b_3), and that could cause difficulty if not corrected.

The quad (b_1) tolerance (0.01) is simply the degree to which the quadrupole component of the dipoles must be controlled to maintain the machine tune within listed tolerances. The b_1 of a combined-function dipole would be 440 units. Typical quad components of ferric dipoles fluctuate by ~1 unit over their operating range. The tolerance of 0.01 implies that trim quads are needed to compensate these systematic fluctuations.

The listed tolerance on b_2 is also approximately the tolerance one needs to set the sextupole component for chromaticity, if one is relying on pole tip shaping to correct chromaticity in a combined function lattice. A more direct calculation shows that the sextupole component needed to correct chromaticity is $b_2 \sim 1.5\times10^{-4}$ cm^{-2}. Typical dipole designs fluctuate in b_2 by at least that much over their operating range, and it therefore appears impractical to rely on b_2 for chromaticity correction; separate correcting sextupoles should be added.

References

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Table 1: Parameter list for a 200 TeV p-p Collider

Parameter	Symbol	Value
Energy per beam	E_p	100 TeV
Luminosity	$L=f_0\, n_B\, N_p^2/4\pi\sigma^2$	10^{35} $cm^{-2}s^{-1}$
Number of p/bunch	N_p	4.1×10^{10}
Number of bunches	n_B	2.5×10^4
Collision frequency	f_o	300 Hz
Circumference	$2\pi R$	1000000m
Normalized emittance	ε_N	10^{-6} m-rad
p-beam emittance	$\varepsilon_t=\varepsilon_N/\gamma$	10^{-11} m-rad
Interaction focus	β_0	0.1 m
Beam size at interaction	$\sigma = (\varepsilon_t\beta_0)^{1/2}$	1 μm
Beam-beam Tune Shift	$\Delta\nu$	0.005

Table 2: FD (combined function) Lattice Cell parameters for 100 TeV proton beam (1000km circumference)

$L_{½cell}$(m)	$\theta_{½cell}$°	μ_{cell}°	β_{max}	β_{min}	η_{max}	B′(T/m)	ν(machine)
200m	0.072	90	612m	127m	0.65m	20.6	625
200	0.072	60	644	246	1.22	14.9	417
200	0.072	30	971	613	4.17	15.3	209
250	0.09	90	765	160	1.02	12.6	500
250	0.09	60	805	307	1.90	8.85	333
250	0.09	30	1214	766	6.51	4.57	167
300	0.108	90	919	191	1.46	9.2	417
300	0.108	60	965.4	368	2.74	6.6	278
300	0.108	30	1456	919	9.38	3.4	139

Table 3: FODO (separated-function) Lattice Cell parameters for 100 TeV beam (1000km circumference)

B′ assumes quad length is 5% of $L_{½cell}$

$L_{½cell}$(m)	$\theta_{½cell}$°	μ_{cell}°	β_{max}	β_{min}	η_{max}	B′(T/m)	ν(machine)
200m	0.072	90	683m	117m	0.68m	236	625
200	0.072	60	693	231	1.26	167	417
200	0.072	30	1007	593	4.24	86	209
250	0.09	90	853	146	1.06	151	500
250	0.09	60	866	289	1.96	107	333
250	0.09	30	1259	741	6.62	55.3	167
300	0.108	90	1024	176	1.53	105	417
300	0.108	60	1039	346	2.83	74.1	278
300	0.108	30	1511	889	9.53	38.4	139

Table 4: Field Quality Requirements

The field quality requirements here are based on the tune shift criteria: $\Delta\nu < 0.005$ at an rms amplitude of 0.4 cm and $\delta = 0.0005$. Tolerances are in units of 10^{-4} at 1cm. Random tolerances are obtained from systematic tolerances simply by multiplying by a factor of $(4000)^{1/2}$.

Multipole	$\Delta\nu$	Tolerance (systematic) $10^{-4}cm^{-n}$	Tolerance (random) $10^{-4}cm^{-n}$
b_1	$\beta b_1/2$	0.0017	0.11
b_2	$\beta b_2 \eta\delta$	0.011	0.70
b_3	$3\beta b_3(\eta^2\delta^2/2 + A_x^2/8)$	0.012	0.76
b_4	$\beta\eta\delta b_4(2\eta^2\delta^2 + 3A_x^2/2)$	0.042	2.7
b_5	$5\beta b_5(\eta^4\delta^4/2 + 3A_x^2\eta^2\delta^2/4 + A_x^4/16)$	0.067	4.2
b_6	$3\beta\eta\delta b_6(\eta^4\delta^4 + 5A_x^2\eta^2\delta^2/2 + 5A_x^4/8)$	0.185	12