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The Effect of Beam Intensity on the Estimation Bias of Beam Position

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Abstract

For the signals of the beam position monitor (BPM), the signal-to-noise ratio (SNR) is directly related to the beam intensity. Low beam intensity results in poor SNR. The random noise has a modulation effect on both the amplitude and phase of the BPM signals. Therefore, the beam position measurement has a certain random error. In the currently used BPM, time-averaging and waveform clipping are used to improve the measurement. The nonlinear signal processing results in a biased estimate of beam position. A statistical analysis was made to examine the effect of the SNR, which is determined by the beam intensity, on the estimation bias. The results of the analysis suggest that the estimation bias has a dependence not only on the beam position but also on beam intensity. Specifically, the dependence gets strong as the beam intensity decreases. This property has set a lower limit of the beam intensity range which the BPM's can handle. When the beam intensity is below that limit the estimation bias starts to vary dramatically, resulting in the BPMs failure. According to the analysis, the lowest beam intensity is that at which the SNR of the generated BPM signal is about 15 dB. The limit for NSEP BPM, for instance, is about 6E11. The analysis may provide the BPM designers with some idea about the potential of the current BPM's.

I. Introduction

The general effects of Gaussian noise on the performance of the BPM's using an analog phase detector were analyzed in reference [1]. The analysis was based upon a numerical simulation under the condition of wide band noise. The result indicates that the measurement accuracy of BPM's degrade as beam intensity decreases as predicted. In practice, the noise in the BPM's is narrow band due to the multistage band-pass filters in the systems. To see the specific behavior of the current BPM's under the condition of narrow-band noise, a statistical analysis is to be made in this note. The procedure of the analysis is to first derive the mathematical expression of the phase detector outputs. Next, the statistical average of the output is calculated between the amplitudes of the signals A and B from the detector. The estimated phase difference is then converted into the estimate of the beam position. Finally, the bias of the beam position estimation is to be observed, which may be considered as an indicator of the BPM performance.

II. Beam Position Estimation

1. Quadrature Expression of Narrow-Band Noise

As well known, the noise of a narrow-band system n(t) may be expressed in the form of signal envelope

$$n(t) = A \cdot \mathrm{Cos}(\omega t + \theta)$$

Where

A is the random amplitude of Rayleigh distribution,
θ is the random phase of uniform distribution $(-\pi, +\pi)$,
ω is the central frequency of the pass-band.

It can be shown that the narrow band noise n(t) may also be expressed in a quadrature form [3]

$$n(t) = n_c(t) \cdot \mathrm{Cos}(\omega t) - n_s(t) \cdot \mathrm{Sin}(\omega t)$$

Where the random variable (or process) nc(t) and ns(t) are stationary, Gaussian, and uncorrelated with each other (also statistically independent in the case of Gaussian process). Their distribution has zero-mean and variance N, that is, the normal distribution (0,N).

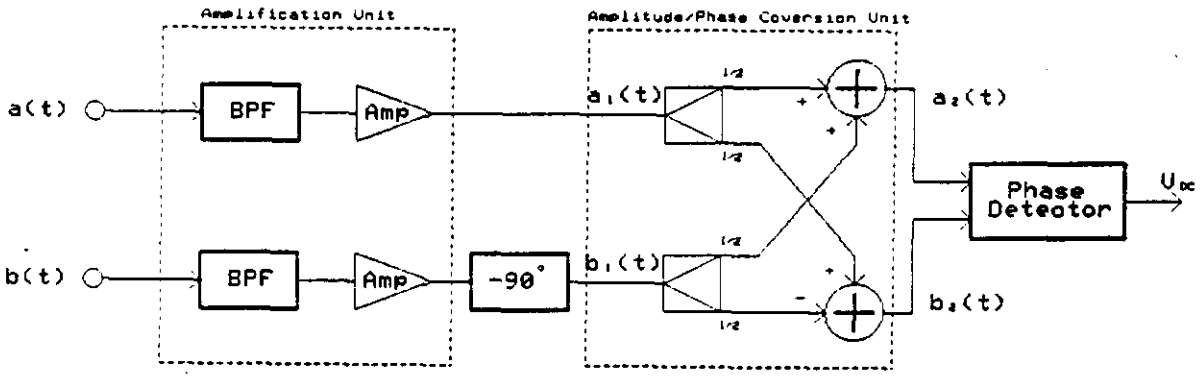

Figure 1. Block Diagram of a BPM System

2. Procedure of Beam Position Detection

To detect the difference between the amplitudes of the A channel signal and the B channel signal, the signals are first processed by an amplitude/phase converter. Ideally, when no noise is present, the amplitude/phase converter should convert the $a_1(t)$ and $b_1(t)$ signal into

$$a_2(t) = A_2 \cdot \mathrm{Cos}(\omega t + \theta), \quad \text{and} \quad b_2 = B_2 \cdot \mathrm{Cos}(\omega t - \theta)$$

$$A_2 = B_2 = \sqrt{A^2 + B^2}$$

where the tangential of the phase θ is solely determined by the amplitude A and B through Tan(θ)=A/B.
The beam position x can finally be calculated from the relation

$$x = \frac{20}{0.67}\mathrm{Log}\left(\frac{A}{B}\right) = \frac{20}{0.67}\mathrm{Log}(\mathrm{Tan}(\theta))$$

3. Estimating Beam Position From Noisy Signals

Back to the practical situation, the noisy inputs of the amplitude/phase conversion stage may be expressed as

$$a_1(t) = A \cdot \mathrm{Cos}(\omega t + \theta) + n_a(t) \tag{4}$$

$$b_1(t) = B \cdot \mathrm{Cos}(\omega t - \tfrac{\pi}{2}) + n_b(t) = B \cdot \mathrm{Sin}(\omega t) + n_b(t) \tag{5}$$

where $n_a(t)$ and $n_b(t)$ are narrow band random noise in channels A and B.
If we substitute the quadrature expression of narrow band noise into the two equations, then

$$a_1(t) = A \cdot \mathrm{Cos}(\omega t) + n_{ca}(t) \cdot \mathrm{Cos}(\omega t) - n_{sa}(t) \cdot \mathrm{Sin}(\omega t) \tag{4}$$

$$b_1(t) = B \cdot \mathrm{Sin}(\omega t) + n_{cb}(t) \cdot \mathrm{Cos}(\omega t) - n_{sb}(t) \cdot \mathrm{Sin}(\omega t) \tag{5}$$

The amplitude/phase converter transforms the signal amplitude difference into the phase difference. Thus, two outputs, $a_2(t)$ and $b_2(t)$ should be

$$a_2(t) = \tfrac{1}{2}a_1(t) + \tfrac{1}{2}b_1(t)$$

$$= \tfrac{1}{2}[A + n_{ca}(t) + n_{cb}(t)]\mathrm{Cos}(\omega t) + \tfrac{1}{2}[B - n_{sa}(t) - n_{sb}(t)]\mathrm{Sin}(\omega t)$$

$$b_2(t) = \tfrac{1}{2}a_1(t) - \tfrac{1}{2}b_1(t)$$

$$= \tfrac{1}{2}[A + n_{ca}(t) - n_{cb}(t)]\mathrm{Cos}(\omega t) - \tfrac{1}{2}[B + n_{sa}(t) - n_{sb}(t)]\mathrm{Sin}(\omega t)$$

Express a2(t) and b2(t) in the envelope form, then

$$a_2(t) = A_2 \cdot \mathrm{Cos}(\omega t + \theta_a) \tag{12}$$

$$b_2(t) = B_2 \cdot \mathrm{Cos}(\omega t + \theta_b) \tag{13}$$

Where

$$A_2 = \tfrac{1}{2}\sqrt{[A + n_{ca}(t) + n_{cb}(t)]^2 + [B - n_{sa}(t) - n_{sb}(t)]^2} \tag{14}$$

$$B_2 = \tfrac{1}{2}\sqrt{[A + n_{ca}(t) - n_{cb}(t)]^2 + [B + n_{sa}(t) - n_{sb}(t)]^2} \tag{15}$$

$$\mathrm{Tan}(\theta_a) = t_a = \frac{A + n_{ca}(t) + n_{cb}(t)}{B - n_{sa}(t) - n_{sb}(t)} \tag{16}$$

$$\mathrm{Tan}(\theta_b) = t_a = \frac{A + n_{ca}(t) - n_{cb}(t)}{B + n_{sa}(t) - n_{sb}(t)} \tag{17}$$

In the analog phase detector used in the current BPM's, the information contained in the signal envelope is discharged by waveform clipping, and only the information in the instantaneous phase is utilized for estimating the wanted amplitude ratio A/B. Thus, the averaged phase tangential is estimated from the phase detector in the form

$$\mathrm{Tan}(\theta_{avg}) = \frac{\mathrm{Tan}(\theta_a) + \mathrm{Tan}(\theta_b)}{2} \tag{18}$$

The value $\mathrm{Tan}(\theta_{avg})$ is comprised of the true value of $\mathrm{Tan}(\theta)$ and the random error due to the noise at the moment t. To get the estimate of the $\mathrm{Tan}(\theta)$, a statistical average is taken (in the BPM's, time-average or low-pass filtering is used to obtain the statistical mean.). Thus, the estimate of the $\mathrm{Tan}(\theta)$ is

$$E\{\mathrm{Tan}(\theta)\} = \frac{E\{\mathrm{Tan}(\theta_a)\} + E\{\mathrm{Tan}(\theta_b)\}}{2} \tag{19}$$

With the obtained estimate of the phase tangential, the estimate of the amplitude ratio and further the beam position can be therefore calculated from the relation

$$E\left\{\frac{A}{B}\right\} = E\{\mathrm{Tan}(\theta)\}$$

$$E\{x\} = \frac{20}{0.67}\mathrm{Log}\left[E\left\{\frac{A}{B}\right\}\right] = \frac{20}{0.67}\mathrm{Log}\left[E\{\mathrm{Tan}(\theta)\}\right] \tag{21}$$

and the bias of the estimate is defined as

$$b_x = E\{x\} - x \tag{22}$$

III. Evaluation of Estimate Bias

To evaluate Equation(21) and (22), one must know the probability distribution functions (pdf) of all random variables concerned. It is generally difficult. Fortunately, most random variables we deal with have Gaussian distribution. This has made our life much easier. For simplicity, let us rewrite Eq. (16) and (17)

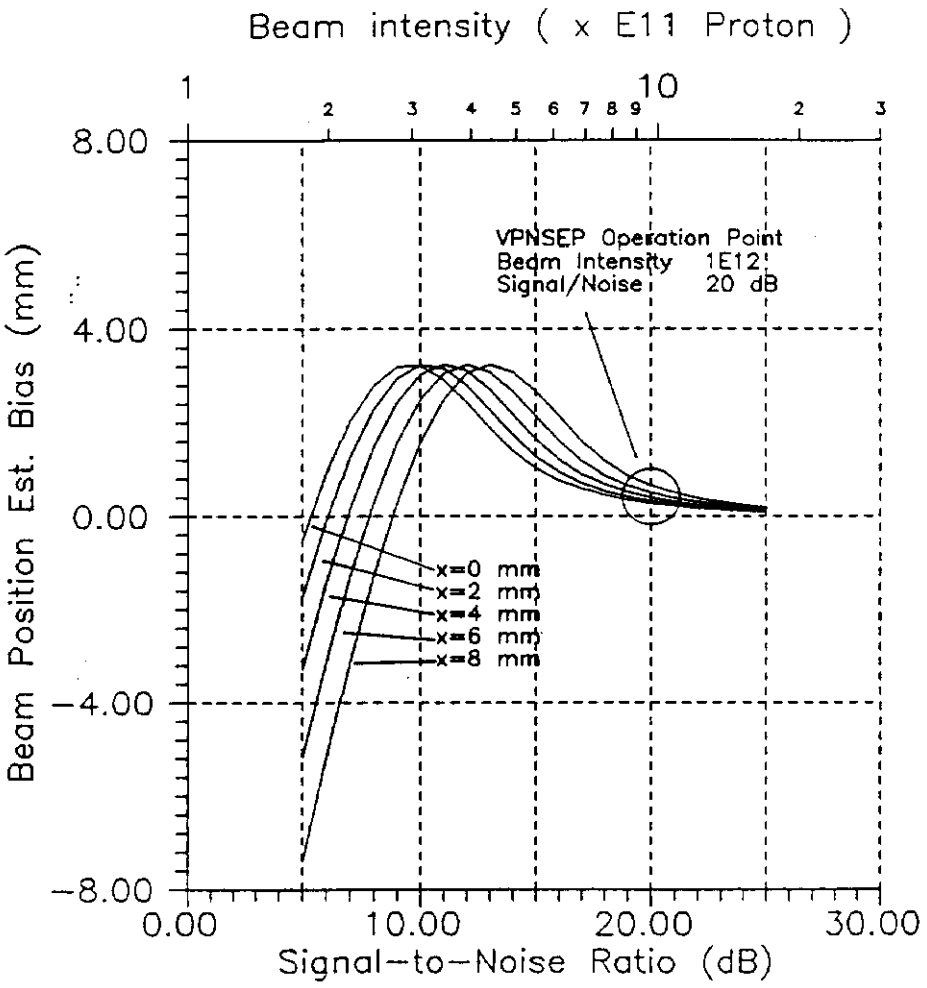

Figure 2. The dependence of beam position estimation bias on the beam intensity and beam position. (Note: The beam intensity axis is scaled based upon the observation taken on the NSEP BPM)

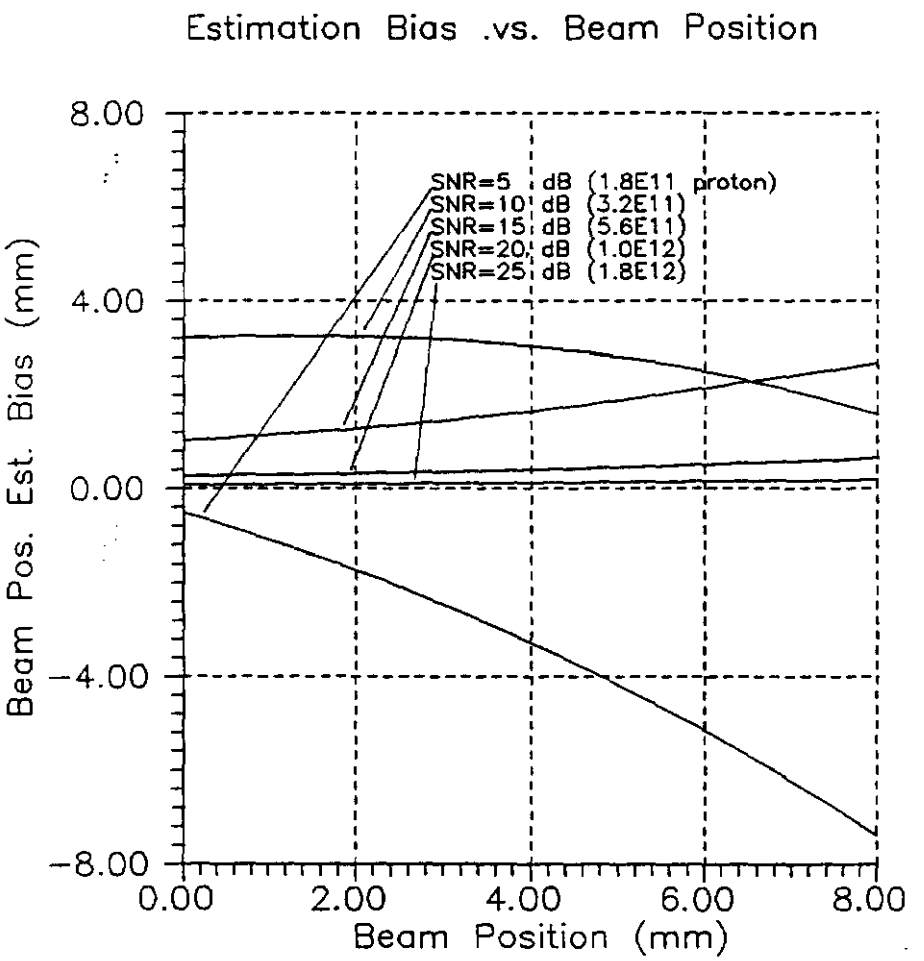

Figure 3. The variation of the estimation bias as function of the beam position and intensity.

$$\mathrm{Tan}(\theta_a) = \frac{A + n_{ca}(t) + n_{cb}(t)}{B - n_{sa}(t) - n_{sb}(t)} = \frac{A+u}{B-v} = t_a \tag{23}$$

$$\mathrm{Tan}(\theta_b) = \frac{A + n_{ca}(t) - n_{cb}(t)}{B + n_{sa}(t) - n_{sb}(t)} = \frac{A+w}{B+z} = t_b \tag{24}$$

where

$$u = n_{ca}(t) + n_{cb}(t), \qquad v = n_{sa}(t) + n_{sb}(t)$$
$$w = n_{ca}(t) - n_{cb}(t), \qquad z = n_{sa}(t) - n_{sb}(t)$$

It has been shown that for the narrow band noise $n_a(t)$, the random process $n_{ca}(t)$ and $n_{sa}(t)$ are two uncorrelated Gaussian processes of (0,N). The same conclusion also applies to $n_{cb}(t)$ and $n_{sb}(t)$. We may further assume that the noise of channel A and the noise of channel B are independently generated in their own electronic devices. Thus, the random process u(t) is uncorrelated with the process v(t), and w(t) is uncorrelated with z(t). The four processes are still Gaussian processes with the distribution of (0,2N). Therefore, their pdf's are in the form

$$f_{uv}(u,v) = \frac{1}{\sqrt{2\pi \cdot (2\,N)}} e^{-\frac{u^2+v^2}{2\cdot(2\,N)}} \tag{25}$$

$$f_{wz}(w,z) = \frac{1}{\sqrt{2\pi \cdot (2\,N)}} e^{-\frac{w^2+z^2}{2\cdot(2\,N)}} \tag{26}$$

Substituting Eq. (25) and (26) in Eq. (19), we get

$$E\{\mathrm{Tan}(\theta)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\mathrm{Tan}(\theta_a)}{2} f_{uv}(u,v)\,du\,dv \;+\; \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\mathrm{Tan}(\theta_b)}{2} f_{wz}(w,z)\,dw\,dz$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2}\frac{A+u}{B-v}\frac{1}{\sqrt{2\pi\cdot(2\,N)}} e^{-\frac{u^2+v^2}{2\cdot(2\,N)}}\,du\,dv \;+\; \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2}\frac{A+w}{B+z}\frac{1}{\sqrt{2\pi\cdot(2\,N)}} e^{-\frac{w^2+z^2}{2\cdot(2\,N)}}\,dw\,dz \tag{30}$$

The values of E{Tan(θ)} expressed in Eq. (30) were calculated as the function of SNR and beam position with a numerical method. The signal-to-noise ratio varies from 25 dB to 5 dB, and the beam position varies from 0 to 8 mm. The calculation was performed by first fixing B=1. Then the value of A was determined from the relation x=20/0.67 Log(A/B) with the specified beam position x. Finally, the noise variance N is determined from the specified signal-to-noise ratio SNR=10 Log($(A^2+B^2)/(2N)$). The obtained values were then plugged in Eq. (21) and (22) to get the estimation bias. The results were plotted in Figure 2 and Figure 3 respectively.

IV. Comments

The plots in Figure 2 and 3 clearly indicate that the estimation bias of beam position is dependent of both the beam position and the signal-to-noise ratio. The latter is related to the beam intensity. The SNR is directly proportional to the beam intensity as mentioned before. To get some idea about the scale of the beam intensity to SNR, an observation of the NSEP BPM in G2 service building was taken. It has been seen that when the beam intensity is 1E12, the SNR of the NSEP BPM is about 20 dB.

When the SNR is better than 20 dB (equivalently, the beam intensity is higher than1E12 for NSEP BPM), the maximum bias is less than 1 mm. The contribution of the bias to the measurement error is negligible. Therefore, the BPM's can work effectively under the beam intensity higher than this level.

When the SNR is poorer than 10 dB (equivalently, the beam intensity of about 1E12 for NSEP BPM), the estimation bias varies dramatically as the beam intensity or position changes. Specifically, as the beam intensity changes from 3E11 to 1.8E11, the bias variation can be as high as 8 mm for the beam position x=8mm, and 3.2 mm for x=0mm.

In the same region, the sensitivity of the bias to the beam position $\sigma(\text{bias})/\sigma x$ is about 0.6 for x=0mm, and 1.0 for x=8mm. The bias variation is so sensitive to both the beam intensity and beam position that the beam position measurement becomes very difficult, if not impossible.

Within the transient region of SNR=20 dB to 10 dB (beam intensity=1E12 to 3E11 for the SNEP BPM), the performance of the BPM's is uncertain.

Finally, it should be pointed out that the above analysis is made under the assumption that the electronic and mechanical hardware of the BPM's are perfect. Therefore, the measurement error due to the bias is solely caused by the imperfection in devices also contributes a significant portion to the measurement error. The actual errors should be greater than those calculated in this note. Hence, the performance curves in Figure 2 and 3 actually reflect the maximum potential the current BPM's can reach.

V. References

[1] H. Ma, "A note on the robustness of Switchyard BPM to noise, Part 1," Technical Note #1, Dec., 1991.

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[5] All the Switchyard BPM and Tevatron BPM notes, 1982-present.