A Generalized TRL Algorithm for S-Parameter De-Embedding

P. Colestock and M. Foley

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

April 1993
Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
A GENERALIZED TRL ALGORITHM FOR S-PARAMETER DE-EMBEDDING

P. Colestock
Fermi National Accelerator Laboratory

M. Foley
Fermi National Accelerator Laboratory

FNAL Technical Memo TM-1781

Abstract

At FNAL bench measurements of the longitudinal impedance of various beamline components have been performed using stretched wire methods. The basic approach is to use a network analyzer (NWA) to measure the transmission and reflection characteristics (s-parameters) of the beam line component. It is then possible to recover the effective longitudinal impedance from the s-parameters. Several NWA calibration procedures have been implemented in an effort to improve the accuracy of these measurements. These procedures are mathematical techniques for extracting the s-parameters of a test device from external NWA measurements which include the effect of measurement fixtures. The TRL algorithm has proven to be the most effective of these techniques. This method has the advantage of properly accounting for the nonideal calibration standards used in the NWA measurements.

1 Introduction

The objective of this work is to recover an equivalent impedance for a given device-under-test (DUT) using a bi-directional reflectometer, otherwise known
as a network analyzer (NWA). The basic algorithm consists of applying an incident wave to the DUT, which is characterized as a general two-port network, and measuring the vector voltages scattered into the forward and reverse directions. The resulting data can be used to calculate s-parameters. However, the measurements are complicated by the fact that transitions occur between the NWA and the DUT. The diagram below is a schematic representation of the measurement setup. A and B are general, linear networks representing the errors occurring in the s-parameter measurements of the DUT. The influence of error networks A and B must be removed from the data in order to accurately evaluate the s-parameters of the DUT. Using standard circuit analysis, it is possible to recover the effective longitudinal, as well as transverse, impedance of the DUT from the de-embedded s-parameters.

The method described in this Technical Memo is based on a generalization of the Thru-Reflect-Line (TRL) algorithm [1,2]. The calibration standards required are two lengths of transmission line and two shorts with equal reflection coefficient. The lengths of the transmission lines and the value of the reflection coefficient for the shorts are not required to be known. However, the ratio of the lengths of the two transmission lines is required.

Assuming the transmission lines used for calibration are nonreflecting, the s-parameter matrices for line 1 and line 2 are defined by

\[
[S_{L1}] = \begin{bmatrix} 0 & L_1^+ \\ L_1^- & 0 \end{bmatrix} \tag{1}
\]

and

\[
[S_{L2}] = \begin{bmatrix} 0 & L_2^+ \\ L_2^- & 0 \end{bmatrix} \tag{2}
\]
The s-parameter matrix for both shorts is

\[
[S_{\text{SHORT}}] = \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}
\]  \hspace{1cm} (3)

2 Derivation of Equations Relating S-Parameters Measured at NWA Ports to S-Parameters of Test Device

The object of this section is to find the expressions which relate the s-parameters of the DUT, \( S_{ij} \), to the NWA measurements, \( S_{ijm} \), where it is assumed that the s-parameters of the networks A and B have been determined, including the complex phase factor \( L_t^\dagger \). In Section 4 the expressions associated with the TRL calibration method which yield these network s-parameters are derived.

The network flow graph for the generalized TRL calibration is shown in Figure 1. The reference planes for this calibration method are located at the middle of the shorter transmission line. Therefore, half the length of the shorter line is included on each side of the DUT.

In order to develop expressions relating the s-parameters measured at the NWA ports, \( S_{ijm} \), to the s-parameters of the DUT, \( S_{ij} \), one follows the procedure of [3,4]. From the network flow graph in Figure 1:

\[
b_0 = S_{11A}a_0 + S_{12A}a_1
\]  \hspace{1cm} (4)
\[
b_1 = S_{21A}a_0 + S_{22A}a_1
\]  \hspace{1cm} (5)
\[
a_1 = L_t^\dagger S_{11}b_1 + L_t^\dagger S_{12}b_2
\]  \hspace{1cm} (6)
\[
a_2 = L_t^\dagger S_{21}b_1 + L_t^\dagger S_{22}b_2
\]  \hspace{1cm} (7)
\[
b_2 = S_{11B}a_2 + S_{12B}a_3
\]  \hspace{1cm} (8)
\[
b_3 = S_{21B}a_2 + S_{22B}a_3
\]  \hspace{1cm} (9)

By definition, the s-parameters measured at the NWA ports are:

\[
S_{11m} = \frac{b_0}{a_0} \bigg|_{a_3=0}
\]
Figure 1: Network Flow Graph for Generalized TRL Calibration

and

\[ S_{21m} = \frac{b_3}{a_0} \bigg|_{a_3=0} \]

From (8) and (9), for \( S_{11m} \) and \( S_{21m} \) \((a_3 = 0)\).

\[ b_2 = S_{11B} a_2 \quad (10) \]
\[ b_3 = S_{21B} a_2 \quad (11) \]

Therefore,

\[ a_2 = \frac{b_3}{S_{21B}} \quad (12) \]

From (10) and (12),

\[ b_2 = \frac{S_{11B}}{S_{21B}} b_3 \quad (13) \]

From (6) and (13),

\[ a_1 = L_1^+ S_{11} b_1 + L_1^+ S_{12} \frac{S_{11B}}{S_{21B}} b_3 \quad (14) \]

From (5) and (14),

\[ a_1 = L_1^+ S_{11} (S_{21A} a_0 + S_{22A} a_1) + L_1^+ S_{12} \frac{S_{11B}}{S_{21B}} b_3 \quad (15) \]
and
\[ a_1(1 - L^+_1 S_{11} S_{22A}) = L^+_1 S_{11} S_{21A} a_0 + L^+_1 S_{12} \frac{S_{11B}}{S_{21B}} b_3 \]  \hspace{1cm} (16)

From (4) and (16),
\[ (1 - L^+_1 S_{11} S_{22A}) b_0 = S_{11A}(1 - L^+_1 S_{11} S_{22A}) a_0 + \\
L^+_1 S_{11} S_{12A} S_{21A} a_0 + L^+_1 S_{12} \frac{S_{12A} S_{11B}}{S_{21B}} b_3 \]  \hspace{1cm} (17)

Dividing through both sides of (17) by \( a_0 \) yields,
\[ (1 - L^+_1 S_{11} S_{22A}) S_{11m} = S_{11A}(1 - L^+_1 S_{11} S_{22A}) + \\
L^+_1 S_{11} S_{12A} S_{21A} + L^+_1 S_{12} \frac{S_{12A} S_{11B}}{S_{21B}} S_{21m} \]  \hspace{1cm} (18)

and
\[ S_{11m} = S_{11A} + L^+_1 S_{11}(S_{12A} S_{21A} + S_{22A} S_{11m} - S_{11A} S_{22A}) + \\
L^+_1 S_{12} \frac{S_{12A} S_{11B}}{S_{21B}} S_{21m} \]  \hspace{1cm} (19)

Multiplying the last term in (19) by \( \frac{L^+_1 S_{21A}}{L^+_1 S_{21A}} \) and simplifying, one obtains,
\[ (S_{11m} - S_{11A}) S_{21A} S_{21B} L^+_1 = \\
(S_{12A} S_{21A} L^+_1 + S_{22A} L^+_1 S_{11m} - S_{11A} S_{22A} L^+_1) S_{21A} S_{21B} L^+_1 S_{11} + \\
S_{12A} S_{21A} L^+_1 S_{11B} L^+_1 S_{21m} S_{12} \]  \hspace{1cm} (20)

In order to determine \( S_{22m} \) in terms of the s-parameters of the DUT, make the following substitutions in (20):

- Replace \( S_{11A} \) by \( S_{22B} \)
- Replace \( S_{12A} \) by \( S_{21B} \)
- Replace \( S_{21A} \) by \( S_{12B} \)
- Replace \( S_{22A} \) by \( S_{11B} \)
- Replace \( S_{11B} \) by \( S_{22A} \)
- Replace \( S_{21B} \) by \( S_{12A} \)
- Replace \( S_{11m} \) by \( S_{22m} \)
- Replace \( S_{21m} \) by \( S_{12m} \)
- Replace \( S_{11} \) by \( S_{22} \)
- Replace \( S_{12} \) by \( S_{21} \)
Equation (20) becomes,

\[
(S_{22m} - S_{22B})S_{12A}S_{12B}L_1^+ = \\
(S_{12B}S_{21B}L_1^+ + S_{11B}L_1^+S_{22m} - S_{11B}S_{22B}L_1^+)S_{12A}S_{12B}L_1^+S_{22} + \\
S_{12B}S_{21B}L_1^+S_{22A}L_1^+S_{13m}S_{21}
\]  

(21)

From (4),

\[
S_{22A}b_0 = S_{22A}S_{11A}a_0 + S_{22A}S_{12A}a_1
\]

(22)

From (5),

\[
S_{12A}b_1 = S_{12A}S_{21A}a_0 + S_{12A}S_{22A}a_1
\]

(23)

From (22) and (23),

\[
S_{22A}b_0 - S_{22A}S_{11A}a_0 = S_{12A}b_1 - S_{12A}S_{31A}a_0
\]

(24)

From (24),

\[
S_{22A}b_0 + (S_{12A}S_{21A} - S_{11A}S_{22A})a_0 = S_{12A}b_1
\]

(25)

From (25),

\[
b_1 = \frac{S_{22A}}{S_{12A}}b_0 + \left( \frac{S_{21A}}{S_{12A}} - \frac{S_{11A}S_{22A}}{S_{12A}} \right) a_0
\]

(26)

From (26) and (7),

\[
a_2 = L_1^+S_{21A}\frac{S_{22A}}{S_{12A}}b_0 + L_1^+S_{21A}\left( \frac{S_{21A}}{S_{12A}} - \frac{S_{11A}S_{22A}}{S_{12A}} \right) a_0 + L_1^+S_{23b_2}
\]

(27)

From (27) and (10),

\[
(1 - L_1^+S_{11B}S_{22})a_2 = L_1^+S_{21A}\frac{S_{22A}}{S_{12A}}b_0 + L_1^+S_{21A}\left( S_{21A} - \frac{S_{11A}S_{22A}}{S_{12A}} \right) a_0
\]

(28)

From (28) and (12),

\[
L_1^+ \frac{S_{22A}}{S_{12A}}b_0 + L_1^+S_{21A}\left( S_{21A} - \frac{S_{11A}S_{22A}}{S_{12A}} \right) a_0
\]

(29)

6
Dividing through both sides of (29) by $a_0$ yields,

$$\frac{(1 - L_1^+ S_{11B} S_{22})}{S_{21B}} S_{21m} = L_1^+ S_{21} \frac{S_{22A}}{S_{12A}} S_{11m} + L_1^+ S_{21} \left( S_{21A} - \frac{S_{11A} S_{22A}}{S_{12A}} \right)$$

(30)

and

$$S_{21m} = L_1^+ S_{21} S_{21A} S_{21B} \left( 1 + \frac{S_{22A}}{S_{12A} S_{21A}} S_{11m} - \frac{S_{11A} S_{22A}}{S_{12A} S_{21A}} \right) + L_1^+ S_{22} S_{11B} S_{21m}$$

(31)

Multiplying both sides of (31) by $S_{12A} S_{21A} L_1^+$ yields,

$$S_{21m} = S_{21A} S_{21B} L_1^+ S_{12A} S_{21A} S_{21B} (1 + \frac{S_{22A}}{S_{12A} S_{21A}} S_{11m} - \frac{S_{11A} S_{22A}}{S_{12A} S_{21A}}) + S_{12A} S_{21A} L_1^+ S_{11B} L_1^+ S_{21m} S_{21}$$

(32)

In order to determine $S_{12m}$ in terms of the s-parameters of the DUT, make the same substitutions as before in (32):

$$S_{12m} = S_{12B} S_{21B} L_1^+ = (S_{12B} S_{21B} L_1^+ + S_{11B} L_1^+ S_{22m} - S_{11B} S_{22B} L_1^+) S_{12A} S_{12B} L_1^+ S_{12} + S_{12B} S_{21B} L_1^+ S_{22A} L_1^+ S_{12m} S_{11}$$

(33)

Equations (20), (21), (32) and (33) relate the s-parameters measured at the NWA ports, $S_{ijm}$, to the s-parameters of the DUT, $S_{ij}$.

### 3 Standard NWA Error Model

In order to find the s-parameters of the error networks A and B, it is useful to define a set of error terms which represent forward and reverse coupling factors at each network.

The network flow graph of error terms for the generalized TRL calibration is shown in Figure 2. The corresponding error terms are given by [2]:

\[ \text{Figure 2: Network flow graph of error terms.} \]
Figure 2: Network Error Model for Generalized TRL Calibration
Using the error terms defined above, equations (20), (21), (32) and (33) become

\[
(S_{11m} - E_{df}) E_{tf} = (E_{rf} + E_{sf} S_{11m} - E_{df} E_{sf}) E_{tf} S_{11} + \frac{E_{rf} E_{tf} S_{21m} S_{12}}{E_{rf} E_{tf} S_{21m} S_{22}} \quad (34)
\]

\[
S_{21m} E_{rf} = (E_{rf} + E_{sf} S_{11m} - E_{df} E_{sf}) E_{sf} S_{21} + \frac{E_{rf} E_{tf} S_{21m} S_{22}}{E_{rf} E_{tf} S_{21m} S_{22}} \quad (35)
\]

\[
(S_{22m} - E_{dr}) E_{tr} = (E_{rr} + E_{sr} S_{22m} - E_{dr} E_{sr}) E_{tr} S_{22} + \frac{E_{rr} E_{tr} S_{12m} S_{21}}{E_{rr} E_{tr} S_{12m} S_{21}} \quad (36)
\]

\[
S_{12m} E_{rr} = (E_{rr} + E_{sr} S_{22m} - E_{dr} E_{sr}) E_{tr} S_{12} + \frac{E_{rr} E_{tr} S_{12m} S_{11}}{E_{rr} E_{tr} S_{12m} S_{11}} \quad (37)
\]

4 Calculation of Error Terms

The purpose of this section is to evaluate the error terms defined in the preceding section using a set of calibration standards. First a transmission line is connected between the networks A and B, and a set of s-parameters are measured at the NWA ports. Then a second transmission line with a known incremental length relative to line 1 is connected and the measurements are repeated. Third, a short with an unknown reflection coefficient is connected at each network in turn, and the reflection coefficients at the NWA are measured. The relevant expressions which yield the error terms defined above
are derived in this section.

For a general two-port network of the form

\[
\begin{align*}
\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} &= [R] \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}
\end{align*}
\]

Note that, in terms of the s-parameters of the two-port network,

\[
[R] = \frac{1}{S_{21}} \begin{bmatrix} -\Delta & S_{11} \\ -S_{22} & 1 \end{bmatrix}
\]  

where \( \Delta = S_{11}S_{22} - S_{12}S_{21} \).

If the wave cascade matrices of the error boxes A and B are denoted by \([R_A]\) and \([R_B]\) respectively, and those for line 1 and line 2 are \([R_{L1}]\) and \([R_{L2}]\), then successively connecting line 1 and line 2 between error boxes A and B yields,

\[
\begin{align*}
[R_{D1}] &= [R_A][R_{L1}][R_B] \\
[R_{D2}] &= [R_A][R_{L2}][R_B]
\end{align*}
\]

Note that from (1), (2) and (39),

\[
[R_{L1}] = \begin{bmatrix} L_1^+ & 0 \\ 0 & 1/L_1^+ \end{bmatrix} = \begin{bmatrix} L_1^+ & 0 \\ 0 & L_1^- \end{bmatrix}
\]

and

\[
[R_{L2}] = \begin{bmatrix} L_2^+ & 0 \\ 0 & 1/L_2^+ \end{bmatrix} = \begin{bmatrix} L_2^+ & 0 \\ 0 & L_2^- \end{bmatrix}
\]
Eliminating $[R_B]$ from (40) and (41), one obtains

\[
[R_{D2}]^{-1}[R_D] = [R_A][R_{L2}][R_B][R_{L1}][R_A]^{-1} \quad \text{and} \quad [R_A][R_{L2}][R_{L1}][R_A]^{-1} = [R_A][L]
\]

where

\[
[L] = [R_{L2}][R_{L1}]^{-1} = \begin{bmatrix} L_1^+ & L_1^- & 0 \\ 0 & L_2^+ & L_2^- \end{bmatrix} = \begin{bmatrix} L^+ & 0 \\ 0 & L^- \end{bmatrix}
\]

Defining

\[
[P] = [R_{D2}]^{-1}
\]

equation (44) becomes

\[
[P][R_A] = [R_A][L]
\]

or

\[
\begin{bmatrix} P_{11}R_{A11} + P_{12}R_{A21} \\ P_{21}R_{A11} + P_{22}R_{A21} \end{bmatrix} = \begin{bmatrix} L^+R_{A11} & L^-R_{A12} \\ L^+R_{A21} & L^-R_{A22} \end{bmatrix}
\]

Solve for the ratios below using (39) and (46):

\[
\frac{R_{A11}}{R_{A21}} = \frac{-P_{12}}{P_{11} - L^+} = \frac{L^+ - P_{22}}{P_{21}} = \frac{\Delta}{S_{22A}} = \frac{S_{12A}S_{21A}}{S_{22A}}
\]

\[
\frac{R_{A12}}{R_{A22}} = \frac{-P_{12}}{P_{11} - L^-} = \frac{L^- - P_{22}}{P_{21}} = S_{11A}
\]

Eliminating $[R_A]$ from (40) and (41) following a procedure similar to that above, one obtains,

\[
[R_B][Q] = [L][R_B]
\]

where $[Q] = [R_{D1}]^{-1}[R_{D2}]$, and

\[
\frac{R_{B11}}{R_{B12}} = \frac{-Q_{21}}{Q_{11} - L^+} = \frac{L^+ - Q_{22}}{Q_{12}} = \frac{\Delta}{S_{11B}} = \frac{S_{12B}S_{21B}}{S_{11B}}
\]

\[
\frac{R_{B21}}{R_{B22}} = \frac{-Q_{21}}{Q_{11} - L^-} = \frac{L^- - Q_{22}}{Q_{12}} = -S_{22B}
\]
From (47)-(50),

\[(L^+)^2 - (P_{11} + P_{22})L^+ + \Delta P = 0\]  
\[(L^-)^2 - (P_{11} + P_{22})L^- + \Delta P = 0\]

\[(L^+)^2 - (Q_{11} + Q_{22})L^+ + \Delta Q = 0\]

\[(L^-)^2 - (Q_{11} + Q_{22})L^- + \Delta Q = 0\]

where \(\Delta P = P_{11}P_{22} - P_{12}P_{21}\) and \(\Delta Q = Q_{11}Q_{22} - Q_{12}Q_{21}\). Note that

\[\Delta P = [R_{D2}][R_{D1}]^{-1} = [R_{D1}]^{-1}[R_{D2}] = \Delta Q\]

Subtracting (53) from (51) or (54) from (52) implies,

\[P_{11} + P_{22} = Q_{11} + Q_{22}\]

Therefore, \(L^+\) and \(L^-\) are the two roots of the quadratic equation

\[(L^\pm)^2 - TPPL^\pm + \Delta P = 0\]  

(55)

In the idealized case where there are no losses, \(L^+\) and \(L^-\) form a conjugate pair of roots.

Equation (55) can be solved and the ratios (47)-(50) evaluated if the elements of \([P]\) and \([Q]\) are known. These are determined from the NWA s-parameter measurements made by successively connecting line 1 and line 2 between error boxes A and B. This procedure is illustrated in Appendix A.

Now insert a short with unknown reflection coefficient, \(\gamma\), at each reference plane, as shown in Figure 3 and Figure 4.

From the diagram in Figure 3,

\[
\begin{bmatrix}
\rho_A \\
1
\end{bmatrix} = \begin{bmatrix}
R_{A11} & R_{A12} \\
R_{A21} & R_{A22}
\end{bmatrix} \begin{bmatrix}
\gamma b_2 \\
b_2
\end{bmatrix}
\]

Therefore,

\[\rho_A = (\gamma R_{A11} + R_{A12})b_2\]  
\[1 = (\gamma R_{A21} + R_{A22})b_2\]  

(56)  

(57)
Figure 3: Reflect at reference plane of DUT

Figure 4: Reflect at reference plane of DUT
Eliminating $b_2$ from (56) and (57), one obtains

$$\rho_A(\gamma R_{A21} + R_{A22}) = \gamma R_{A11} + R_{A12} \quad (58)$$

From the diagram in Figure 4,

$$\begin{bmatrix} b_1 \\ \gamma b_1 \end{bmatrix} = \begin{bmatrix} R_{B11} & R_{B12} \\ R_{B21} & R_{B22} \end{bmatrix} \begin{bmatrix} 1 \\ \rho_B \end{bmatrix}$$

Therefore,

$$b_1 = R_{B11} + \rho_B R_{B22} \quad (59)$$

$$\gamma b_1 = R_{B21} + \rho_B R_{B22} \quad (60)$$

Eliminating $b_1$ from (59) and (60), one obtains

$$\gamma(R_{B11} + \rho_B R_{B22}) = R_{B21} + \rho_B R_{B22} \quad (61)$$

Eliminating $\gamma$ from (58) and (61), one obtains

$$\frac{R_{A22}}{R_{A21}} = \beta \frac{R_{B22}}{R_{B12}} \quad (62)$$

where

$$\alpha = \frac{\frac{R_{A12}}{R_{A22}} - \rho_A}{\rho_A - \frac{R_{A11}}{R_{A21}}} \quad (63)$$

and

$$\beta = \frac{\frac{R_{B21}}{R_{B22}} + \rho_B}{\rho_B + \frac{R_{B11}}{R_{B12}}} \quad (64)$$

Note that $\rho_A$ and $\rho_B$ are known from the NWA measurements made by successively inserting short 1 and short 2 at each reference plane.

Consider the NWA measurement with line 1 inserted. The reflection coefficient for this measurement is:

$$S_{11D1} = \frac{R_{D1}^2}{R_{D1}} = \frac{R_{A11}R_{B12}L_1^+ + R_{A12}R_{B22}L_1^-}{R_{A21}R_{B12}L_1^+ + R_{A22}R_{B22}L_1^-}$$

$$= \frac{\frac{R_{A11}}{R_{A21}} + \frac{R_{A12}}{R_{A21}} \frac{L_1^-}{R_{B12} L_1^+}}{1 + \frac{R_{A22}}{R_{A21}} \frac{L_1^-}{R_{B12} L_1^+}} \quad (65)$$
From (62),
\[ \frac{R_{B22}}{R_{B12}} = \frac{\alpha}{\beta} \frac{R_{A22}}{R_{A21}} \]

Substituting in (65), one obtains
\[
S_{11D1} = \frac{\frac{R_{A11}}{R_{A21}} + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^-}{L_1^+} \right) \frac{R_{A12}}{R_{A21}} \frac{R_{A22}}{R_{A21}}}{1 + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^-}{L_1^+} \right) \frac{R_{A12}}{R_{A21}} \left( \frac{R_{A11}}{R_{A21}} \right)^2 \left( \frac{R_{A22}}{R_{A11}} \right)^2}
\]

(66)

From (66) the ratio \( \frac{R_{A22}}{R_{A11}} \) can be determined
\[
\frac{R_{A22}}{R_{A11}} = \pm \sqrt{\frac{1 - \frac{\left( \frac{R_{A11}}{R_{A21}} \right)}{S_{11D1}} \frac{R_{A11}}{R_{A21}}}{\left( 1 - \frac{1}{S_{11D1}} \right) \left( \frac{\alpha}{\beta} \right) \left( \frac{R_{A11}}{R_{A21}} \right)^2 \left( \frac{R_{A22}}{R_{A11}} \right)^2}}
\]

(67)

Using (62) the ratio \( \frac{R_{B22}}{R_{B11}} \) can be determined
\[
\frac{R_{B22}}{R_{B11}} = \frac{\frac{R_{B22}}{R_{B12}}}{\frac{R_{B11}}{R_{B12}}} = \frac{\alpha}{\beta} \frac{R_{A22}}{R_{A21}} \frac{R_{A22}}{R_{A11}} = \frac{\alpha}{\beta} \frac{R_{A12}}{R_{A21}} \frac{R_{A22}}{R_{A21}}
\]

(68)

In order to evaluate (67), (68) and the error terms in (34)-(37), the values of \( L_1^+ \) and \( L_1^- \) must be calculated. Defining
\[
\xi = \frac{L_2}{L_1} \text{ where } L_2 > L_1
\]
as the ratio of the two lengths of transmission line, it is shown in Appendix B that
\[
L_1^- = (L^-)^{\frac{1}{\xi-1}}
\]
and
\[
L_1^+ = (L^+)^{\frac{1}{\xi-1}}
\]

where \( L^+ \) and \( L^- \) are the two roots of (55).
It is also necessary to select the proper root when evaluating (67) and (68). This is accomplished by estimating the phase of the reflect, as shown in Appendix C. Physically, the selection corresponds to distinguishing between an open or a short at the reference plane of the DUT.

Using (39),

\[ S_{11A} = S_{21A} R_{A12} = \frac{R_{A12}}{R_{A22}} \]  \hspace{1cm} (69)

\[ S_{22A} = -S_{21A} R_{A21} = -\frac{R_{A21}}{R_{A22}} = -\frac{R_{A21} R_{A11}}{R_{A11} R_{A22}} \]  \hspace{1cm} (70)

and

\[ S_{12A} S_{31A} = S_{21A} R_{A11} + S_{11A} S_{22A} = S_{11A} S_{22A} + \frac{R_{A11}}{R_{A22}} \]  \hspace{1cm} (71)

From (69), (70) and (71), one obtains

\[ S_{12A} S_{31A} = \frac{R_{A12}}{R_{A22}} \left( -\frac{R_{A21} R_{A11}}{R_{A11} R_{A22}} \right) + \frac{R_{A11}}{R_{A22}} \]

\[ = \frac{R_{A11} R_{A12} R_{A21}}{R_{A22} R_{A22} R_{A11}} \]  \hspace{1cm} (72)

Similarly,

\[ S_{11B} = \frac{R_{B12}}{R_{B11}} \frac{R_{B11}}{R_{B22}} \]  \hspace{1cm} (73)

\[ S_{22B} = -\frac{R_{B21}}{R_{B22}} \]  \hspace{1cm} (74)

and

\[ S_{31B} S_{12B} = \frac{R_{B11}}{R_{B22}} - \frac{R_{B11} R_{B21} R_{B12}}{R_{B22} R_{B22} R_{B11}} \]  \hspace{1cm} (75)

The transmission coefficient for the NWA measurement with line 1 inserted is given by

\[ S_{21D1} \doteq \frac{1}{R_{D1}^2} = \frac{1}{R_{A21} R_{B12} L_1^+ + R_{A22} R_{B22} L_1^-} \]

\[ = \frac{1}{R_{A21} R_{B12} L_1^+ \left[ 1 + \frac{R_{A22} R_{B22}}{R_{A21} R_{B12}} \left( \frac{L_1^-}{L_1^+} \right) \right]} \]  \hspace{1cm} (76)
From (39) and (62),

\[
S_{21D1} = \frac{1}{\frac{S_{22A} S_{11B} L_1^+}{S_{21A} S_{21B}} \left( 1 + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^+}{L_1^-} \right) \left( \frac{R_{A22}}{R_{A21}} \right)^2 \right)}
\]

\[
= \frac{S_{21A} S_{21B}}{-S_{22A} S_{11B} L_1^+ \left( 1 + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^+}{L_1^-} \right) \left( \frac{R_{A11}}{R_{A21}} \right)^2 \left( \frac{R_{A22}}{R_{A11}} \right)^2 \right)}
\]  

(77)

From (77),

\[
S_{21A} S_{21B} = -S_{21D1} S_{22A} S_{11B} L_1^+ \left[ 1 + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^-}{L_1^+} \right) \left( \frac{R_{A11}}{R_{A21}} \right)^2 \left( \frac{R_{A22}}{R_{A11}} \right)^2 \right]
\]  

(78)

From (40),

\[
[R_{D1}] = [R_A] [R_{L1}] [R_B]
\]

Therefore,

\[
|R_{D1}| = |R_A||R_{L1}||R_B| = \frac{S_{12A} S_{12B}}{S_{21A} S_{21B}} L_1^+ L_1^-
\]  

(79)

By definition,

\[
L_1^+ L_1^- = L_1^+ \frac{1}{L_1^-} = 1
\]  

(80)

From (79) and (80),

\[
S_{12A} S_{12B} = |R_{D1}| S_{21A} S_{21B} = \frac{S_{12D1}}{S_{21D1}} S_{21A} S_{21B}
\]  

(81)

From (78) and (81),

\[
S_{12A} S_{12B} = -S_{12D1} S_{22A} S_{11B} L_1^+ \left[ 1 + \left( \frac{\alpha}{\beta} \right) \left( \frac{L_1^-}{L_1^+} \right) \left( \frac{R_{A11}}{R_{A21}} \right)^2 \left( \frac{R_{A22}}{R_{A11}} \right)^2 \right]
\]  

(82)

From (69)-(75), (70) and (82) the relevant s-parameters of error networks A and B are expressed in terms of known ratios, and therefore the error terms in equations (34)-(37) can be evaluated.
5 De-Embedding the S-Parameters of the Test Device

The purpose of this section is to summarize the results obtained thus far, and give the expressions which show how to de-embed the s-parameters of the DUT from the NWA measurements.

Equations (34)-(37) are expressed in the form

\[ A_1 = A_2 S_{11} + A_3 S_{12} \]  \hspace{1cm} \text{(83)}
\[ B_1 = B_2 S_{21} + B_3 S_{22} \]  \hspace{1cm} \text{(84)}
\[ C_1 = C_2 S_{22} + C_3 S_{21} \]  \hspace{1cm} \text{(85)}
\[ D_1 = D_2 S_{12} + D_3 S_{11} \]  \hspace{1cm} \text{(86)}

where

\[ A_1 = (S_{11m} - E_{df}) E_{lf} \]
\[ A_2 = (E_{rf} + E_{sf} S_{11m} - E_{df} E_{sf}) E_{lf} \]
\[ A_3 = S_{21m} E_{rf} E_{lf} \]
\[ B_1 = S_{21m} E_{rf} \]
\[ B_2 = (E_{rf} + E_{sf} S_{11m} - E_{df} E_{sf}) E_{lf} \]
\[ B_3 = S_{21m} E_{rf} E_{lf} \]
\[ C_1 = (S_{22m} - E_{dr}) E_{lr} \]
\[ C_2 = (E_{rr} + E_{sr} S_{22m} - E_{dr} E_{sr}) E_{lr} \]
\[ C_3 = S_{12m} E_{rr} E_{lr} \]
\[ D_1 = S_{12m} E_{rr} \]
\[ D_2 = (E_{rr} + E_{sr} S_{22m} - E_{dr} E_{sr}) E_{lr} \]
\[ D_3 = S_{12m} E_{rr} E_{lr} \]

The s-parameters of the test device are obtained by solving the linear, simultaneous equations (83)-(86) to yield,

\[ S_{11} = \frac{A_1 D_2 - D_1 A_3}{A_2 D_2 - D_3 A_3} \]
A listing of the FORTRAN code to implement the generalized TRL algorithm is given in Appendix D.

6 Calculation of Impedance from De-Embedded S-Parameters

It is worthwhile to note the relationship between the s-parameters just found, and the concept of shunt impedance which is usually applied to beamline components.

The relation between the scattering matrix, \([S]\), and the impedance matrix, \([Z]\), for the DUT is given by [5],

\[
[S] = ([Z] + [I])^{-1} ([Z] - [I])
\]  \hspace{1cm} (87)

From (87),

\[
([Z] + [I])[S] = [Z] - [I]
\]

\[
[Z] - [Z][S] = [S] + [I]
\]

\[
[Z]([I] - [S]) = [S] + [I]
\]

and therefore

\[
[Z] = ([S] + [I])([I] - [S])^{-1}
\]  \hspace{1cm} (88)
Consider the equivalent networks

\[ Z_{11} - Z_{12} \quad Z_{22} - Z_{12} \]

\[ Z_{12} \]

and

\[ Y_{12} \]

\[ Y_{11} - Y_{12} \quad Y_{22} - Y_{12} \]

Assuming shunt losses are not negligible, interpret the longitudinal impedance
as

$$\frac{Z}{Z_0} = \frac{1}{Y_{12}}$$

By definition, the admittance matrix, $[Y]$, is given by

$$[Y] = [Z]^{-1} \quad (89)$$

From (88) and (89),

$$[Y] = ([I] - [S])([S] + [I])^{-1}$$

$$= \frac{1}{1 + S_{11} + S_{22} + \Delta} \begin{bmatrix} 1 - S_{11} & -S_{12} \\ -S_{21} & 1 - S_{22} \end{bmatrix} \begin{bmatrix} S_{22} + 1 & -S_{12} \\ -S_{21} & S_{11} + 1 \end{bmatrix} \quad (90)$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$.

From (90),

$$Y_{12} = \frac{2S_{12}}{1 + S_{11} + S_{22} + \Delta} = \frac{2S_{21}}{1 + S_{11} + S_{22} + \Delta} = Y_{21}$$

Therefore, the impedance, $Z$, is given by

$$Z = \frac{Z_0(1 + S_{11} + S_{22} + \Delta)}{2S_{21}} \quad (91)$$

A listing of the FORTRAN code to implement (91) is given in Appendix E.
7 Conclusions

An algorithm has been derived for de-embedding the impedance parameters of a general 2-port network from a realistic set of s-parameter measurements including the effects of external impedance transformations. The method requires the separate measurement of inserted delays of two different lengths (optimally different by $\lambda/2$), and the measurement of identical, but possibly nonideal, reflects. Moreover, the algorithm has been implemented in the form of a FORTRAN computer code, which can be used with standard NWA output data to provide comparatively accurate values for the de-embedded impedance of a given device over as much as an octave in frequency. This method has the advantage of properly taking into account the often-experienced nonideal transmission line standards encountered in these measurements. The details of the comparison of this algorithm with synthesized data, as well as with an actual device whose impedance is known theoretically, are covered in a separate document [6].
References


Figure 5: Calibration Networks D1 and D2

Appendix

A Evaluation of P and Q Matrices

Consider the networks D1 and D2 which are formed by successively connecting transmission line 1 and transmission line 2 between error networks A and B, as shown in Figure 5.

NWA measurements on D1 and D2 yield the s-parameter matrices \([S_{D1}]\) and
respectively. Using (39), the corresponding wave cascade matrices are generated,

\[
[R_D_1] = \frac{1}{S_{21D_1}} \begin{bmatrix}
-\Delta_{D_1} & S_{11D_1} \\
-S_{22D_1} & 1
\end{bmatrix}
\]

\[
[R_D_2] = \frac{1}{S_{21D_2}} \begin{bmatrix}
-\Delta_{D_2} & S_{11D_2} \\
-S_{22D_2} & 1
\end{bmatrix}
\]

where \(\Delta_{D_1} = S_{11D_1}S_{22D_1} - S_{12D_1}S_{21D_1}\) and \(\Delta_{D_2} = S_{11D_2}S_{22D_2} - S_{12D_2}S_{21D_2}\).

\[
[R_{D_1}]^{-1} = \frac{1}{S_{22D_1}} \begin{bmatrix}
1 & -S_{11D_1} \\
S_{22D_1} & -\Delta_{D_1}
\end{bmatrix}
\]

The elements of \([P]\) are obtained from

\[
[P] = [R_D_2][R_D_1]^{-1} = \frac{1}{S_{21D_2}} \begin{bmatrix}
-\Delta_{D_2} & S_{11D_2} \\
-S_{22D_2} & 1
\end{bmatrix} \frac{1}{S_{12D_1}} \begin{bmatrix}
1 & -S_{11D_1} \\
S_{22D_1} & -\Delta_{D_1}
\end{bmatrix}
\]

\[
= \frac{1}{S_{12D_1}S_{21D_2}} \begin{bmatrix}
(S_{11D_2}S_{22D_1} - \Delta_{D_2}) & (S_{11D_1}\Delta_{D_2} - S_{11D_2}\Delta_{D_1}) \\
(S_{22D_1} - S_{22D_2}) & (S_{11D_1}S_{22D_2} - \Delta_{D_1})
\end{bmatrix}
\]

Similarly,

\[
[Q] = [R_{D_1}]^{-1}[R_D_2] = \frac{1}{S_{13D_1}} \begin{bmatrix}
1 & -S_{11D_1} \\
S_{22D_1} & -\Delta_{D_1}
\end{bmatrix} \frac{1}{S_{12D_2}} \begin{bmatrix}
-\Delta_{D_2} & S_{11D_2} \\
-S_{22D_2} & 1
\end{bmatrix}
\]

\[
= \frac{1}{S_{12D_1}S_{21D_2}} \begin{bmatrix}
(S_{11D_1}S_{22D_2} - \Delta_{D_2}) & (S_{11D_2} - S_{11D_1}) \\
(S_{22D_2}\Delta_{D_1} - S_{22D_1}\Delta_{D_2}) & (S_{11D_2}S_{22D_1} - \Delta_{D_1})
\end{bmatrix}
\]
B Calculation of $L_1^+$ and $L_1^-$

Define $\xi$ as the ratio of the lengths of the two transmission line calibration standards.

$$\xi = \frac{L_2}{L_1} \quad \text{where} \quad L_2 > L_1$$

By definition $L^+ = L_2^+ L_1^-$ and $L^- = L_1^+ L_2^-$. Therefore,

$$L^+ = e^{-a(L_2-L_1)} e^{-jk(L_2-L_1)} = e^{-(a+jk)(L_2-L_1)} = e^{-\sigma(L_2-L_1)} \quad (92)$$

and

$$L^- = \frac{1}{L^+} = e^{\sigma(L_2-L_1)} \quad (93)$$

where $\sigma = (a + jk), a > 0$ is the complex propagation constant for the two transmission line standards. From (92) and (93),

$$L_1^+ = e^{-\sigma L_1} = e^{-\sigma(L_2-L_1)/L_2 L_1} = (L^+)_{\frac{1}{\xi+1}}$$

Similarly,

$$L_1^- = e^{\sigma L_1} = e^{\sigma(L_2-L_1)/L_2 L_1} = (L^-)_{\frac{1}{\xi-1}}$$

Therefore,

$$L_1^+ = (L^+)_{\frac{1}{\xi+1}}$$

$$L_1^- = (L^-)_{\frac{1}{\xi-1}}$$
C Proper Root Choice

The proper choice of root in evaluating $\frac{R_{A22}}{R_{A11}}$ is accomplished by estimating the phase of the reflect, as shown below.

From (58),

$$\rho_A = \frac{\gamma R_{A11} + R_{A12}}{\gamma R_{A21} + R_{A22}} = \frac{\gamma R_{A11} + R_{A12}}{\gamma R_{A21} + 1}$$

(94)

From (94),

$$\gamma \left( \frac{R_{A11}}{R_{A22}} + \frac{R_{A12}}{R_{A22}} \right) = \gamma \rho_A \frac{R_{A21}}{R_{A22}} + \rho_A$$

and

$$\gamma \left( \frac{R_{A11}}{R_{A22}} - \rho_A \frac{R_{A21}}{R_{A22}} \right) = \rho_A - \frac{R_{A12}}{R_{A22}}$$

Therefore, the unknown reflection coefficient, $\gamma$, is given by

$$\gamma = \frac{\rho_A - \frac{R_{A12}}{R_{A22}}}{\frac{R_{A11}}{R_{A22}} \left[ 1 - \rho_A \left( \frac{R_{A11}}{R_{A21}} \right) \right]} = \frac{\frac{R_{A22}}{R_{A11}} \left[ \rho_A - \frac{R_{A12}}{R_{A22}} \right]}{1 - \rho_A \left( 1 \frac{R_{A11}}{R_{A21}} \right)}$$

(95)

Using (47), (48) and the value of $\rho_A$ from NWA measurements, $\gamma$ can be evaluated from (95) for each choice of root in (67). Since $\gamma$ represents the reflection coefficient for a short, the proper root choice is that value of $\frac{R_{A22}}{R_{A11}}$ for which the corresponding value of $\gamma$ lies in the shaded region of the complex plane.
$\gamma$ - Plane
D TRL Algorithm FORTRAN Code
The purpose of D EMBED is to extract the s-parameters of a test device from NWA measurements. D EMBED implements the generalized TRL algorithm. See Fermi National Accelerator Laboratory Technical Memo No. 1781 for the theoretical development.

Author: Michael Foley
708/840-2505
AD/Mechanical Engineering Support

program d_embed
common/spard/sl1(2,2),sl2(2,2),ss1(2,2),ss2(2,2),sm(2,2)
common/sparc/s(2,2)
complex•16 sll,sl2,ssl,ss2,sm,s
real•8 freq,ratio
integer npoints,data_format

Open the input data files and an output file

call file_open(ratio,npoints,data_format)

De-embed the s-parameters of the test device

do 11 i=1,npoints
  call file_read(freq,data_format)
  call s_parameter(freq,ratio)
  write(*,100) freq,s(1,1),s(2,1),s(1,2),s(2,2)
11 continue

close(unit=91)
close(unit=92)
close(unit=93)
close(unit=94)
close(unit=95)
close(unit=96)

100 format(1p9e12.4)

stop
end

subroutine file_open(ratio,npoints,data_format)
character•20 file_linel,file_line2,file_shortl,
  x file_short2,file_data,file_output
real•8 ratio
integer data_format
character•100 title(22)

type 11

11 format(1,'','Enter the name of the Line 1 calibration file (use single quotes to enclose the file name):',100)
type 12
format(' ', 'Enter the name of the Line 2 calibration file (use single quotes to enclose the file name):', $)
    accept*, file_line2

accept*, file_line2

format(' ', 'Enter the name of the Short 1 calibration file (use single quotes to enclose the file name):', $)
    accept*, file_short1

accept*, file_short1

format(' ', 'Enter the name of the Short 2 calibration file (use single quotes to enclose the file name):', $)
    accept*, file_short2

accept*, file_short2

format(' ', 'Enter the name of the file containing the measured s-parameters for the test device (use single quotes):', $)
    accept*, file_data

accept*, file_data

format(' ', 'Enter the name of the output file for the computed s-parameters for the test device (use single quotes):', $)
    accept*, file_output

accept*, file_output

format(' ', 'Enter the number of data points in the files:', $)
    accept*, npoints

accept*, npoints

format(' ', 'Enter the ratio of the lengths of the two transmission lines (L2/L1) - If Line 1 is a direct connection enter 0:', $)
    accept*, ratio

accept*, ratio

format(' ', 'Is calibration file data in mag,arg format(1) or re,im format(2): ', $)
    accept*, data_format

accept*, data_format

open(unit=91, file=file_line1, status='old')
open(unit=92, file=file_line2, status='old')
open(unit=93, file=file_short1, status='old')
open(unit=94, file=file_short2, status='old')
open(unit=95, file=file_data, status='old')
open(unit=96, file=file_output, status='new')

c Read data file headers

do 20 i=1,22
    read(91,100) title(i)
    read(92,100) title(i)
    read(93,100) title(i)
    read(94,100) title(i)
    read(95,100) title(i)
    continue

20

format(a100)

return end
subroutine file_read(freq, data_format)
    common/spard/sl1(2,2), sl12(2,2), ss1(2,2), ss2(2,2), sm(2,2)
    complex*16 sl1, sl12, ss1, ss2, sm
    real*8 freq
    real*8 s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
    integer data_format

    if (data_format.eq.1) then
        c Data in magnitude, argument format
        read(91,*) freq, s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
        sl1(1,1) = dcmplx(s1lm*dcosd(s1la), s1lm*dsind(s1la))
        sl1(2,1) = dcmplx(s2lm*dcosd(s2la), s2lm*dsind(s2la))
        sl1(1,2) = dcmplx(s12m*dcosd(s12a), s12m*dsind(s12a))
        sl1(2,2) = dcmplx(s22m*dcosd(s22a), s22m*dsind(s22a))

        read(92,*) freq, s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
        sl2(1,1) = dcmplx(s1lm*dcosd(s1la), s1lm*dsind(s1la))
        sl2(2,1) = dcmplx(s2lm*dcosd(s2la), s2lm*dsind(s2la))
        sl2(1,2) = dcmplx(s12m*dcosd(s12a), s12m*dsind(s12a))
        sl2(2,2) = dcmplx(s22m*dcosd(s22a), s22m*dsind(s22a))

        read(93,*) freq, s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
        ss1(1,1) = dcmplx(s1lm*dcosd(s1la), s1lm*dsind(s1la))
        ss1(2,1) = dcmplx(s2lm*dcosd(s2la), s2lm*dsind(s2la))
        ss1(1,2) = dcmplx(s12m*dcosd(s12a), s12m*dsind(s12a))
        ss1(2,2) = dcmplx(s22m*dcosd(s22a), s22m*dsind(s22a))

        read(94,*) freq, s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
        ss2(1,1) = dcmplx(s1lm*dcosd(s1la), s1lm*dsind(s1la))
        ss2(2,1) = dcmplx(s2lm*dcosd(s2la), s2lm*dsind(s2la))
        ss2(1,2) = dcmplx(s12m*dcosd(s12a), s12m*dsind(s12a))
        ss2(2,2) = dcmplx(s22m*dcosd(s22a), s22m*dsind(s22a))

        read(95,*) freq, s1lm, s1la, s2lm, s2la, s12m, s12a, s22m, s22a
        sm(1,1) = dcmplx(s1lm*dcosd(s1la), s1lm*dsind(s1la))
        sm(2,1) = dcmplx(s2lm*dcosd(s2la), s2lm*dsind(s2la))
        sm(1,2) = dcmplx(s12m*dcosd(s12a), s12m*dsind(s12a))
        sm(2,2) = dcmplx(s22m*dcosd(s22a), s22m*dsind(s22a))
    else if (data_format.eq.2) then
        c Data in real, imaginary format
        read(91,99) freq, sl1(1,1), sl1(2,1), sl1(1,2), sl1(2,2)
        read(92,99) freq, sl2(1,1), sl2(2,1), sl2(1,2), sl2(2,2)
        read(93,99) freq, ss1(1,1), ss1(2,1), ss1(1,2), ss1(2,2)
        read(94,99) freq, ss2(1,1), ss2(2,1), ss2(1,2), ss2(2,2)
        read(95,99) freq, sm(1,1), sm(2,1), sm(1,2), sm(2,2)
    endif

79 format(9e12.4)

return
end

subroutine s_parameter(freq, ratio)
    implicit complex*16 (a-h, o-z)
common/sparc/s(2,2)
dimension p(2,2),q(2,2)
complex*16 s11,s12,ss1,ss2,sm
complex*16 p,q,s,lplus,lminus,gamma,z
real*8 freq,ratio,power,garg
real*8 lpmag,lparg

c Evaluate P and Q matrices

deltasl1=s11(1,1)*s11(2,2)-s11(2,1)*s11(1,2)
deltasl2=s12(1,1)*s12(2,2)-s12(2,1)*s12(1,2)
p(1,1)=s12(1,1)*s11(2,2)-deltasl2
p(2,1)=s11(2,2)-s12(2,2)
p(1,2)=s11(1,1)*deltasl2-s12(1,1)*deltasl1
p(2,2)=s11(1,1)*s12(2,2)-deltasl1
q(1,1)=s11(1,1)*s12(2,2)-deltasl2
q(2,1)=s12(2,2)*deltasl1-s11(2,2)*deltasl2
q(1,2)=s12(1,1)-s11(1,1)
q(2,2)=s12(1,1)*s11(2,2)-deltasl1

do 11 i=1,2
do 11 j=1,2
   p(i,j)=(1./(s11(1,2)*s12(2,1)))*p(i,j)
   q(i,j)=(1./(s11(1,2)*s12(2,1)))*q(i,j)
11 continue

c Calculate the pair of roots (lplus and lminus) of
the quadratic equation

      L**2 - TrP*L + DELTAP = 0

deltap=p(1,1)*p(2,2)-p(2,1)*p(1,2)
deltaq=q(1,1)*q(2,2)-q(2,1)*q(1,2)
lplus=0.5*((p(1,1)+p(2,2))+sqrt((p(1,1)+p(2,2))**2-4.*deltap))
lminus=0.5*((p(1,1)+p(2,2))-sqrt((p(1,1)+p(2,2))**2-4.*deltap))
lpmag=abs(lplus)
lparg=57.2958*atan2(dimag(lplus),dreal(lplus))

c Assign roots of quadratic equation to proper location in
[L] matrix

if(lparg.gt.0.) then
   z=lplus
   lplus=lminus
   lminus=z
endif

***************************************************************************
* * Note that the code is valid only for K(L2-L1) < PI. If the * *
* frequency range is such that K(L2-L1) exceeds PI, then the * *
* code must be modified by creating the appropriate selection * *
* structure. * *
***************************************************************************
Calculate wave cascade matrix element ratios

\[ ra11ra21 = \frac{1}{p(2,2)} - \frac{p(2,1)}{p(2,1)} \]
\[ ra12ra22 = \frac{1}{lplus - q(2,2)} - \frac{q(1,2)}{q(1,2)} \]
\[ rb11rb22 = \frac{1}{lminus - q(2,2)} - \frac{q(1,2)}{q(1,2)} \]

Evaluate reflections from Short 1 and Short 2 respectively

\[ \rho_a = s_{11}(1,1) \]
\[ \rho_b = s_{22}(2,2) \]
\[ \alpha = \frac{ra12ra22 - \rho_a}{\rho_a - ra11ra21} \]
\[ \beta = \frac{rb21rb22 + \rho_b}{\rho_b + rb11rb12} \]

Calculate \( L_{lplus} \) and \( L_{lminus} \)

\[
\text{if} (\text{ratio} > 1) \text{ then}
\begin{align*}
\text{power} & = 1. / (\text{ratio} - 1.) \\
\text{lplus} & = \text{lplus} ** \text{power} \\
\text{lminus} & = \text{lminus} ** \text{power}
\end{align*}
\]
\[
\text{else} \quad \begin{align*}
\text{lplus} & = 1.0 \\
\text{lminus} & = 1.0
\end{align*}
\]
\[
\text{endif}
\]
\[ ra22ra11 = -\sqrt{\frac{\left( ra11ra21 * (s_{11}(1,1)) - 1. \right)}{(1. - ra12ra22 * (s_{11}(1,1)) * ra11ra21 ** 2) \left( \frac{\alpha}{\beta} * (lminus / lplus) \right)}} \]

Select proper root by checking phase of reflect

\[ \gamma = ra22ra11 * (\rho_a - ra12ra22) / (1. - \rho_a * (1./ra11ra21)) \]
\[ \text{garg} = 57.2958 \times \text{atan2}(\text{dimag}(\gamma), \text{dreal}(\gamma)) \]
\[
\text{if} (\text{abs} (\text{garg}) < 90.) \text{ then}
\begin{align*}
\text{ra22ra11} & = -\text{ra22ra11}
\end{align*}
\]
\[
\text{endif}
\]
\[ rb22rb11 = (\alpha / \beta) * ra11ra21 * ra22ra11 / rb11rb12 \]

Evaluate appropriate s-parameters of A and B networks

\[ s_{11} = ra12ra22 \]
\[ s_{22} = 1. / (ra11ra21 * ra22ra11) \]
\[ s_{12} = ra22ra11 / (ra22ra11 - ra12ra22) \]
\[ s_{11} = 1. / (rb11rb12 * rb22rb11) \]
\[ s_{22} = rb21rb22 \]
\[ s_{21} = 1. / (rb22rb11 - rb21rb22) \]
\[ s_{21} = (s_{21} * s_{22}) * s_{11}(2,1) * (1. + \alpha / \beta) * (lminus / lplus) * (ra11ra21 * ra22ra11)**2 \]
\[ s_{12} = s_{21} * s_{21} * (s_{11}(1,2) / s_{11}(2,1)) \]

Evaluate error terms for standard NWA error model

\[ \text{edf} = s_{11} \]
\[ \text{esf} = s_{22} * lplus \]
\[ \text{erf} = s_{12} \times lplus \]
elf=sblhl*1plus
etf=sa21_sb21*1plus

edr=sb22
esr=sblhl*1plus
err=sb21_sb12*1plus
elr=sa22*1plus
etr=sa12_sb12*1plus

c Solve for de-embedded s-parameters of the test device

\[ a1 = (sm(1,1) - edf) \cdot etf \]
\[ a2 = (erf + esf \cdot sm(1,1) - edf \cdot esf) \cdot etf \]
\[ a3 = sm(2,1) \cdot erf \cdot elf \]
\[ b1 = sm(2,1) \cdot erf \]
\[ b2 = (erf + esf \cdot sm(1,1) - edf \cdot esf) \cdot etf \]
\[ b3 = sm(2,1) \cdot elf \cdot erf \]
\[ c1 = (sm(2,2) - edr) \cdot etr \]
\[ c2 = (err + esr \cdot sm(2,2) - edr \cdot esr) \cdot etr \]
\[ c3 = sm(1,2) \cdot err \cdot elr \]
\[ d1 = sm(1,2) \cdot err \]
\[ d2 = (err + esr \cdot sm(2,2) - edr \cdot esr) \cdot etr \]
\[ d3 = sm(1,2) \cdot elr \cdot err \]

\[ s(1,1) = \frac{(a1 \cdot d2 - d1 \cdot a3)}{(a2 \cdot d2 - d3 \cdot a3)} \]
\[ s(1,2) = \frac{(a2 \cdot d1 - d3 \cdot a1)}{(a2 \cdot d2 - d3 \cdot a3)} \]
\[ s(2,1) = \frac{(b1 \cdot c2 - c1 \cdot b3)}{(b2 \cdot c2 - c3 \cdot b3)} \]
\[ s(2,2) = \frac{(b2 \cdot c1 - c3 \cdot b1)}{(b2 \cdot c2 - c3 \cdot b3)} \]

return
end
E  Impedance FORTRAN Code
program z_calc
    c    Calculate magnitude and phase of impedance from de-embedded s-parameters

    complex*16 s11(1000), s21(1000), s12(1000), s22(1000), zz(1000)
    complex*16 deltas
    real*8 freq(1000), zmag(1000), zphase(1000)
    real*8 sllm, slla, s21m, s21a, s12m, s12a, s22m, s22a
    character*20 data_file

    type 8
    8 format(' ', 'Enter name of s-parameter file (single quotes):' $)
       accept*, data_file

    open(unit=91, file=data_file, status='old')
    open(unit=92, file='zmag.dat', status='new')
    open(unit=93, file='zphase.dat', status='new')

    type 11
    11 format(' ', 'Enter number of data points: ' $)
       accept*, npoints

    do 12 i=1, npoints
        c Data in real, imaginary format
            read(91,100) freq(i), s11(i), s21(i), s12(i), s22(i)
        c Data in magnitude, phase(degrees) format
            s11(i) = dcmplx(s11(i), slla)
            s21(i) = dcmplx(s21(i), s21a)
            s12(i) = dcmplx(s12(i), s12a)
            s22(i) = dcmplx(s22(i), s22a)

            deltas = s11(i) * s22(i) - s21(i) * s12(i)
            zz(i) = (1.0 + s11(i) + s22(i) + deltas) / (2.0 * s21(i))
            zz(i) = 266.0 * zz(i)
    12 continue

    do 20 i=1, npoints
        zmag(i) = abs(zz(i))
        zphase(i) = 57.2958 * atan2(dimag(zz(i)), dreal(zz(i)))
    20 continue

    do 40 i=1, npoints
        write(92, *) freq(i), zmag(i)
        write(93, *) freq(i), zphase(i)
    40 continue

    100 format(9e12.4)

    stop
end