Fermi National Accelerator Laboratory

TM-1675

Calculation of Tune *vs* Amplitudes for the New Low Beta Quadrupoles

Vladimir V. Višnjić
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

July 18, 1990

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

TM-1675
July 18, 1990

CALCULATION OF TUNE *vs.* AMPLITUDE FOR THE NEW LOW BETA QUADRUPOLES

Vladimir Visnjic
Fermi National Accelerator Laboratory
Batavia, IL 60510

Abstract

I calculate the tune as function of amplitude due to higher multipole errors (in particular the dodecapole) in the new low beta quadrupoles. The results indicate that these multipoles are not expected to give rise to serious problems for the next Tevatron Collider run with two interaction regions.

1. INTRODUCTION

Recently magnetic field measurements have been done in 8 trim quads intended to be used in the B0 and D0 low beta insertions. Some of the higher multipoles (in particular the normal dodecapole b_5) turned out to be very large (up to 15 times the corresponding values for the present low beta quadrupoles) and there is a concern as to their performance in the 1991 run. Also the measurements were done on two 6 m quads and one 3.3 m quad belonging to the triplet. The quality of these magnets appears to be much better than that of the trim quads, however, the corresponding beta functions are about an order of magnitude larger. I calculate here the tune shift *vs.* emitance due to the normal dodecapole of these quadrupoles.

2. THE TRIM QUADRUPOLES

The measured coefficients are given in the following table (8 magnets, 13 measured coefficients for each):

MORMAL MULTIPOLES:

	1	2	3	4	5	6	7	8
2	15.89	-6.82	2.70	-5.16	-2.00	2.35	-3.32	-0.83
3	-0.52	-0.58	0.74	1.09	-0.90	1.15	-0.66	-2.01
4	-0.70	2.68	-2.22	0.27	-0.52	3.36	2.69	-0.20
5	15.64	18.02	17.98	16.58	16.31	17.18	16.51	15.62
6	0.24	0.45	-0.02	0.16	0.29	1.05	0.24	0.15
7	-0.27	0.00	0.03	0.34	-0.41	0.21	-0.63	-0.06
8	-0.39	-0.49	-0.25	-0.09	-0.01	0.35	-0.22	-0.03
9	-1.10	-1.17	-1.69	-0.85	-0.99	-1.16	-0.92	-1.31
10	0.33	0.04	-0.19	0.05	-0.03	0.20	0.14	0.41
11	0.27	0.11	0.47	0.12	0.18	0.29	-0.16	0.93
12	0.04	-0.17	0.51	0.06	0.13	0.18	-0.32	-0.03
13	0.07	-0.05	0.29	0.49	0.05	0.30	0.17	-1.15
14	-0.19	-0.17	-0.23	0.04	-0.09	-0.15	0.41	-0.75

SKEW MULTIPOLES:

2	3.51	1.44	-5.30	-3.11	-5.91	1.73	1.51	4.03
3	0.70	0.59	-3.74	-0.45	0.81	-3.80	-6.90	-2.76
4	0.96	2.54	-1.09	0.83	0.05	0.47	-1.19	2.11
5	1.13	-0.83	1.29	0.46	-1.27	0.82	1.10	-0.19
6	0.92	0.66	1.13	1.12	0.64	0.46	1.06	1.76
7	0.11	0.81	0.11	0.62	0.05	0.67	-0.51	0.37
8	0.12	0.46	-0.01	0.17	0.05	-0.07	0.15	0.28
9	0.71	0.56	0.80	0.71	0.17	0.83	0.95	0.66
10	0.03	-0.09	0.55	0.16	-0.20	0.09	-0.04	0.40
11	-0.21	0.06	0.25	-0.12	0.03	-0.13	-0.20	-0.32
12	-0.45	0.06	-0.20	-0.01	-0.19	-0.24	0.04	-1.46
13	-0.40	-0.26	-0.55	-0.22	-0.24	-0.59	0.05	-0.93
14	0.07	-0.18	-0.01	-0.07	0.06	-0.15	-0.01	0.40

I will use the Hamiltonian approach [1]. The Hamiltonian due to higher multipoles is

$$H = -\frac{RA_3}{B\rho},$$

where

$$A_3 = -B_0 \sum_{m,n}^{\infty} c_{mn} x^m y^n,$$

and the coefficients c_{mn} are given by

$$c_{mn} = \frac{1}{m+n} \begin{pmatrix} m+n \\ n \end{pmatrix} \begin{cases} (-1)^{n/2} & b_{m+n-1}, \text{ even } n \\ (-1)^{(n+1)/2} & a_{m+n-1}, \text{ odd } n \end{cases}$$

Consider now the *normal dodecapole, i.e.* b_5.
n is even and $m+n=6$, so m and n can take values 0,2,4,6. Then

$$c_{06} = -c_{60} = -\frac{1}{6} b_5 \text{ and } c_{24} = -c_{42} = -\frac{5}{2} b_5.$$

Plugging these into the definition of A_3, we obtain

$$A_3 = \frac{1}{6} B_1 b_5 (x^6 - 15x^4y^2 + 15x^2y^4 - y^6).$$

We use

$$x = \sqrt{2\beta_x I_x}\sin\phi_x \quad \text{and} \quad y = \sqrt{2\beta_y I_y}\sin\phi_y,$$

where

$$I = \frac{\epsilon}{2\pi} = \frac{\epsilon_N}{2\pi\gamma}.$$

Averaging over ϕ gives

$$< A_3 >= \frac{5}{12}B_1 b_5[(\beta_x I_x)^3 - 9(\beta_x I_x)^2\beta_y I_y + 9\beta_x I_x(\beta_y I_y)^2 - (\beta_y I_y)^3].$$

The Hamiltonian is now

$$< H >= \frac{5}{12}\frac{1}{2\pi}\sum_{i=1}^{12}\frac{B_1\tilde{b}_5}{B\rho}[(\beta_x I_x)^3 - 9(\beta_x I_x)^2\beta_y I_y + 9\beta_x I_x(\beta_y I_y)^2 - (\beta_y I_y)^3].$$

Here the integration over the arc length s is implicit–this is the origin of the summation sign and $\tilde{b}_5^i$ here is $\int b_5 dl$ over the i-th magnet– which is actually the number given by the measurement. Also the β functions here are the average beta functions in given magnets.

This Hamiltonian can be written as $C_1 I_x^3 + C_2 I_x^2 I_y + C_3 I_x I_y^2 + C_4 I_y^3$. Now

$$\Delta\nu_x = \frac{\partial < H >}{\partial I_x} \quad \text{and} \quad \Delta\nu_y = \frac{\partial < H >}{\partial I_y},$$

i.e.

$$\Delta\nu_x = 3C_1 I_x^2 + 2C_2 I_x I_y + C_3 I_y^2$$

and

$$\Delta\nu_y = C_2 I_x^2 + 2C_3 I_x I_y + 3C_4 I_y^2.$$

Finally, expressing I_x and I_y in terms of the (normalized) emittances, we obtain

$$\Delta\nu_x = A_1(\frac{\epsilon_x}{\pi})^2 + 2A_2\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} + A_3(\frac{\epsilon_y}{\pi})^2$$

and

$$\Delta\nu_y = A_2(\frac{\epsilon_x}{\pi})^2 + 2A_3\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} + A_4(\frac{\epsilon_y}{\pi})^2,$$

where

$$A_1 = \frac{5}{8\pi}\frac{1}{(2\gamma)^2}\frac{1}{B\rho}\sum_{i=1}^{12} B_1^{(i)}\tilde{b}_5^{(i)}\beta_x^{(i)3}$$

$$A_2 = -\frac{15}{8\pi}\frac{1}{(2\gamma)^2}\frac{1}{B\rho}\sum_{i=1}^{12} B_1^{(i)}\tilde{b}_5^{(i)}\beta_x^{(i)2}\beta_y^{(i)}$$

$$A_3 = \frac{15}{8\pi}\frac{1}{(2\gamma)^2}\frac{1}{B\rho}\sum_{i=1}^{12} B_1^{(i)}\tilde{b}_5^{(i)}\beta_x^{(i)}\beta_y^{(i)2}$$

$$A_4 = -\frac{5}{8\pi}\frac{1}{(2\gamma)^2}\frac{1}{B\rho}\sum_{i=1}^{12} B_1^{(i)}\tilde{b}_5^{(i)}\beta_y^{(i)3}.$$

For the numerical evaluation I assigned *all* magnets the largest measured value of $\tilde{b}_5$, 18.02×10^{-4} in^{-3}. The values of γ and $B\rho$ at $E = 900$GeV are 959 and 3×10^4 kGm, respectively. The B0 and D0 sections are identical, so I do the sum over one of them (sum from 1 to 6) and multiply the result by 2. The gradients and the beta functions (calculated using SYNCH) are given in the following table:

i	1	2	3	4	5	6
$B_1^{(i)}$ [kG/m]	-84.6	-335.0	416.5	-416.5	335.0	84.6
β_x^i [m]	33.48	112.85	189.16	34.62	22.57	93.47
β_y^i [m]	95.77	23.93	33.20	198.40	99.44	31.23

The numerical computation gives

$$A_1 = 9.31 \times 10^{-7}\text{mm}^{-2} \quad A_2 = -3.79 \times 10^{-7}\text{mm}^{-2}$$

$$A_3 = -5.21 \times 10^{-7}\text{mm}^{-2} \quad A_4 = 1.16 \times 10^{-6}\text{mm}^{-2},$$

and, finally

$$\Delta\nu_x = 9.31 \times 10^{-7}(\frac{\epsilon_x}{\pi})^2 - 7.59 \times 10^{-7}\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} - 5.21 \times 10^{-7}(\frac{\epsilon_y}{\pi})^2$$

and

$$\Delta\nu_y = -3.79 \times 10^{-7}(\frac{\epsilon_x}{\pi})^2 - 1.04 \times 10^{-6}\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} + 1.16 \times 10^{-6}(\frac{\epsilon_y}{\pi})^2.$$

The values of $\Delta\nu_x$ and $\Delta\nu_y$ for several values of ϵ_x and ϵ_y are given in the following table:

ϵ_x/π	ϵ_y/π	$\Delta\nu_x$	$\Delta\nu_y$
[mm mrad]			
15.00	15.00	-7.84E-11	-5.81E-11
20.00	20.00	-1.39E-10	-1.03E-10
25.00	25.00	-2.18E-10	-1.61E-10
30.00	30.00	-3.14E-10	-2.32E-10
35.00	35.00	-4.27E-10	-3.16E-10
40.00	40.00	-5.58E-10	-4.13E-10
45.00	45.00	-7.06E-10	-5.23E-10
50.00	50.00	-8.71E-10	-6.46E-10
55.00	55.00	-1.05E-09	-7.81E-10
60.00	60.00	-1.25E-09	-9.30E-10
65.00	65.00	-1.47E-09	-1.09E-09
70.00	70.00	-1.71E-09	-1.27E-09
75.00	75.00	-1.96E-09	-1.45E-09
80.00	80.00	-2.23E-09	-1.65E-09
85.00	85.00	-2.52E-09	-1.87E-09
90.00	90.00	-2.82E-09	-2.09E-09
95.00	95.00	-3.15E-09	-2.33E-09

The reason I show $\Delta\nu$ for the emittances approaching 100 π mm mrad is that the orbits at these magnets are separated and the beam probes larger amplitudes.

3. THE TRIPLET QUADRUPOLES

The measurements have been done on two 230 inch quadrupoles and one 130 inch one. At the maximal current (4622 A) $\tilde{b}_5$ is -1.74, -0.59 and -2.9 in^{-3}, respectively. We do the same calculation as in Section 1, with the following values of the parameters:

i	1	2	3	4	5	6
$B_1^{(i)}$ [kG/m]	1400.8	-1377.6	1400.8	-1400.8	1377.6	-1400.8
β_x^i [m]	1391.3	598.9	295.8	488.75	1075.6	733.6
β_y^i [m]	708.1	1039.7	473.1	282.5	574.6	1337.1

In this calculation, all four 230 inch quadrupoles were assigned $\tilde{b}_5$ of -1.74 and all eight 130 inch ones -2.9.

The numerical computation gives

$$A_1 = -2.86 \times 10^{-4}\mathrm{mm}^{-2} \quad A_2 = 2.39 \times 10^{-4}\mathrm{mm}^{-2}$$

$$A_3 = 2.27 \times 10^{-4}\mathrm{mm}^{-2} \quad A_4 = 1.01 \times 10^{-4}\mathrm{mm}^{-2},$$

and

$$\Delta\nu_x = -2.86 \times 10^{-4}(\frac{\epsilon_x}{\pi})^2 + 4.77 \times 10^{-4}\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} + 2.27 \times 10^{-4}(\frac{\epsilon_y}{\pi})^2$$

$$\Delta\nu_y = 2.39 \times 10^{-4}(\frac{\epsilon_x}{\pi})^2 + 4.54 \times 10^{-4}\frac{\epsilon_x}{\pi}\frac{\epsilon_y}{\pi} + 1.01 \times 10^{-4}(\frac{\epsilon_y}{\pi})^2.$$

The values of $\Delta\nu_x$ and $\Delta\nu_y$ for several values of ϵ_x and ϵ_y are given in the following table:

ϵ_x/π [mm mrad]	ϵ_y/π [mm mrad]	$\Delta\nu_x$	$\Delta\nu_y$
10.00	10.00	4.19E-08	7.94E-08
15.00	15.00	9.42E-08	1.79E-07
20.00	20.00	1.68E-07	3.17E-07
25.00	25.00	2.62E-07	4.96E-07
30.00	30.00	3.77E-07	7.14E-07
35.00	35.00	5.13E-07	9.72E-07
40.00	40.00	6.70E-07	1.27E-06
45.00	45.00	8.48E-07	1.61E-06
50.00	50.00	1.05E-06	1.98E-06

These tune shifts are significantly larger than those generated by the trim quads, but still comfortably small to cause any real problems.

4. CONCLUSIONS

The calculation of the tune *vs.* amplitude due to the normal dodecapole harmonic of the new low beta quadrupoles does not indicate any serious problems for the Tevatron beam when these magnets are in place. Other harmonics are significantly smaller and are not likely to alter this conclusion.

REFERENCES

1. G. Guignard, CERN 78–11 (1978)
   L. Michelotti, *AIP Conference Proceedings 153*, p. 236, American Institute of Physics (1987).