Piping Instability Resulting from Bellows Misalignment

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Introduction

The failure of the single phase bellows and magnet test stand during quench testing of SSC dipole magnet DD0011 has lead to much speculation about the inherent stability of operating SSC magnets. This note addresses the problem of instabilities resulting from both translational and angular misalignment between pipes connected by bellows in the general sense and with respect to the SSC single phase system specifically. Note that none of the instabilities referenced here result from bellows ‘squirm’. Inelastic bellows failure is not within the scope of this work. The failure mode referenced here is an elastic instability.

Translational Misalignment

The notation used in the analysis of a translationally misaligned bellows system is shown in Figure 1. Potential instabilities result from any initial relative translation between the two connected pipes. It will be shown that any non-zero misalignment, regardless of how small, can lead to an unstable system.

Essentially there are three forces at work; the force resulting from internal bellows pressure, the restoring force resulting from deflection of the pipes and a restoring force resulting from deflection of the bellows itself. The effect of the force due to the applied pressure is to drive the interconnected pipes apart in the direction of the initial misalignment. The effects of the restoring forces are to resist that motion. Using the direction notation in Figure 1, equilibrium requires that:

\[ -PA \sin(\alpha) - K_{1b}\delta_1 + K_{b1}(\delta_1 - \delta_1 + \delta_2) = 0 \]  \hspace{1cm} [1]

and

\[ PA \sin(\alpha) - K_{2b}\delta_2 - K_{b1}(\delta_1 - \delta_1 + \delta_2) = 0 \]  \hspace{1cm} [2]

where: 

- \( P \) = internal pressure
- \( A \) = effective bellows cross sectional area
\[ \alpha = \text{misalignment angle} = \frac{(3/2)(\delta_1 - \delta_1 + \delta_2)}{l} \]

- \( K_{1b} \) = lateral stiffness of pipe 1
- \( K_{2b} \) = lateral stiffness of pipe 2
- \( K_{bl} \) = lateral stiffness of bellows
- \( \delta_i \) = initial translational misalignment
- \( \delta_1 \) = displacement of pipe 1
- \( \delta_2 \) = displacement of pipe 2
- \( l \) = bellows length

Further, recognize that the addition of [1] and [2] yields:

\[ \delta_2 = -\delta_1 \left( \frac{K_{1b}}{K_{2b}} \right) \]  \[3\]

Instability of the system occurs whenever the force due to the internal pressure exceeds the sum of the two restoring forces. The solution amounts to using [3] to solve [1] and [2] for \( \delta_1 \) and \( \delta_2 \) at a given internal pressure, \( P \). All of the system stiffnesses are assumed constant and are determined either analytically or experimentally. The initial misalignment is measured after installation of the bellows.

The following parameters are taken from measurements made on SSC dipole magnets and on the stand used during testing of DD0011.

- \( A = 118.8 \text{ inches}^2 \) \( (r = 6.15 \text{ inches}) \)
- \( K_{1b} = 3100 \text{ lb/inch (dipole)} \)
- \( K_{2b} = 1000 \text{ lb/inch (test stand)} \)
- \( K_{2b} = 3100 \text{ lb/inch (dipole)} \)
- \( K_{bl} = 3000 \text{ lb/inch} \)
- \( l = 10.000 \text{ inches (dipole to test stand cold length)} \)
- \( 11.000 \text{ inches (dipole to dipole cold length)} \)

The initial misalignment is not known, but it is safe to assume it is not exactly equal to zero. Using these parameters and solving for \( \delta_1 \) and \( \delta_2 \), we can plot the displacement of the magnet and test stand pipes over any range of pressures. Figure 2 shows a plot of \( \delta_1 \) and \( \delta_2 \) resulting from these parameters with an initial misalignment of 0.010 inches. Case 1 is for a dipole to test stand interconnection. Case 2 is for a dipole to dipole interconnection. Using Figure 2, instability of the dipole to test stand connection occurs at approximately 210 psi. Instability of the dipole to dipole connection occurs at approximately 280 psi.
As one might expect, the point at which instability occurs is sensitive to the connection geometry, system stiffnesses, and to the initial misalignment. Figures 3 and 4 illustrate the sensitivity to the pressure at which instability occurs for varying magnet and bellows stiffnesses and for varying degrees of initial misalignment. All parameters except those being varied are the same as those used for the dipole to dipole interconnection from Figure 2. Using Figure 3, a 100% increase in $K_{bl}$ yields a 32% increase in the pressure at which instability occurs. The same increase in $K_{bl}$ yields 67%. In the case of $\delta$, a 100% change in the degree of initial misalignment changes the critical pressure by less than 1% so clearly control of the installed position is not an effective means of controlling stability.

Data recorded during the test of SSC dipole DD0011 indicated a maximum pressure in the interconnection of 250 psi. Given the results from this analysis and assuming that the parameters used are reasonably close to actual, the test stand failure likely occurred before the highest pressure was reached.

**Angular Misalignment**

It seems unlikely that two SSC components could be angularly misaligned to any significant amount, but for the sake of completeness the analysis is included below. The notation used in the analysis of an angularly misaligned bellows system is shown in Figure 5. Potential instabilities result from any initial relative rotation between the two connected pipes.

As in the case of translational misalignment there are three forces at work; the force resulting from internal bellows pressure, the restoring force resulting from deflection of the pipes and a restoring force resulting from deflection of the bellows itself. The effect of the force due to the applied pressure is to drive the interconnected pipes apart in the direction of the initial misalignment. The effects of the restoring forces are to resist that motion. Using the direction notation in Figure 5, equilibrium requires that:

\[
PA \sin(\theta_1) - K_{1b} \delta_1 - K_{ba} e_1 \sin(\theta_1) = 0 \tag{4}
\]

and

\[
PA \sin(\theta_2) - K_{2b} \delta_2 - K_{ba} e_2 \sin(\theta_2) = 0 \tag{5}
\]

where: $P$, $A$, $K_{1b}$, $K_{2b}$, $\delta_1$, $\delta_2$ are as defined above

$\theta_1 = \text{misalignment angle of pipe } 1 = \theta_i + \tan^{-1}(\delta_1 / l_1)$

$\theta_2 = \text{misalignment angle of pipe } 2 = \theta_i + \tan^{-1}(\delta_2 / l_2)$

$\theta_i = \text{initial misalignment angle}$

$K_{ba} = \text{axial stiffness of bellows}$
$l_1 = \text{length from interconnection to pivot point of tube 1}$

$l_2 = \text{length from interconnection to pivot point of tube 2}$

$e_i = \text{initial bellows extension}$

Assume: $e_i \gg \text{extension due to change in } \theta_1 \text{ and } \theta_2$

As in the case of translational misalignment, equations [4] and [5] are solved for $\delta_1$ and $\delta_2$. Figure 6 illustrates the displacement of an angularly misaligned dipole interconnection using the following parameters:

- $A$, $K_{1b}$, $K_{2b}$ are those used for the translational case above
- $\theta_i = 5$ degrees (much greater than any likely misalignment)
- $K_{ba} = 1500 \text{ lb/inch}$
- $l_1 = 60 \text{ inches}$
- $l_2 = 60 \text{ inches}$
- $e_i = 2 \text{ inches}$

Clearly from Figure 6, instability is not an issue with angular misalignment. Figure 7 illustrates the increase in the pressure induced force as a function of internal bellows pressure for both misalignment cases. Since both pipes deflect in the same direction, $\theta$ simply does not increase fast enough to generate the extreme pressure induced force that one sees in the case of a translational misalignment.

**Summary and Conclusions**

It is clear not just from this analysis, but from the test of DD0011 that the instability mechanism is a very real concern for SSC magnet interconnections. It will need to be addressed within the scope of current work and when future devices, e.g. quads and spools, are being developed. Given an interconnection design like that used in SSC dipoles, it isn’t enough to avoid instability. We must also assure ourselves that we avoid any condition that permanently deforms the single phase bellows. Any yielding of this bellows will cause permanent misalignment of the bore tube.

The pressure rating of the single phase system is 300 psi. Given the predicted pressure at which instability occurs of 280 psi, a means must be found to build in some margin of safety. The most direct way to achieve it is to increase either the cold mass lateral spring constant, ($K_{1b}$ and $K_{2b}$) or the lateral stiffness of the single phase bellows, ($K_{bl}$).
Either of these two alternatives is straightforward, but not without some cost. Increasing the lateral stiffness of the cold mass and suspension system increases both cost and heat load. The most effective means is to increase the lateral stiffness of the single phase bellows or to add some lateral restraint in the interconnection region. From Figure 3, this clearly serves to increase the stability of the system more than increasing the cold mass stiffness. Such a change would obviously add to the cost, but has few other adverse affects assuming that other pertinent bellows parameters can be met, e.g. extension and compression requirements, operating cycles, etc.

Finally, the single phase bellows in SSC dipoles is not unique in its tendency towards instability. Any bellows system may potentially be affected by the failure modes discussed in this note. Hopefully this work will serve as a guide in reexamining existing designs and in designing new systems.

References

1. Mazur, P.O., private communications.
Assume the shape of a fixed-guided beam subjected to a concentrated end load, i.e.:

\[ \frac{\Delta}{\delta_{\text{total}}} = \frac{Wl^2}{8EI} \Rightarrow \frac{\Delta}{\delta_{\text{total}}} = \frac{12}{8l} \text{ or } \Delta = \frac{3\delta_{\text{total}}}{2l} \]
Displacement (pipes 1,2) vs. Internal Pressure

Figure 2 - Dple/Tat Stnd (cs 1) & Dple/Dple (cs 2)
Instability Pressure vs. K1b and Kbl

Figure 3 - Sensitivity to System Stiffnesses
Instability Pressure vs. Initial Misalignment

Figure 4 - Sensitivity to Initial Misalignment

Instability Pressure (psi) vs. $d_1$ (inches)

$P$ vs $d_1$
Figure 5: Angular Misalignment Nomenclature

\[ K_{ba} e_i \sin \theta_i \]

\[ K_{ba} e_i \sin \theta_2 \]

\[ K_{ib}, l_1 + \delta_1 \theta_i \]

\[ K_{ib}, l_1 \]

\[ \theta_i \]

\[ \theta_2 \]

\[ \delta_2 \]

\[ \delta_1 \]

\[ K_{2b}, l_2 \]

\[ K_{2b}, l_2 \]

\[ K_{2b}, l_2 \]

\[ P A \sin \theta_i \]

\[ P A \sin \theta_2 \]
Pressure Induced Force vs. Internal Pressure

Figure 7 - Translational and Angular Misalignment

Pressure Induced Force (lb)

P (psi)

--- Offset

--- Angled