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MICROWAVE STABILITY LIMITS FOR THE MAIN RING AND GROWTH ACROSS TRANSITION*

King-Yuen Ng

January 1986

*Based on a talk given November 4, 1985 at Fermilab

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I. INTRODUCTION

The Fermilab Main Ring is an important link of the TeV I project. The present duties of the Main Ring are[1]:

(1) To accelerate proton bunches up to 150 GeV and coalesce seven of them to form an intense bunch of $\sim 10^{11}$p to be injected into the Energy Saver for further acceleration and eventual collision with an antiproton bunch. In order to achieve a high luminosity, the proton bunches must be kept at small bunch areas ~0.2 eV-sec before coalesence.

(2) To accelerate intense proton bunches up to 120 GeV for antiproton production. To maximize the production efficiency, these bunches must be of high intensity $\sim 2.4\text{x}10^{10}$p/bunch and small bunch area ~0.1 eV-sec.

(3) To accelerate antiprotons starting at 8 GeV after being cooled in the accumulator. At 150 GeV, 13 p bunches are coalesced to form a bunch of intensity $\sim 10^{11}$. The bunch area before coalescence has to be ~0.20 eV-sec to ensure a high luminosity in p-p collision.

*Based on a talk given on 4 November 1985.

We see that, in each performance, the bunch area has to be small, unlike the operation in the fixed target mode where the bunch area was deliberately increased by the bunch spreader in order to have a better duty cycle. As a result, we have to pay close attention to the longitudinal impedance per unit harmonic (Z/n) of the ring so as to avoid any undesirable microwave blowup of the phase space. The purpose of this paper is to estimate the phase space blowup across transition and give critical $|Z/n|$ limits at each stage of performance. It turns out that the most stringent limit is $|Z/n|$~1.3 Ω which occurs during the RF manipulation of the proton bunches at 120 GeV in preparation of $\bar{p}$ production.

II. GROWTH ACROSS TRANSITION

The growth of the microwave amplitude across transition is unavoidable because, for a certain time interval, the frequency dispersion parameter $\eta = 1/\gamma_t^2 - 1/\gamma^2$ is too small to provide enough frequency spread for Landau damping. It has been shown by Courant and Snyder[1,2] as well as by Herrera[3] that, if one assumes η/E to be a linear function of time in that interval, the invariant of the longitudinal phase space can be solved analytically in terms of Bessel function and Neumann function of order 2/3. Assuming that the bunch is Gaussian in shape, a dispersion relation can be set up and the growth rate can then be solved numerically[4]. If we further assume that Z/n is real, i.e., omitting space charge, the problem can be solved approximately resulting in a handy formula (derivation is given in Appendix). The total growth transition is $\exp(S_b+S_a)$ where

$$S_{b,a} = \int \Im m\,\Delta\Omega \; dt \,\Big|_{\Im m\,\Delta\Omega > 0} \qquad (1)$$

represent the integrated growth rate $\mathrm{Im}(\Delta\Omega)$ before and after transition. Our handy formula gives

$$\frac{S_b}{n} = \frac{S_a}{n} = \frac{F_1 (eN \frac{Z}{n} \gamma_t^2)^2}{V \sin\phi_0} \frac{(E_0/e)^2 \sigma_\tau}{(A/e)^3}, \qquad (2)$$

where e is the electric charge, N is the number of particles per bunch, n is the harmonic of the microwave frequency, γ_t is gamma at transition, V is the RF voltage at transition, ϕ_0 is the synchronized phase at transition, E_0 is the rest energy of the particle concerned, A is the bunch area enclosing 95%, $F_1 = 8.735$ a numerical constant, and σ_τ is the RMS time spread of the bunch at the moment when stability is lost (before transition) or when stability is regained (after transition). The time t_0 from loss of stability to transition or from transition to regain of stability is given by

$$|t_0| = \frac{F_2\, eN\, Z/n\, \gamma_t^4}{\omega_0 V \sin\phi_0} \frac{(E_0/e)^2 \sigma_\tau}{(A/e)^2}, \qquad (3)$$

where ω_0 is the angular frequency and $F_2 = 49.42$ is a constant. A characteristic time

$$T = \left[\frac{4\pi^2(E_0/e)^2\beta^2\gamma_t^4}{h\omega_0^3 V^2 \sin 2\phi_0}\right]^{\frac{1}{3}} \tag{4}$$

can be defined where β is the velocity of the bunch in unit of c, the light velocity and h the RF harmonic. Then, the RMS time spread σ_τ in Eq. (2) can be written as

$$\frac{\sigma_\tau}{T} = \begin{cases} \dfrac{2}{3^{1/3}\Gamma(\frac{1}{3})}\left(\dfrac{AeV\omega_0\sin\phi_0}{6\pi E_0^2\beta^2\gamma_t^4}\right)^{\frac{1}{2}}\left(1 + 0.6859\,\dfrac{|t_0|}{T}\right) & \dfrac{|t_0|}{T} \ll 1\,, \\[2ex] \left(\dfrac{AeV\omega_0\sin\phi_0}{6\pi^2 E_0^2\beta^2\gamma_t^4}\right)^{\frac{1}{2}}\left|\dfrac{t_0}{T}\right|^{\frac{1}{4}} & \dfrac{|t_0|}{T} \gtrsim 1\,. \end{cases} \tag{5}$$

For the Main Ring, we take $N = 1.2\text{x}10^{10}$, $\gamma_t = 18.75$, $\omega_o = 3.00\text{x}10^5\ \text{sec}^{-1}$, $Z/n = 10$ ohms, and at transition $A = 0.15$ eV-sec, $V = 2.5$ MV, $\phi_o = 50^\circ$. The result is unstable time interval $2t_o = 52$ ms and integrated growth rate $S_b/n + S_a/n = 3.6\text{x}10^{-5}$. Exact numerical solution of the dispersion relation including space-charge effects gives 55 ms and $3.86\text{x}10^{-5}$. Thus, the accuracy of our handy formula is remarkable when taking into account that we are in the region $t_o \approx T = 28.5$ ms where the computation of σ_t is least accurate.

Microwave signals at 1.65 GHz have been observed. This corresponds to a harmonic of $3.46\text{x}10^4$. Thus, the growth across transition is $\exp(S_b+S_a) = 3.86$ times. However, this is the growth of the microwave amplitude, which will dilute the bunch area only if it is big compared to the bunch density. The initial microwave amplitude comes from two sources: the noise background in the machine and the frequency spectrum of the bunch. For a well-behaved bunch, the amplitude in the microwave region is very tiny, so all the contribution to the microwave amplitude comes from the noise background. Under such situation, it is possible that the bunch area grows very little when the microwave amplitude grows by 3.86 times. However, if the bunch is ill-behaved and does not fit the bucket well, its amplitude at the microwave region can be big and its growth will become a growth of the bunch area. Growth across transition in the Main Ring has been reported[6] to be 20% to ~4 times. We believe that this variation is mainly due to the behavior of the bunch before transition.

When the Main Ring accelerates proton bunches for $\bar{p}$ production, the number of protons per bunch $N \sim 2.4\times10^{10}$ is bigger and the bunch area $A \sim 0.1$ eV-sec is smaller. According to Eq. (2), S_b+S_a scales as $(Z/n)^2N^2A^{-9/4}$. Thus, S_b+S_a increases by ten times and the growth of the microwave amplitude becomes almost 7×10^5 which is certainly too big to tolerate.

In deriving our formula, the Vlasov equation has been linearized. This implies that the result is correct only when the final microwave amplitude is still small compared with the unperturbed phase-space density. If not, nonlinear effects will set in and overshoots will occur.

III. BUNCH COALESCENCE

After passing through transition, the Z/n limit is roughly proportional to $(\eta/E)^{3/4}|V\cos\phi|^{1/4}$ during acceleration. After transition, η becomes bigger and bigger and $|V\cos\phi_o|$ still increases; thus the bunch becomes more stable. Later, for higher energies, η approaches a constant and $|V\cos\phi_o|$ is constant or decreasing; thus the $|Z/n|$ limit becomes smaller as E increases. Figure 1 shows a typical acceleration cycle. We see that the limit is most stringent at the highest energy of 120 GeV or 150 GeV depending on the situation. Thus, we need to discuss stability at these top energies only.

Seven proton bunches each of intensity $N = 1.2\times10^{10}$ and $A = 0.20$ eV-sec are left in the Main Ring at 150 GeV (here, we assume that the bunch area increases to 0.20 eV-sec across transition). They are allowed to coalesce into one bunch of intensity $\sim8\times10^{10}$. The procedure consists of lowering the RF voltage adiabatically from 0.68 MV to ~2.30 kV so that the bunch will just fill the bucket.

This h = 1113 RF is then turned off while a h = 53 RF of 22 kV is turned on. The 7 bunches will lie inside the h = 53 RF bucket and rotate. After 1/4 rotation, this RF is turned off while a h = 1113 RF of 0.12 MV is turned on to capture these 7 bunches into a single bucket. The RF voltage is increased slowly to 1 MV so that the coalesced high-intensity bunch is matched to the bucket. The $|Z/n|$ limit of each stage during the coalescence is listed in Table I. We see that the lowest limit is $|Z/n| = 6.86\ \Omega$ which occurs when the h = 1113 RF voltage is lowered adiabatically to match the bunch of area 0.2 eV-sec and intensity 1.2×10^{10}.

When the impedance takes the form of a sharp resonance narrower than the frequency spread of the bunch, what the bunch sees (or the Z appears in the usual microwave stability limit) is not the peak impedance but only an effective one average over the bunch spectrum. However, the usual microwave stability limit transforms to a neat form

$$\frac{Z}{Q} < \frac{4|\eta|(E/e)}{I_{AV}}\left(\frac{\sigma_E}{E}\right)^2, \tag{6}$$

where Z is the peak impedance of the resonance with quality factor Q, E the particle energy, σ_E the energy spread and I_{AV} the average bunch current. Here, we want to point out that Eq. (6) has never been proved rigorously. In our situation for p-bunch coalescence, the worst limit is $Z/Q = 8.12\ \mathrm{k\Omega}$.

The antiprotons are coalesced at 150 GeV similarly. The only difference is that 13 adjacent bunches each of intensity 8×10^9 are involved. Because of this lower intensity, the lowest limit is $|Z/n| = 10.3\ \Omega$ instead. For a sharp resonance narrower than the bunch spectrum, this corresponds to $Z/Q = 38.8\ \mathrm{k\Omega}$. This and other data for the coalescence are listed in Table II.

The microwave driving Z/n is usually real (the peak of a resonance). Then, the stability limits in Tables I and II (and also Table III below) should be 1/0.697 or 1.43 times bigger. This is because real Z/n corresponds to the point B in the stability curve of Figure 3 while the usual stability formula corresponds to the point A where Z/n is capacitive (see Appendix).

IV. PREPARATION OF PROTON BUNCHES FOR ANTIPROTON PRODUCTION

The production of antiprotons is done by bombarding a target with protons that have been accelerated to 120 GeV in the Main Ring. For a fixed antiproton momentum spread, the bunch area of the antiproton is minimized by making the time spread of the extracted proton bunches as narrow as possible. To maximize the production efficiency the proton bunch must be of intensity 2.4×10^{10} and of area 0.1 eV-sec. The maneuvering is as follows[7]: at the 120 GeV flat-top, the RF voltage is maintained at its maximum value of 4 MV and the bunches, about 0.1 eV-sec, are matched to the large bucket (ΔE ~300 MeV). The RF voltage is reduced to ~117 kV within two turns, a time very short compared to a phase oscillation period of ~32 msec (Figure 2). The now mismatched bunches begin phase oscillation which results in a maximum time spread of $\sqrt{6}\sigma_{\tau}$ ~4 ns, about half the bucket length in one quarter phase oscillation period (8 msec). The RF voltage is then increased quickly again to its maximum value whereupon the mismatched bunches rotate in one quarter phase oscillation (~1.3 msec) to a large momentum spread and narrow time spread configuration ($\sqrt{6}\sigma_{\tau}$ ~0.5 ns) and are extracted. The various bunch parameters and $|Z/n|$ limits are summarized in Table III. We see that $|Z/n|$ reaches 1.26 Ω when the time spread of the proton bunch is biggest. For a sharp resonance narrower than the bunch spectrum, this corresponds to Z/Q = 58.3 kΩ. If only one batch of 82 proton bunches are required for antiproton production, these bunches will stay at this most dangerous position for a very short time, ~one or two msec, and will be rotated again and extracted. Even if the Main Ring broad-band Z/n is bigger than this limit, there will not be enough time for the microwave amplitude to grow appreciably before it is damped again. However, in the future, we may want to have three batches of proton bunches accelerated in the Main Ring and each batch is extracted every one or two seconds for antiproton production. Each bunch will then oscillate between maximum and minimum time spreads for a few seconds. If there is a growth of the microwave amplitude, the bunch will certainly be unstable.

APPENDIX

The shift in coherent microwave frequency $\Delta\Omega$ at a certain time is given by the dispersion relation[5,8]

$$1 = -\left(\frac{\Delta\Omega_0}{n}\right)^2 \int \frac{F'(\omega)}{\Delta\Omega/n - \omega}\, d\omega\,, \tag{A.1}$$

where $\Delta\Omega_o$ is the shift without Landau damping

$$\left(\frac{\Delta\Omega_0}{n}\right)^2 = \frac{ie\eta\omega_0^2 I_p}{2\pi\beta^2 E}\cdot\frac{Z}{n} \tag{A.2}$$

and

$$F(\omega) = \frac{1}{\sqrt{2\pi}\,\sigma_\omega}\, e^{-\omega^2/2\sigma_\omega^2} \tag{A.3}$$

with

$$\sigma_\omega = \frac{|\eta|\omega_0\sigma_E}{\beta^2 E} \tag{A.4}$$

the RMS frequency spread in the bunch and $I_p = eN/\sqrt{2\pi}\sigma_\tau$ the peak current. If we define

$$u = \omega/\sigma_\omega\ , \quad z = \Delta\Omega/n\sigma_\omega\,, \tag{A.5}$$

Eq. (A.1) becomes

$$1 = -\left(\frac{\Delta\Omega_0}{n\sigma_\omega}\right)^2 \int \frac{G'(u)}{z-u}\, du \tag{A.6}$$

with

$$G(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}. \tag{A.7}$$

Putting in

$$\frac{\eta}{E} = \frac{2eV\omega_0 \sin\phi_0}{2\pi\gamma_t^4 E_0} t , \tag{A.8}$$

we get

$$1 = -i \frac{a}{t} \int \frac{G'(u)}{z-u} du, \tag{A.9}$$

where

$$a = \frac{eN(Z/n)\gamma_t^4 (E_0/e)\beta^2}{2\sqrt{2\pi}\,\sigma_\tau \sigma_E^2 V\omega_0 \sin\phi_0} \tag{A.10}$$

is a slowly varying function of t. Written in this form, the integral in Eq. (A.9) is independent of any machine or bunch parameter. At t_0 when stability is regained, since we assume Z/n to be real, the real part of the integral must vanish and we get

$$t_0 = 0.697285\, a(t_0). \tag{A.11}$$

If we further neglect the time dependence in Eq. (A.10), the dispersion relation becomes

$$1 = -\frac{i}{0.697285t'}\int \frac{G'(u)}{z-u}\,du\,, \tag{A.12}$$

where $t' = t/t_o$ varies from zero to one. Note that Eq. (A.12) is now independent of any parameter; it is universal for all Gaussian bunches. Thus, what we need to solve is only one dispersion equation. The integrated growth rate is

$$\frac{S_a}{n} = \int_0^{t_0} \mathcal{I}m\,\frac{\Delta\Omega}{n}\,dt = t_o\int_0^1 \sigma_\omega\,\mathcal{I}m\,z\,dt'$$

$$= \frac{\sigma_E\,eV\omega_o^2\sin\phi_o\,t_o^2}{\pi\beta^2\gamma_t^4E_0}\int_0^1 t'\,\mathcal{I}m\,z\,dt' \tag{A.13}$$

The last integral equals 0.21179 by solving Eq. (A.12) numerically. Putting Eqs. (A.10) and (A.11) into Eq. (A.13) we arrived at

$$\frac{S_a}{n} = \frac{(eN\,Z/n\,\gamma_t^2)^2(E_0/e)^2\beta^2}{8\pi^2\sigma_\tau^2\sigma_E^3V\sin\phi_o}\,(0.69729)^2(0.21179)$$

which is just Eq. (2). The formulas for σ_τ are given in Ref. 5.

In fact, we can relax the condition that Z/n is real. In that case, the numerical constant in Eq. (A.11) will change and there is a different constant for a different phase in Z/n. As a result, the final formula will not be so handy. In reality, it is always the peak of a broad resonance that is responsible for microwave instability and in that situation Z/n is real. Above transition, when Z/n is completely capacitive, the numerical constant is exactly unity and Eq. (A.11) just corresponds to the usual microwave stability limit[8] for a Gaussian bunch or point A in the stability plot in Figure 3. Equation (A.11) with the constant 0.697285 corresponds to point B.

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Table I. Coalescence of 7 proton bunches at 150 GeV

	Start	V_{RF} lowered to match bunch	7 bunches in h=53 bucket	End of 90° rotation	Recapture into h=1113 bucket	V_{RF} increases before extraction
Longitudinal emittance $6\pi\sigma_{\tau}\sigma_E$	0.20 eV-sec	0.20 eV-sec	1.39 eV-sec	2.19 eV-sec	1.45 eV-sec	1.45 eV-sec
$\sqrt{6}\sigma_{\tau}$	1.63 ns	9.4 ns	65.8 ns	8.89 ns	9.4 ns	3.99 ns
$\sqrt{6}\sigma_E$ ($\sqrt{6}\sigma_E/E$)	39.0 MeV (2.60×10^{-4})	8.31 MeV (5.54×10^{-5})	8.31 MeV (5.54×10^{-5})	61.5 MeV (4.10×10^{-4})	60.3 MeV (4.02×10^{-4})	116 MeV (7.7×10^{-4})
V_{RF}	0.68 MV	2.30 kV	22 kV	22 kV	0.12 MV	1.0 MV
h	1113	1113	53	53	1113	1113
ν_s	1.51×10^{-3}	8.72×10^{-5}	5.91×10^{-5}	5.91×10^{-5}	6.37×10^{-4}	1.83×10^{-3}
Number per bunch	1.2×10^{10}	1.2×10^{10}	8.4×10^{10}	8.4×10^{10}	8×10^{10}	8×10^{10}
Microwave **stability** limit						
$Z_{\parallel}/n$ (broad band)	26.2 Ω	6.86 Ω	6.86 Ω	50.8 Ω	54.2 Ω	84.4 Ω
$Z_{\parallel}/Q$ (narrow)	206 kΩ	9.4 kΩ	9.4 kΩ	73 kΩ	73 kΩ	272 kΩ

Table II. Coalescence of 13 antiproton bunches at 150 GeV

	Start	V_{RF} lowered to match bunch	13 bunches in h=53 bucket	End of 90^o rotation	Recapture into h=1113 bucket	V_{RF} increases before extraction
Longitudinal emittance $6\pi\sigma_\tau\sigma_E$	0.20 eV-sec	0.20 eV-sec	2.60 eV-sec	4.08 eV-sec	2.76 eV-sec	2.76 eV-sec
$\sqrt{6}\sigma_\tau$	1.63 ns	9.42 ns	122 ns	8.89 ns	9.42 ns	5.5 ns
$\sqrt{6}\sigma_E$ ($\sqrt{6}\sigma_E/E$)	39.0 MeV (2.60×10^{-4})	8.31 MeV (5.54×10^{-5})	8.31 MeV (5.54×10^{-5})	114 MeV (7.61×10^{-4})	115 MeV (7.67×10^{-4})	160 MeV (1.06×10^{-3})
V_{RF}	0.68 MV	2.30 kV	22 kV	22 kV	.438 MV	1.0 MV
h	1113	1113	53	53	1113	1113
ν_s	1.51×10^{-3}	8.72×10^{-5}	5.91×10^{-5}	5.91×10^{-5}	1.21×10^{-3}	1.83×10^{-3}
Number per bunch	8×10^{9}	8×10^{9}	1.04×10^{11}	1.04×10^{11}	1×10^{11}	1×10^{11}
Microwave instability limit						
$Z_{\parallel}/n$ (broad band)	39.3 Ω	10.3 Ω	10.3 Ω	141 Ω	158 Ω	176 Ω
$Z_{\parallel}/Q$ (narrow)	309 kΩ	14.0 kΩ	14.0 kΩ	204 kΩ	266 kΩ	417 kΩ

Table III. RF maneuvering of proton bunches for $\bar{p}$ production

	120 GeV flat top	V_{RF} reduced and bunch rotates to max time spread	V_{RF} raised to 4 MV bunch rotates for 90°
Longitudinal emittance $6\pi\sigma_{\tau}\sigma_{E}$	0.1 eV-sec	0.1 eV-sec	0.1 eV-sec
$\sqrt{6}\sigma_{\tau}$	.78 ns	4 ns	0.5 ns
$\sqrt{6}\sigma_{E}$ $(\sqrt{6}\sigma_{E}/E)$	49.1 MeV (4.06×10^{-4})	6.98 MeV (5.77×10^{-5})	239 MeV (1.98×10^{-3})
V_{RF}	4 MV	117 kV	4 MV
Bucket height	314 MeV	53.7 MeV	314 MeV
ν_{s}	0.00405	0.000693	0.00405
Number per bunch	2.4×10^{10}	2.4×10^{10}	2.4×10^{10}
Microwave stability limit			
$Z_{\parallel}/n$ (broad band)	12.2 Ω	1.27 Ω	186 Ω
$Z_{\parallel}/Q$ (narrow resonance)	201 kΩ	4.06 kΩ	4.76 MΩ

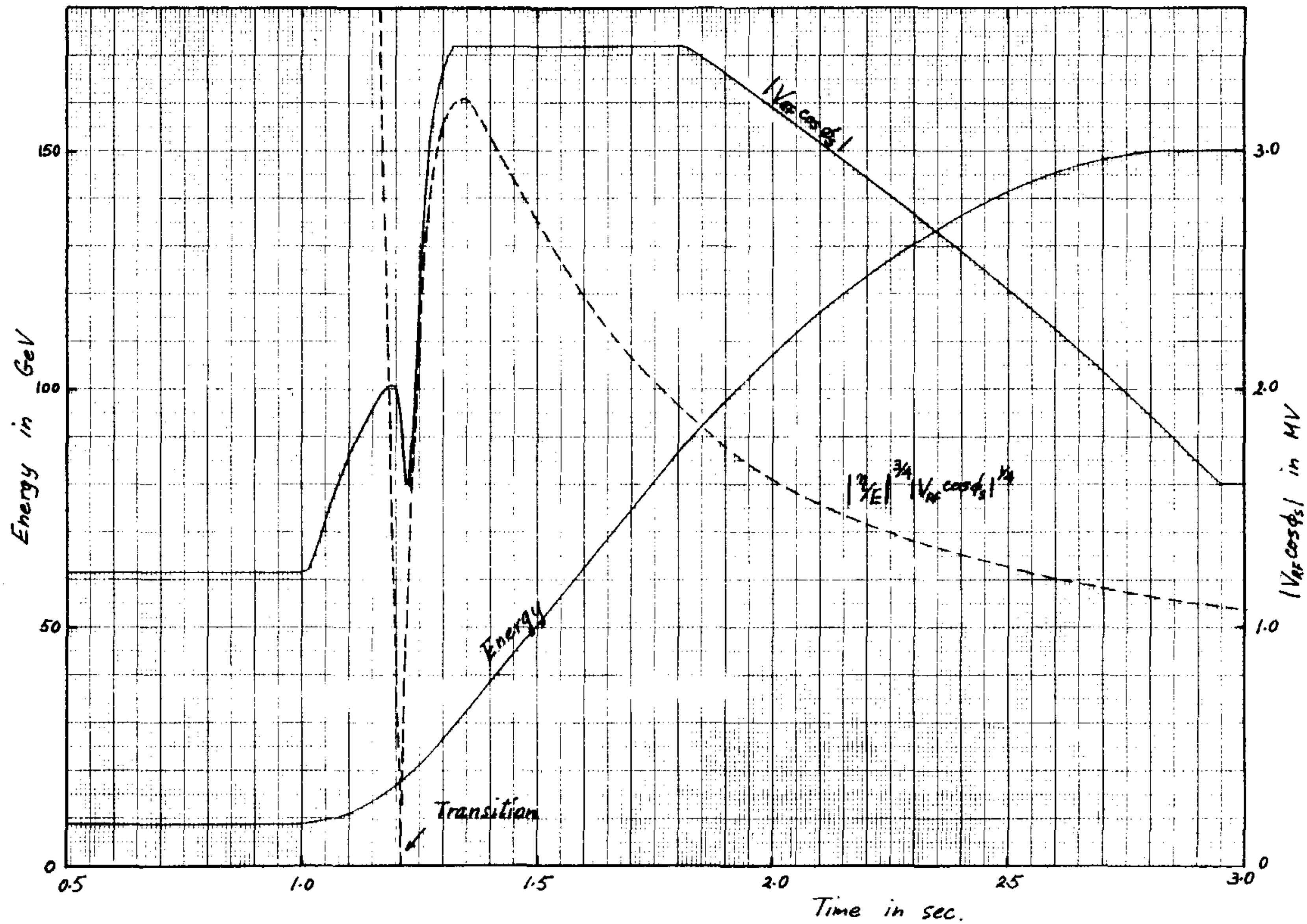

Fig. 1. A typical acceleration cycle in the Main Ring. The scale of the dashed curve is arbitrary.

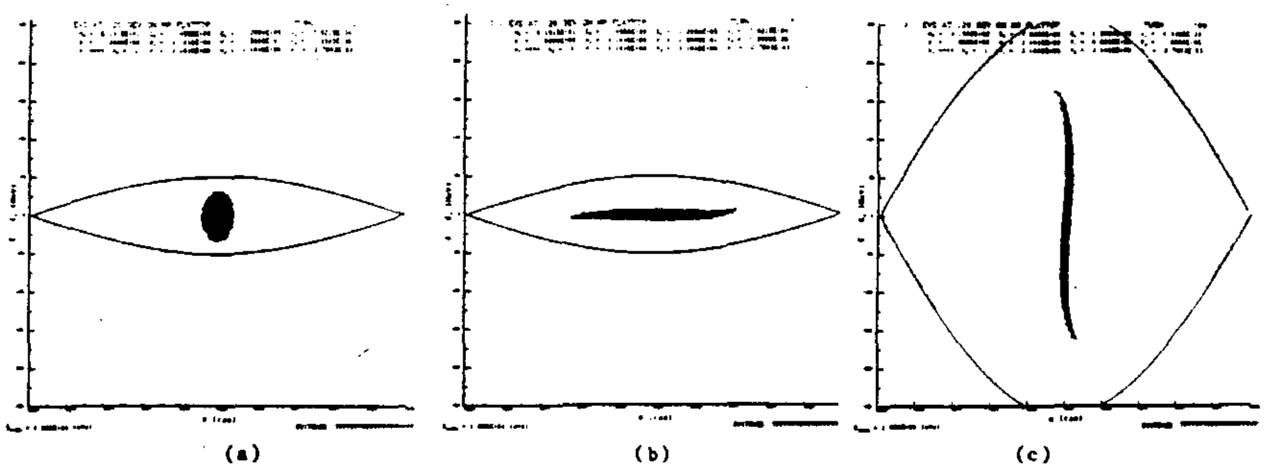

Fig. 2. Computer simulation of fast bunch narrowing.

(a) 0.1 eV-sec bunch in small mismatched bucket just after voltage reduction.

(b) Same bunch after broadening in small bucket.

(c) Same bunch after one quarter phase oscillation in large bucket.

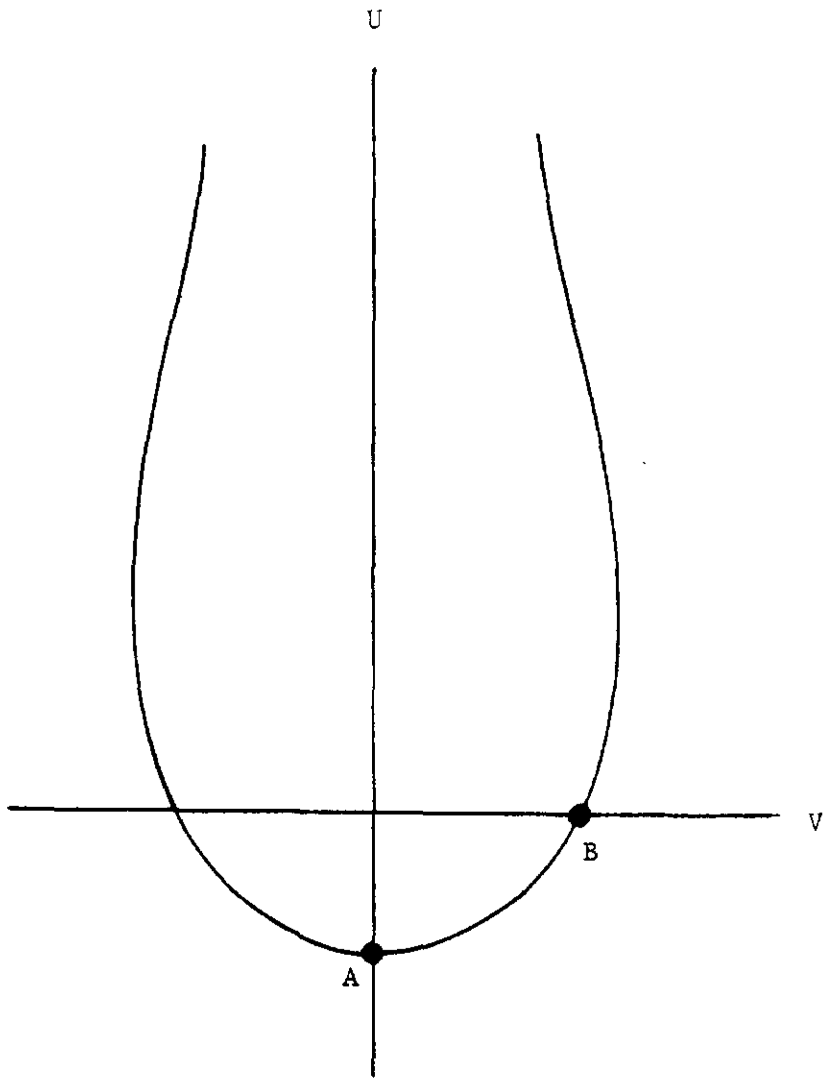

Fig. 3. Stability curve for Gaussian distribution.