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COUPLING IMPEDANCE AND ENERGY LOSS WITH MAGNET LAMINATIONS

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I. Introduction

The purpose of this note is to review the longitudinal coupling impedance for the Fermilab Booster, and to estimate the energy loss in the magnet laminations. The work is a review of earlier calculations by Snowdon[1] and Ruggiero.[2]

II. Coupling Impedance

The longitudinal coupling impedance of beam pipe has been calculated by many persons[3]. The result for a pipe wall of arbitrary surface impedance can be written[4], for a beam radius a, and pipe radius b, as

$$(Z_L/nZ_0) = (2i/\beta\gamma^2)\left(\frac{1 - 2\cdot F_1(x)I_1(x)}{x^2}\right) \tag{2.1}$$

where

$$x \equiv \sigma a, \qquad \sigma \equiv (\omega/v\gamma), \qquad n \equiv (R\omega/v) \tag{2.2}$$

and where $I_1(x)$ is the usual modified Bessel Function.

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The function $F_1(x)$ is

$$F_1(x) \equiv K_1(x) + \alpha\, I_1(x) \tag{2.3}$$

where α is determined by the boundary condition at the beam pipe. For a perfect conducting wall, one has

$$\alpha = \frac{K_0(\sigma b)}{I_0(\sigma b)} \tag{2.4}$$

leading to the well-known-result at large γ ($\sigma b \ll 1$)[5]

$$(Z_L^{(o)}/nZ_o) \simeq (i/\beta\gamma^2)(\tfrac{1}{4} + \ln \frac{b}{a}) \tag{2.5}$$

For a wall with small but finite wall impedance Z_w (defined as the ratio $-E_z/H_\theta$), one has

$$\alpha \simeq \frac{K_0(\sigma b)}{I_0(\sigma b)} + i\, \frac{\beta^2\gamma^2 c}{\omega b} \frac{Z_w}{Z_o} \tag{2.6}$$

The corresponding coupling impedance is

$$(Z_L/nZ_o) \simeq \frac{i}{\beta\gamma^2}\left(\tfrac{1}{4} + \ln \frac{b}{a}\right) + \frac{v}{\omega b} \frac{Z_w}{Z_o} \tag{2.7}$$

For a smooth wall with finite conductivitiy, the wall impedance is given by

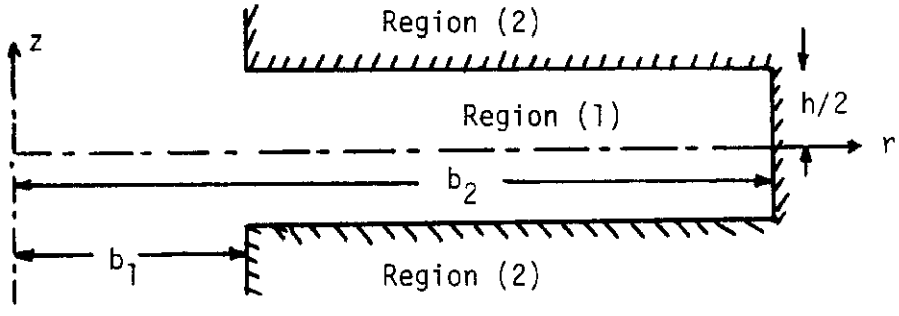
$$Z_w/Z_o = \frac{1-i}{2} \frac{\omega}{c} \delta \tag{2.8}$$

so that the contribution to the coupling impedance is

$$(\Delta Z_L/nZ_o) = \frac{1-i}{2} \frac{\beta\delta}{b} \tag{2.9}$$

III. Wall Impedance

If the wall consists of laminations of thickness D, permeability μ_2, conductivity σ_2 corresponding to a skin depth δ, separated by a dielectric of thickness h, permittivity ε_1, conductivity σ_1, the equivalent wall impedance can be calculated by treating the "crack", which extends from $r = b \equiv b_1$ to $r = b_2$, as a radial transmission line.[1,2] For the parameters appropriate to the Booster one can neglect losses in the dielectric, and consider the end of the transmission line to be a short. The geometry of the crack is as shown in the figure:



The field components for the azimuthally symmetric (outgoing) radial mode are given by

<u>Region (1)</u> $|z| < h/2$

$$E_z = A \cosh(g_1 z)\, H_0^{(1)}(kr)$$

$$E_r = -(g_1 A/k) \sinh(g_1 z)\, H_1^{(1)}(kr) \tag{3.1}$$

$$H_\theta = (-i\, \omega\varepsilon_1 A/k) \cosh(g_1 z)\, H_1^{(1)}(kr)$$

<u>Region (2)</u>

$$E_z = B\, e^{-g_2|z|}\, H_0^{(1)}(kr)$$

$$E_r = (g_2 B/k)\, e^{-g_2|z|} H_1^{(1)}(kr) \tag{3.2}$$

$$H_\theta = (\sigma_2 B/k)\, e^{-g_2|z|} H_1^{(1)}(kr)$$

where

$$k^2 = k_1^2 + g_1{}^2,$$

$$k_1{}^2 = \omega^2 \mu_1 \varepsilon_1, \tag{3.3}$$

$$k^2 = (2i/\delta^2) + g_2{}^2$$

For the parameters of interest

$$|g_2| \gg k_1 \tag{3.4}$$

Continuity of E_r and H_θ at $z = h/2$ leads to

$$g_1^2 \simeq (1 + i)(\mu_2/\mu_1)\, k_1^2(\delta/h), \quad g_2 \simeq \frac{1-i}{\delta} \tag{3.5}$$

Assuming a short at $r = b_2$, we can write for the "crack" impedance

$$\frac{Z_c}{Z_o} = -\frac{E_z(r=b_1)}{Z_o H_\theta(r=b_1)} = -\frac{ikc\varepsilon_o}{\varepsilon_1\omega}\,\frac{H_o^{(1)}(kb_1)H_o^{(2)}(kb_2) - H_o^{(2)}(kb_1)H_o^{(1)}(kb_2)}{H_1^{(1)}(kb_1)H_o^{(2)}(kb_2) - H_1^{(2)}(kb_1)H_o^{(1)}(kb_2)}$$

$$= -\frac{ikc\varepsilon_o}{\omega\varepsilon_1}\,\frac{J_o(kb_2)N_o(kb_1) - J_o(kb_1)N_o(kb_2)}{J_o(kb_2)N_1(kb_1) - J_1(kb_1)N_o(kb_2)} \tag{3.6}$$

For $\mu_2/\mu_1 \simeq 100$, $\cdot 1 < \delta/h < 1$, we can write

$$k^2 \simeq g_1^2 \simeq (1+i)\frac{\mu_2}{\mu_1}\frac{\varepsilon_1}{\varepsilon_o}\frac{\omega^2}{c^2}\frac{\delta}{h} \tag{3.7}$$

Equation (3.6) is very close to that obtained by Ruggiero (FN-220, (17)), but he includes the factor $v/\omega b_1$ in Eq. (2.7) to convert wall impedance to coupling impedance, and also calibrates the impedance to the width of the "crack".

The low frequency limit of Eq. (3.6) is

$$\frac{Z_c(\omega \to 0)}{Z_0} \simeq (1-i)\,\frac{b_1\omega}{c}\,\frac{\delta}{h}\,\frac{\mu_2}{\mu_0}\,\ln(b_2/b_1) \qquad (3.8)$$

while the high frequency limit for $b_2 \gg b_1$ is

$$\frac{Z_c(\omega\to\infty)}{Z_0} \simeq -i\frac{kc\varepsilon_0}{\omega\varepsilon_1}\,\frac{H_0^{(1)}(kb_1)}{H_1^{(1)}(kb_1)} \simeq (1+i)^{\frac{1}{2}}\left(\frac{\mu_2\varepsilon_0\delta}{\mu_0\varepsilon_1 h}\right)^{\frac{1}{2}} - i\,\frac{c\varepsilon_0}{2b_1\omega\varepsilon_1} \qquad (3.9)$$

These results are consistent with the numerical results of Snowdon, suggesting an asymptotic result varying as $\omega^{-1/4}$ with the value at 500 MHz

$$\frac{Z_c(\omega/2\pi = 500\text{MHz})}{Z_0} \simeq 1.4 + i\,0.3 \qquad (3.10)$$

compared with that of Snowdon which is

$$\frac{Z_c}{Z_0} \simeq 1.57(\text{decreasing with } \omega) + i\,0.24\ (\text{increasing with } \omega) \qquad (3.11)$$

at 500 MHz.

The contribution to the longitudinal coupling impedance can be obtained by weighting the "crack" impedance in Eqs. (3.6), (3.8), (3.9) and the wall impedance in Eq. (2.8) proportional to the thickness of each. The justification for this is that the boundary does not disturb H_θ which causes compensating currents to flow in the laminations, but does disturb the electric field pattern in such a way that

$$E_z^{av} = \frac{hE_z^{"crack"} + D\,E_z^{"iron"}}{h + D} \tag{3.12}$$

leading to

$$\frac{Z_{wall}^{equiv}}{Z_o} \simeq \frac{\frac{1-i}{2}\frac{\omega}{c}\delta\, D + (Z_c/Z_o)h}{h + D} \tag{3.13}$$

For $Z_c/Z_o \gg \omega\delta/c$, $D \gg h$, one has

$$\frac{Z_{wall}^{equiv}}{Z_o} \simeq \frac{Z_c}{Z_o}\frac{h}{D} = \begin{cases} (1-i)\dfrac{b_1\omega}{c}\dfrac{\delta}{D}\dfrac{\mu_2}{\mu_o}\,\ell n(b_2/b_1), & \text{low } \omega \\[2ex] (1+i)^{\frac{1}{2}}\left(\dfrac{\mu_2}{\mu_o}\dfrac{\varepsilon_o}{\varepsilon_1}\,\delta h\right)^{\frac{1}{2}}/D, & \text{high } \omega \end{cases} \tag{3.14}$$

The contribution to the coupling impedance is therefore

$$\frac{\Delta Z_L}{nZ_o} = \begin{cases} \hat{\alpha}(1-i)\ \beta\ \frac{\delta}{D}\frac{\mu_2}{\mu_o}\ \ln(b_2/b_1) & \text{low } \omega \\ \hat{\alpha}(1+i)^{\frac{1}{2}}\ \frac{\beta\ c}{\omega b_1}\ (\frac{\mu_2}{\mu_o}\frac{\varepsilon_o}{\varepsilon_1}\ \delta h)^{\frac{1}{2}}/D & \text{high } \omega \end{cases} \tag{3.15}$$

where $\hat{\alpha}$ is the fraction of the circumference occupied by the laminations.

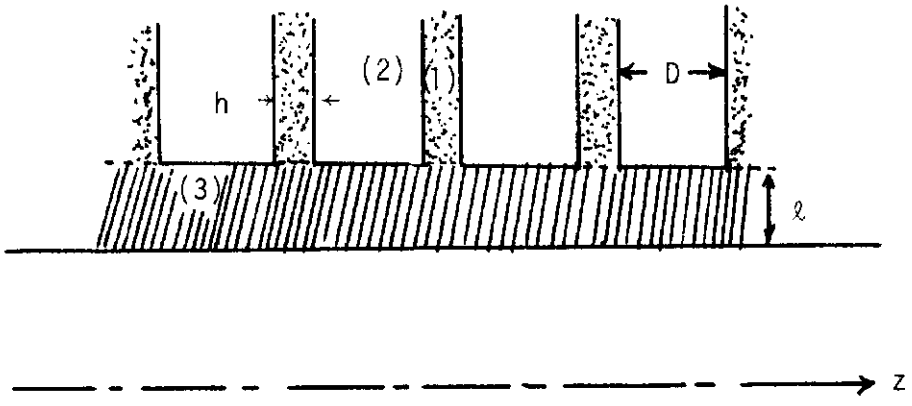
The equivalent longitudinal coupling impedance per unit length is obtained by dividing ΔZ_L by RZ_o giving

$$\frac{\Delta Z_L}{RZ_o} = \begin{cases} \hat{\alpha}\ (1-i)\ \frac{\omega}{c}\frac{\delta}{D}\frac{\mu_2}{\mu_o}\ \ln(b_2/b_1) & \text{low } \omega \\ \hat{\alpha}\ \frac{(1+i)^{\frac{1}{2}}}{b_1 D}\ \ (\frac{\mu_2}{\mu_o}\frac{\varepsilon_o}{\varepsilon_1}\ \delta h)^{\frac{1}{2}}\ , & \text{high } \omega \end{cases} \tag{3.16}$$

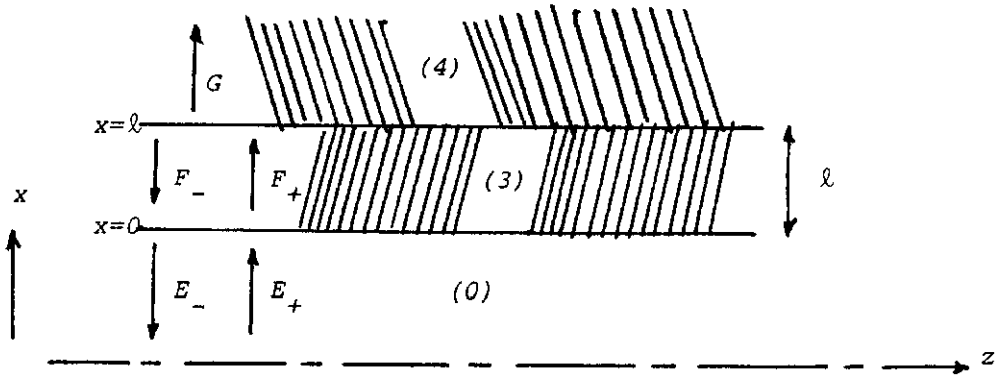
[Equation (3.16) appears to correspond to Ruggiero's Z_n (FN-220, p. 9) but is a factor 2 smaller.]

IV. Effect of a Coating

If the magnet is covered with a coating at the bore, the impedance seen by the beam will be modified.



The details will depend on the relative size of ℓ, h, D. In particular, if $\ell < h \ll D$, one expects modification of the "crack" and "iron" impedances separately due to the coating. The combination will then be weighted as in Eq. (3.13). On the other hand, if $\ell > h$, one expects the coating to modify the equivalent wall impedance in Eqs. (3.13), (3.14) directly. In either event, it is necessary to calculate the impedance seen by the combination shown,



where the medium denoted by (4) is either the crack, the iron, or the combined equivalent. We will assume the validity of rectangular geometry, and will ignore variation with z, which takes place over distances much larger than ℓ.

For plane waves traveling in the x direction, we have

$$E_o = E_+ e^{ik_o x} + E_- e^{-ik_o x}$$
$$Z_o H_o = E_+ e^{ik_o x} - E_- e^{-ik_o x} \tag{4.1}$$

$$E_3 = F_+ e^{ik_3 x} + F_- e^{-ik_3 x}$$
$$Z_3 H_3 = F_+ e^{ik_3 x} - F_- e^{-ik_3 x} \tag{4.2}$$

$$E_4 = G\, e^{ik_4(x-\ell)}$$
$$Z_4 H_4 = G\, e^{ik_4(x-\ell)} \tag{4.3}$$

Continuity of E and H at the interfaces yields

$$\frac{F_+ e^{ik_3\ell}}{F_- e^{-ik_3\ell}} = \frac{Z_4 + Z_3}{Z_4 - Z_3} \tag{4.4}$$

$$\frac{1}{Z_3}\frac{E_o}{H_o} = \left.\frac{E_3}{Z_3 H_3}\right|_{x=0} = \frac{F_+ + F_-}{F_+ - F_-} \tag{4.5}$$

The equivalent impedance Z, seen at $x = 0$ is therefore given by

$$\frac{Z}{Z_3} = \frac{Z_4\cos(k_3\ell) - i\, Z_3\sin(k_3\ell)}{Z_3\cos(k_3\ell) - i\, Z_4\sin(k_3\ell)}$$

$$= \frac{Z_4\{1-i(Z_3/Z_4)\tan(k_3\ell)\}}{Z_3\{1-i(Z_4/Z_3)\tan(k_3\ell)\}} \tag{4.6}$$

Here

$$Z_3 = \sqrt{\frac{-i\omega\mu_3}{\sigma_3-i\omega\varepsilon_3}} \quad , \qquad k_3 = \sqrt{i\omega\mu_3(\sigma_3-i\omega\varepsilon_3)} \tag{4.7}$$

Equation (4.6) can be simplified in various limiting cases. For example, if

1) $|k_3\ell| << 1$

$$\frac{Z}{Z_4} \simeq 1 - ik_3\ell\left(\frac{Z_3}{Z_4} - \frac{Z_4}{Z_3}\right)$$

$$\simeq 1 - i\ell\left\{\frac{\omega\mu_3}{Z_4} - Z_4\,(\omega\varepsilon_3+i\sigma_3)\right\} \tag{4.8}$$

2) $\sigma_3 >> \omega\varepsilon_3$

$$\frac{Z}{Z_4} \simeq \frac{1 - \dfrac{(i+1)\omega\mu_3\delta_3}{2Z_4}\tan\left(\dfrac{1+i}{\delta_3}\ell\right)}{1 + \dfrac{(1-i)Z_4}{\omega\mu_3\delta_3}\tan\left(\dfrac{1+i}{\delta_3}\ell\right)} \tag{4.9}$$

where $k_3 \simeq (1+i)/\delta_3$, $Z_3 \simeq \frac{1-i}{2}\omega\mu_3\delta_3$, $\delta_3{}^2 = \frac{2}{\omega\mu_3\sigma_3}$ (4.10)

In the general case, Eq. (4.6) can be used to calculate the effect of the coating.

IV. Energy Loss

For a beam bunch with longitudinal charge density

$$f(z) = \sum_{n=-\infty}^{\infty} f_n \, e^{inz/R} \, , \quad f_n = \frac{1}{2\pi R} \int dz \, e^{-inz/R} f(z) \tag{5.1}$$

one has for the harmonic distribution of the circulating current

$$I(z,t) = I_0 \sum_{n=-\infty}^{\infty} \frac{f_n}{f_0} \, e^{in(z-vt)/R} \tag{5.2}$$

The energy loss per turn, for a particle located at $z - vt = \xi$ relative to the center of the bunch, is

$$\Delta W = e \, \mathrm{Re} \int_0^{2\pi R} dz \, E_z(z, t=(z-\xi)/v) \tag{5.3}$$

(Re ≡ real part)

Since the coupling impedance is defined by the Fourier decomposition of $E_z(z,t)$ as

$$2\pi R \, E_z(z,t) = I_0 \sum_{n=-\infty}^{\infty} Z_L(n) \frac{f_n}{f_0} \, e^{in(z-vt)/R} \tag{5.4}$$

we obtain

$$\Delta W(\xi) = eI_0 \sum_{n=-\infty}^{\infty} \frac{f_n}{f_0} \, \mathrm{Re} \, \{Z_L(n) \, e^{in\xi/R}\} \tag{5.5}$$

which is close to Ruggiero's result (FN-230 (7)).

Since we have h bunches in the ring, where h is the harmonic number, we have

$$f(z) = \sum_{j=1}^{h} \rho(z - \frac{2\pi R}{h} j) \tag{5.6}$$

where ρ is the density distribution of a single bunch. We therefore have, from Eq. (5.1),

$$f_n = \frac{1}{2\pi R} \int dz\, \rho(z)\, e^{-inz/R} \sum_{j=1}^{h} e^{-i\frac{2\pi jn}{h}} \tag{5.7}$$

Since

$$\sum_{j=1}^{h} e^{-i2\pi jn/h} = \begin{cases} h, & n=0,\pm h,\pm 2h, \ldots \\ 0, & \text{all other } n \end{cases} \tag{5.8}$$

we have

$$\frac{f_n}{f_o} = \begin{cases} \dfrac{\int dz\, \rho(z)\, e^{-inz/R}}{\int dz\, \rho(z)}, & n = 0,\pm h,\pm 2h, \ldots \\ 0, & \text{all other } n \end{cases} \tag{5.9}$$

and the sum in Eq. (5.5) only goes over integral values of n/h. For a beam bunch of length ℓ, the cutoff in n equivalent to Eq. (5.1) is

$$n_o \sim 2\pi R/\ell \tag{5.10}$$

Using the estimates of coupling impedance in Eq. (3.16), and $n_o \sim 1000$, one obtains an energy loss of the order 100 keV per turn at the center of the bunch which is in agreement with that estimated by Ruggiero (FN-230).

The procedure outlined above should be valid as long as the r.f. wave length is longer than the width of the laminations if they extend all around the circumference. The situation is less clear for wave lengths shorter than individual magnet lengths, but we will accept Ruggiero's and Snowdon's concept of a "filling factor" $\hat{\alpha}$, as contained in Eqs. (3.15) and (3.16).

As a final comparison, Ruggiero's result for the energy loss per particle per turn is

$$\Delta\hat{W}(\xi) = Ne^2\beta c \sum_{n=0}^{\infty} \frac{f_n}{f_o} \operatorname{Re} \{\hat{Z}_L(n) e^{in\xi/R}\} \tag{5.11}$$

where $\hat{Z}_L(n)$ corresponds to his definition of impedance. Using Ruggiero's asymptotic value $\operatorname{Re} \hat{Z}_L(n) = 3.5\text{x}10^{-13}$, with $N = 4.2\text{x}10^{10}$ protons per bunch, one obtains for $\beta = 1$,

$$\Delta\hat{W}(\xi) \simeq 60 \text{ eV} \sum_{n=0} \frac{f_n}{f_o} e^{in\xi/R} \tag{5.12}$$

Our analogous result for $I_o = 335$ mA, $\hat{\alpha} = .6$, is given in Eqs. (5.5) and (3.16):

$$\Delta W(\xi) \simeq 8 \text{ keV} \sum_{n=0}^{\infty} \frac{f_n}{f_o} e^{in\xi/R} \tag{5.13}$$

where the sum over n in Eq. (5.13) includes only multiples of h, the harmonic number. Assuming a smooth variation of f_n with n, Eq. (5.13) can be rewritten as

$$\Delta W(\xi) \simeq 95 \text{ eV} \sum_{\text{all } n}^{\infty} \frac{f_n}{f_o} e^{in\xi/R} \tag{5.14}$$

in reasonable agreement with Ruggiero. For n_o ~1000, the energy loss is of order 50 to 100 keV per particle per turn.

V. References

1. S.C. Snowdon, Wave Propagation Between Booster Laminations Induced by Longitudinal Motion of Beam, TM-277, November 3, 1970.

2. A.G. Ruggiero, Longitudinal Space Charge Forces Within Bunched Beams, FN-219, December 15, 1970; Longitudinal Space Charge Forces Within a Bunched Beam in the Presence of Magnetic Laminations, FN-220, January 1971; Energy Loss Due to the Resistive Magnet Lamination in the FNAL Booster, FN-230, May 25, 1971.

3. See, for example, Nielsen, Sessler and Symon, Proc. of the International Conference on High Energy Accelerators, Geneva, 1959, p. 239; A.W. Chao, 1982 Summer School Lectures, SLAC, p. 396.

4. R.L. Gluckstern, Longitudinal Coupling Impedance for a Cavity and Beam Pipe, TM-1268, June 1984.

5. Some authors give a result where the 1/4 is replaced by 1/2. The result in Eq. (2.5) comes from averaging the electric field over the beam cross section, rather than taking its value at $r = 0$.