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A method is given for reducing the most troublesome sextupole harmonics in a ring by measuring the sextupole field in groups of magnets, and ordering them according to a predetermined prescription. The predicted result is a decrease in sextupole related distortions by a factor $\sqrt{2}/(J+1)^{\frac{1}{2}}$ where J magnets, covering one or more betatron periods, are measured at one time. Simulations performed for typical SSC lattices confirm the expected improvements.

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Summary

A method is given for reducing the most troublesome sextupole harmonics in a ring by measuring the sextupole field in groups of magnets, and ordering them according to a predetermined prescription. The predicted result is a decrease in sextupole related distortions by a factor $[2/(J+1)]^{1/2}$ where J magnets, covering one or more betatron periods, are measured at one time. Simulations performed for typical SSC lattices confirm the expected improvements.

Introduction

The presence of unavoidable normal and skew sextupole errors in the dipoles of a superconducting ring are known to cause nonlinear oscillations in the beam size, and a resultant decrease in the dynamic aperture of the ring. Although the growth is not completely described by one or two "resonances", the harmonic description of the effect is useful in pointing out which regions of the harmonic spectrum are most troublesome. Such an analysis in first order in the sextupole amplitudes leads to the conclusion that beam size oscillation due to the $n^{\underline{th}}$ harmonic of the sextupole error, ε_n, are proportional to ε_n and inversely proportional to the resonance denominators $|\nu - n|$ and $|3\nu - n|$, where the x and y tunes, ν_x and ν_y are taken to be equal to ν.

If all sextupole errors are known, it is conceivable that the order of the magnets could be chosen so that all harmonics in broad bands around $n = \nu$ and $n = 3\nu$ could be made sufficiently small, thus reducing the beam size oscillations. Such "shuffling" is impractical, however, since it requires measurements on all magnets before any can be positioned. For this reason, we propose an alternate scheme for measuring and positioning J magnets at a time, which is capable of reducing the expected value of <u>all</u> harmonics in the troublesome band, and therefore the beam size oscillations, by a factor of order $J^{-1/2}$.

Description of Shuffling Scheme

For our analysis, we will consider a regular lattice consisting of MJ magnets, where M is an integer near the tune ν, and where J magnets cover an integral number of magnet focussing periods. After measuring the first group of J magnets, we will place them in an order to be specified later, correlated with the size of the sextupole error. The next group of J magnets are then measured and placed in an order which is anti-correlated with that in the first group, i.e., the magnet with the $j^{\underline{th}}$ lowest (most negative) sextupole error in the second group is placed in the position corresponding to that of the $j^{\underline{th}}$ highest (most positive) sextupole error in the first group. The process is then continued until all magnets have been measured and positioned. Aside from statistical fluctuations in the size of the $j^{\underline{th}}$ highest sextupole error, we then have a sextupole error which repeats with a sign change every J magnets. (Note that the parameters β_x and β_y repeat every J magnets.) Thus, we have now created systematic sextupole harmonics of order M/2, 3M/2, 5M/2, etc. All other harmonics have been reduced in size because the width of the distribution of the $j^{\underline{th}}$ highest sextupole error is reduced from that of the total distribution by a factor of order $J^{-1/2}$.

Analysis of Shuffling Scheme

The standard action-angle treatment of the third integer resonances[1] leads to five driving terms in the Hamiltonian, each of which can be represented as a sum of harmonics[2]. The form of the five coefficients is illustrated by a typical n^{th} harmonic coefficient written in phase amplitude form as

$$B_n e^{i\beta_n} = (i/16\pi) \sum_{m=1}^{M} \sum_{j=1}^{J} S_{mj} (\beta_x)^{3/2} e^{in\theta_{mj} + iQ_j} \tag{1}$$

Here S_{mj} is the integrated strength of the $j^{\underline{th}}$ sextupole in the $m^{\underline{th}}$ group of magnets and the amplitude functions β_x and β_y depend only on j because of the periodicity of the lattice. The phase Q_j reflects the difference between the phase $\nu\theta$ and the phase $\int ds/\beta$ within a cell and is therefore also independent of m. The independent variable θ increases by 2π in one revolution, and θ_{mj} is given by

$$\theta_{mj} = \theta_j + \left(\frac{m-1}{M}\right) 2\pi$$

because of our measurement and placement scheme.

If we now perform the sum over m first in Eq. (1), we have

$$F_j \equiv \sum_{m=1}^{M} S_{mj} e^{2\pi i n m/M} \tag{2}$$

In order to proceed further, we now evaluate $\langle F_j \rangle$ and $\langle F_j F_{j'}^* \rangle$, statistical averages over the distribution of sextupole errors. If we assume a distribution of sextupole errors, p(s), symmetric around s = 0, the distribution of the $j^{\underline{th}}$ highest [$(J+1-j)^{\underline{th}}$ lowest] sextupole error is given by

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$$P_j(s)ds = \frac{J!}{(J-j)!(j-1)!}$$

$$[\int_{-\infty}^{s} dx\, p(x)]^{J-j}[\int_{s}^{\infty} dx\, p(x)]^{j-1}\, p(s)\, ds \quad (3)$$

with

$$P_{J+1-j}(s) = P_j(-s) \quad (4)$$

One then finds, assuming M is even,

$$\langle F_j\rangle = \prod_{\ell=1}^{J} \int_{-\infty}^{\infty} ds_\ell\, P_j((-1)^\ell s_\ell) \sum_{m=1}^{M} S_{mj} e^{2\pi i n m/M} \quad (5)$$

$$= \langle s^{(j)}\rangle \Big[\sum_{\substack{m=2\\ \text{even}}}^{M} e^{2\pi i n m/M} - \sum_{\substack{m=1\\ \text{odd}}}^{M-1} e^{2\pi i n m/M} \Big] , \quad (6)$$

where

$$\langle s^{(j)}\rangle \equiv \int_{-\infty}^{\infty} P_j(s)\, s ds = -\langle s^{(J+1-j)}\rangle \quad (7)$$

It is now clear that the bracket in Eq. (6) vanishes for all n except $n = rM/2$ where r is an odd integer. We then obtain

$$\langle F_j\rangle = \begin{cases} M\langle s^{(j)}\rangle & , n = rM/2,\ r \text{ odd} \\ 0 & , \text{all other } n \end{cases} \quad (8)$$

In a similar way, we find

$$\langle F_j F^*_{j'}\rangle = \langle F_j\rangle\langle F^*_{j'}\rangle + M g_{jj'} \quad (9)$$

where

$$g_{jj'} \equiv \langle s^{(j)} s^{(j')}\rangle - \langle s^{(j)}\rangle\langle s^{(j')}\rangle .$$

For the uniform distribution of sextupole errors

$$p(s) = \begin{cases} 1/(2\Delta) & , |s| < \Delta \\ 0 & , |s| > \Delta \end{cases} ,$$

one finds

$$g_{jj'} = \frac{4 j_< (J + 1 - j_>)\, \Delta^2}{(J+1)^2 (J+2)} \quad (10)$$

where $j_<, j_>$, are the smaller, larger of j, j'.

One now can write Eq. (1) in the form

$$\langle B_n e^{i\beta_n}\rangle =$$

$$\begin{cases} (iM/16\pi) \sum_{j=1}^{J} s^{(j)} \beta_x^{3/2}\, e^{iQ_j + in\theta_j} & , n = rM/2,\ r \text{ odd} \\ 0 & , \text{all other } n \end{cases} \quad (11)$$

and, for all n,

$$\langle B_n^2\rangle = |\langle B_n e^{i\beta_n}\rangle|^2 + \frac{M}{256\pi^2} \sum_{\substack{j=1\\ j'=1}}^{J} (\beta_{xj}\beta_{xj'})^{3/2}\, e^{i(Q_j - Q_{j'})}\, e^{in(\theta_j - \theta_{j'})}\, g_{jj*} \quad (12)$$

We can only evaluate the sum over j in $\langle B_n^2\rangle$ explicitly for a given lattice and ordering arrangement. However, we can estimate the sum by assuming constant β_{xj} and Q_j, and by neglecting the $j \neq j'$ correlation terms in the expectation of some cancellation due to the phase variation of $n(\theta_j - \theta_{j'})$. In this way we find, for $n \neq rM/2$,

$$\langle B_n^2\rangle \simeq \frac{\langle\beta_x^3\rangle}{256\pi^2} \cdot \frac{MJ\Delta^2}{3} \cdot \frac{2}{J+1} \quad (13)$$

The first factor on the right comes from Ohnuma's normalization of B_n, the second comes from the rms value of the sextupole error, and the third is the reduction factor coming from our shuffling scheme, and correctly becomes unity for no shuffling ($J = 1$ magnet in each "group").

Although we have considered here only one of the five distortion parameters, our shuffling scheme will simultaneously reduce all five by approximately the same factor, since the basic reduction comes from the reduced rms width of the distribution $P_j(s)$.

It should also be pointed out here that the specific result in Eq. (13) is valid only for a uniform distribution of sextupole errors. A closed form can also be obtained for a parabolic distribution, but the algebra required is much lengthier.

Shuffling Within a Group

The remaining calculation of $\langle B_n e^{i\beta_n}\rangle$ in Eqs. (11) and (12) depends in detail on the choice of the ordering scheme within a group, which is reflected in the values of θ_j. Although the β dependent factor in Eq. (12) and in one other driving term is β_x^3, the other three driving terms contain the factor $\beta_y^2\beta_x$. The approximate constancy of $\beta_x + \beta_y$ implies that the sensitivity to variations of β_x and β_y is most acute for the driving terms with β_x^3. This suggests that magnets with the highest and lowest sextupole error should be placed where β_x is smallest, that is, near the quadrupole which is defocussing in the x direction. Also, it is desirable to alternate the signs of the sextupole errors in adjacent magnets so that $\langle B_n e^{i\beta_n}\rangle$ in Eq. (11) will be significant only for large r. One possible arrangement between focussing magnets might be

defocussing quad

$j = 1$ dipole, highest S

$j = 2$ dipole, 2nd lowest S

$j = 3$ dipole, 3rd highest S

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focussing quad

$j = J - 2$ dipole, 3rd lowest S

$j = J - 1$ dipole, 2nd highest S

$j = J$ dipole, lowest S

defocussing quad

This scheme concentrates the harmonic content into the

immediate vicinity of $r = J$.

Similar schemes are possible if J is an integral multiple of the number of magnets in a betatron oscillation period.

Approximate Sum Rule

In the approximation of constant β_x^3, it is easy to show from Eq. (1) that

$$\sum_{n=1}^{MJ} B_n^2 = \frac{\langle\beta_x^3\rangle}{256\pi^2} \sum_{m=1}^{M} \sum_{j=1}^{J} S_{mj}^2 \qquad (14)$$

Thus, the sum of the squares of the harmonic amplitudes cannot be changed by shuffling. It is therefore apparent that any shuffling arrangement merely moves the sextupole harmonic content from one region of the spectrum to another. In our scheme we have depleted the harmonic content in all harmonics with $n \neq rM/2$ at the expense of enhancing the ones with $n = rM/2$, which cause much less sextupole distortion. In particular, the shuffling scheme suggested within a group concentrates the harmonic content into the ones close to $n = MJ/2$ which cause little multipole distortion.

Two Parameter Shuffling

There are circumstances where one wants to reduce two independent error families at the same time. One such example occurs in magnets with uncorrelated but comparable normal and skew sextupole errors. Another would occur in any 2-in-1 magnet assembly. A third example might be uncorrelated quadrupole and sextupole errors of comparable magnitude in a single dipole magnet.

We expect that the shuffling process outlined earlier would still work, but now one must order the errors into two parameter "bins". As a result the improvement factor is expected to be significantly reduced from that for one parameter shuffling.

Numerical Results

Simulations have been performed for several SSC lattices, including insertions. In those cases where the number of magnets in a superperiod is not an integral multiple of twice the number of magnets in a betatron period, a few of the "best" magnets are set aside for the unbalanced group, and the remaining ones distributed according to the original prescription. Four* of the five driving terms have been evaluated with ten independent sets of random sextupoles, first with a truly random placement, and then with our suggested ordering. As a final figure of merit, we obtain

$$\lambda_\ell = \sum_n B_{n\ell}^{ordered} / \sum_n B_{n\ell}^{random} \qquad (15)$$

for each of the four ($\ell = 1 - 4$) driving terms, where the sum over harmonics n is taken over the 100 harmonics centered at M or 3M as appropriate. Futhermore, the numerator and denominator in Eq. (15) are averages over the ten independent random sets of errors.

The following table indicates the results for uniform (U) or gaussian (G) error distributions for six (A - F) sample lattices.

	Figure of Merit				Predicted
Lattice	λ_1	λ_2	λ_3	λ_4	$\left(\frac{2}{J+1}\right)^{1/2}$
A_U	.15	.23	.13	.23	.22
A_G	.11	.21	.19	.28	.22
B_U	.08	.09	.33	.28	.18
B_G	.09	.14	.30	.32	.18
C_U	.12	.13	.37	.36	.28
C_G	.12	.19	.32	.37	.28
D_U	.24	.25	.30	.31	.34
D_G	.19	.33	.25	.36	.34
E_U	.13	.16	.43	.40	.28
E_G	.14	.22	.37	.44	.28
F_U	.08	.30	.09	.27	.20
F_G	.08	.15	.08	.10	.20

The prescribed ordering appears to yield improvements which are in most cases as good as predicted, for either uniform or gaussian distributions.

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References

1. See for example S. Ohnuma, AIP Conf. Proceedings No. 123 (1984), p. 415.

2. T.L. Collins (Fermilab-84/114) discusses the need for five distortion parameters in a non-harmonic Green's function type approach. He correctly points out the danger of manipulating a few "dominant" harmonics without worrying about a large number of nearby harmonics. As will be seen later, our scheme reduces all but a few harmonics at the same time.

*The fifth differs only slightly from the fourth in the phase term Qj.