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HEAT CONDUCTION THRU SUPPORTS BETWEEN 300°K AND 4°K

Thomas H. Nicol

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Thomas H. Nicol
Fermi National Accelerator Laboratory
P.O. Box 500
Batavia, Illinois 60510

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INTRODUCTION

Of fundamental importance in the design of any superconducting machinery is a reasonable prediction of the heat flow between the outside world at 300°K and a helium volume at roughly 4°K. As machines grow larger and energy costs higher the accuracy with which these estimates are made becomes more critical. Unfortunately, as cryogenic vessels become more complex so does the procedure for making estimates accurately.

PRELIMINARY ANALYSIS

If the support in question is homogeneous (i.e. of one material) between two known temperatures the calculation of heat flow is straightforward assuming one has k vs. T data for the selected material. That is, thermal conductivity as a function of temperature. From this data the thermal conductivity integral can be approximated and a reasonable estimate of heat flow determined.

Suppose, for example, we have the following data for 304 stainless steel (see ref. 4).

T(°K)	k(W/cm°K)
300	0.15
250	0.14
200	0.13
180	0.123
160	0.120
140	0.110
120	0.103
100	0.095
90	0.090
80	0.083
70	0.076
60	0.068
50	0.058
40	0.047
35	0.040
30	0.033
25	0.026
20	0.0195
15	0.0132
10	0.0077
8	0.0057
6	0.0039
4	0.0024

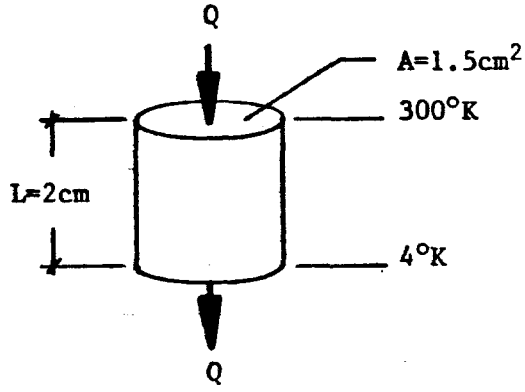
Numerical integration of this data yields the following full and intermediate results.

$$\int_{4}^{300} k\,dT = 30.6 \text{ W/cm}$$

$$\int_{80}^{300} k\,dT = 27.12 \text{ W/cm}$$

$$\int_{4}^{80} k\,dT = 3.48 \text{ W/cm}$$

Given these integrals and the example below,



the heat flow, Q, is determined by

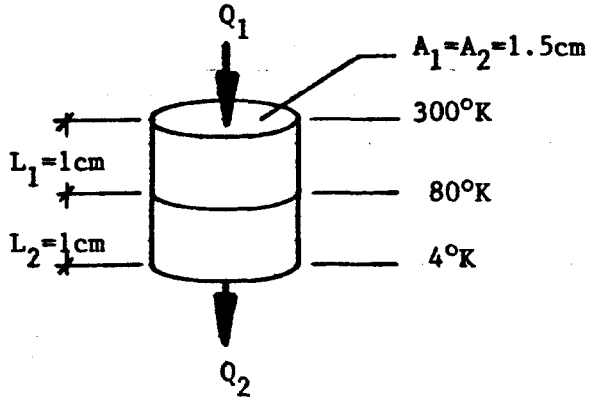
$$Q = \frac{A}{L} \int_{4}^{300} k dT$$

or

$$Q = 22.95 \text{ W}$$

(Note that the effects of radiation and convection heat transfer are ignored in this and all subsequent discussions.)

More realisticaly we might want to tie some intermediate point to an LN_2 sink. This too is easily handled if the material between any two known temperatures is homogeneous. Given the second example below,



heat flows, Q_1, and Q_2, are determined by

$$Q_1 = \frac{A_1}{L_1} \int_{80}^{300} k dT$$

or

$$Q_1 = 40.68 \text{ W}$$

and

$$Q_2 = \frac{A_2}{L_2} \int_{4}^{80} k dT$$

or

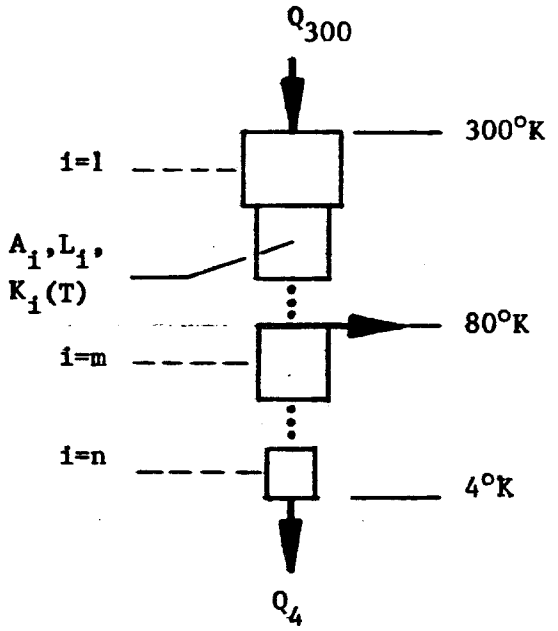
$$Q_2 = 5.22 \text{ W.}$$

In this second example, both materials need not have been the same. It is only necessary that the material between 300°K and 80°K and between 80°K and 4°K be uniform, and that the thermal conductivity integrals for both materials be known. Four typical curves of k vs. T can be found in appendix A of this report. For additional information see references 1 and 4.

GENERALIZATION FOR COMPOSITE SECTIONS

It is oftentimes desirable to use composite sections between known temperature points, generally for structural strength or stability considerations. A composite section being defined as one consisting of more than one material.

Let some arbitrary support between 300°K and 4°K with an optional 80°K strap be depicted as follows:



where the materials in sections 1 through n may be arbitrarily selected.

As in the previous examples, the heat flow through any given section, i, is governed by

$$Q_i = \frac{k_i \; A_i(\Delta T_i)}{L_i} \tag{1}$$

where the conventional minus sign is omitted for clarity of the following discussions. Using the integrated thermal conductivity this becomes

$$Q_i = \frac{A_i}{L_i}\int_{T_{i+1}}^{T_i} k_i \; dT. \tag{2}$$

Unlike the previous two examples, however, the values of T_i, T_{i+1}, and consequently $\int k_i \, dT$ are unknown. All we know immediately is that

$$Q_i = Q_{i+1} = \ldots = Q_m$$

and

$$Q_{m+1} = Q_{m+2} = \ldots = Q_n$$

due to compatibility requirements.

This being the case it becomes convenient to define an element resistance term as

$$R_i = \frac{L_i}{k_i A_i}$$

and to rewrite (1) as

$$\Delta T_i = \frac{Q_i L_i}{k_i A_i} \tag{3}$$

or

$$\Delta T_i = Q_i R_i \quad . \tag{4}$$

Now if we look at the section from i = 1...m, for example, we have

$$\Delta T_{1\ldots m} = Q_{1\ldots m} \sum_{i=1}^{m} R_i$$

or

$$(300°K - 80°K) = Q_{300} \sum_{i=1}^{m} R_i$$

Similarly,

$$(80°K - 4°K) = Q_4 \sum_{i=m+1}^{n} R_i$$

SOLUTION TECHNIQUE

Bear in mind that the unknown thermal conductivities are still embedded in the various R_i terms. As a general rule, analytic functions for $k=f(T)$ do not exist so we must resort to some numerical technique for a solution to (1) or (4).

One such technique involves iterating Q over the known temperature regions until the conditions of equilibrium are satisfied. Or more simply stated, because the governing equations are linear in all parameters, we know that only one value for Q exists which will yield the known ΔT across a given region. Our task is to determine this Q.

This procedure is outlined as follows:

a) Choose a range between which Q must fall. Initially this can be defined by

$$Q_{LO} = 0$$

and

$$Q_{HI} = \frac{\Delta T}{R_i(\min)}$$

where the $R_i(\min)$ terms are evaluated using the maximum values for k between the extreme temperature points. ΔT is the known temperature drop across the entire section. Initially let

$$Q = \frac{Q_{HI} - Q_{LO}}{2} .$$

b) Using Q from (a) and starting at i=1, calculate ΔT_i using (4) where k is initially equal to the maximum k in (a). Calculate a new k_{mean} by interpolation of k vs. T using ΔT_i. Compute a new ΔT_i based on k_{mean}. Repeat until successive ΔT_i values differ by some small value, ε.

c) Repeat (b) for each section, 1 through m for example, summing all final values of ΔT_i.

d) Given the actual and computed ΔT's determine a new range for Q. That is, if $\Delta T_{calc} > \Delta T_{actual}$ than set $Q_{HI} = Q$, otherwise set $Q_{LO} = Q$.

e) Again, let Q equal the average of the newly specified range. Repeat (b), (c), and (d) using this interval halving procedure until $\Delta T_{calc} \approx \Delta T_{actual}$

If a strap to LN_2 is specified, steps (a) - (e) must be performed twice, once for i = 1...m (ΔT_{actual} = 220°K), another for i = m+1...n (ΔT_{actual} = 76°K). If no strap is specified they need be performed only once, for i = 1...n (ΔT_{actual} = 296°K).

COMPUTER SOLUTION

It should be clear from the preceding discussion that performing such a set of steps by hand is a tedious and error prone task. For this reason an interactive FORTRAN program has been written which performs the operations in steps (a) through (e) for a given geometry and selected materials for the case with or without an LN_2 strap. It resides publicly in UN=93380 on the Fermilab CYBER system and can be executed by the following control statements.

```
GET, CRYOSPT/UN=93380
CRYOSPT
```

At this writing, thermal conductivity data is available for the following materials (300°K -4°K):

Stainless, 304	(STN)
OFHC copper	(COP)
Manganin	(MNG)
Brass	(BRS)
Aluminum, 6063	(ALM)
Steel, 1020	(STL)
Teflon	(TFE)
Glass	(GLS)
G-10	(G10)

Abbreviations used in the program are shown in parentheses. Sections may be rectangular, circular or tubular with a maximum of 10 allowable. Prompts should be self explanatory. If not, contact this author with comments or questions.

Sample problems can be found in appendix B. Two of these provide data which may be used to compare the numerical method described and the conventional method described in the preliminary analysis section of this report. As can be seen in these cases, the results agree to within less than one percent, clearly better than the certainty with which we know k vs. T for most materials. Some validation of results, of course, is always required, particularly in the case of complex geometries, however, in the majority of cases this method is expected to yield valid heat flow estimates.

REFERENCES

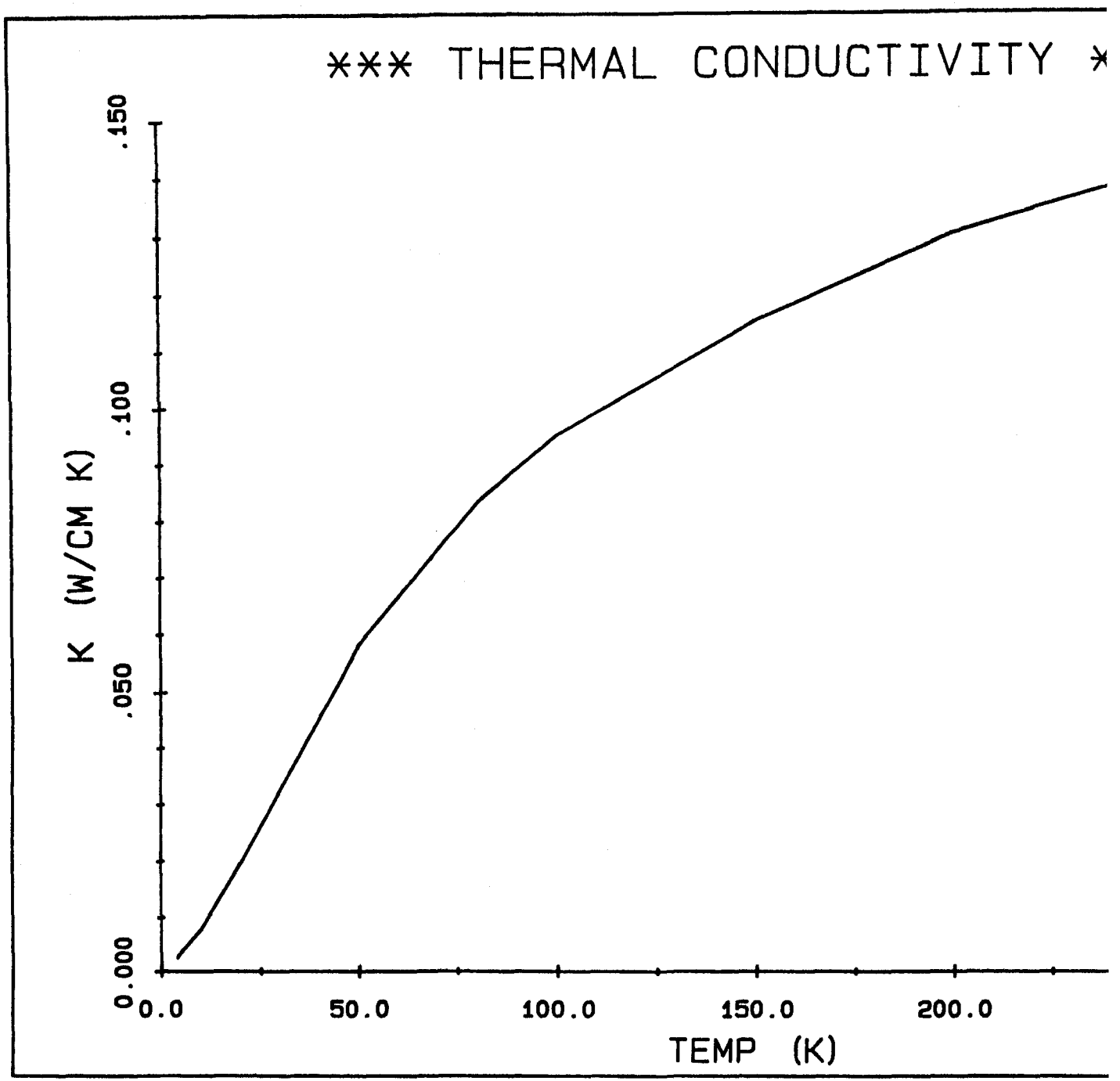
1. Battelle Columbus Laboratories, HANDBOOK ON MATERIALS for SUPERCONDUCTING MACHINERY, January 1977.

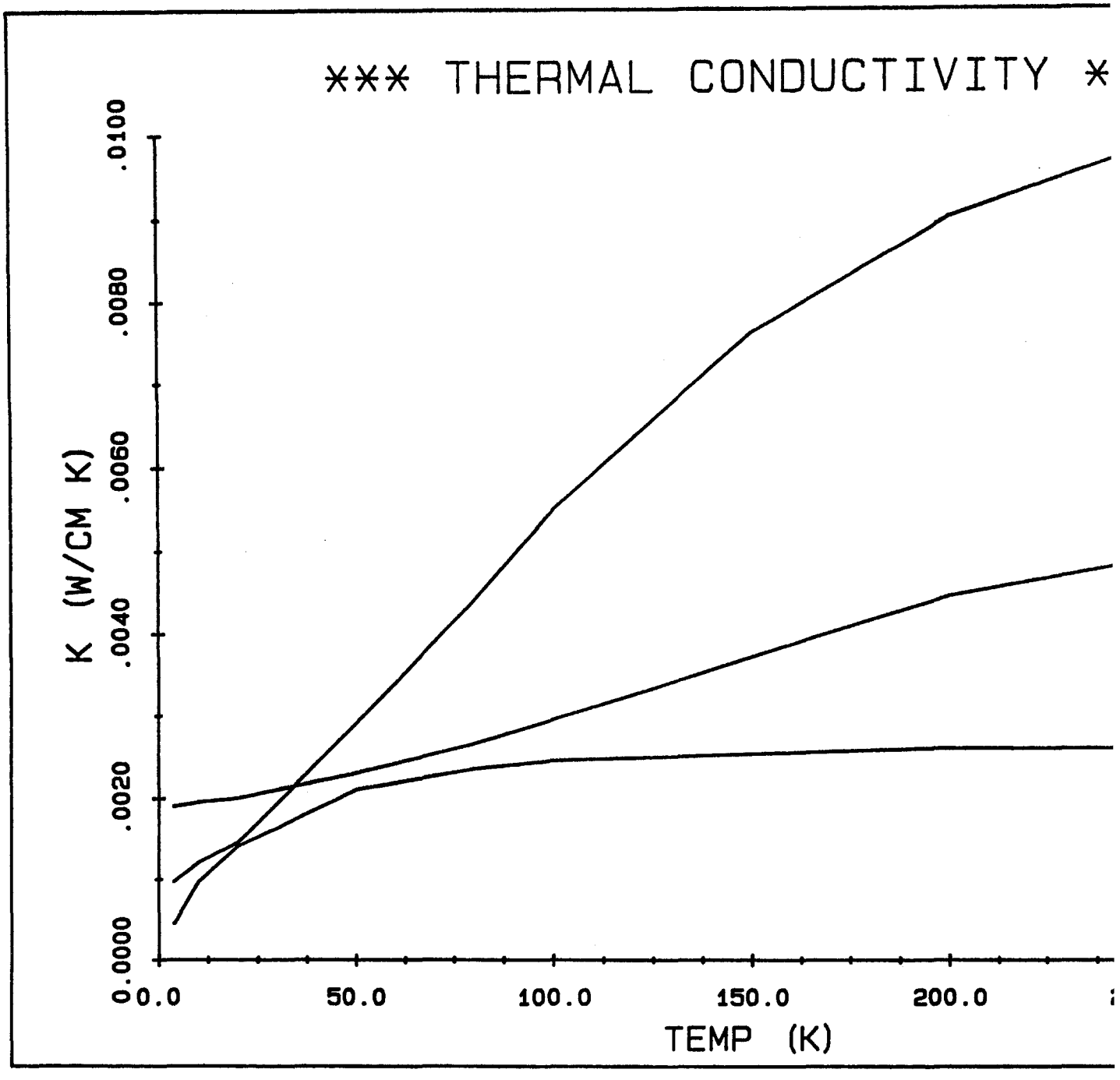
2. Granet, Irving, THERMODYNAMICS AND HEAT POWER, Virginia: Reston Publishing Co., Inc., 1974.

3. Scott, Russel B., CRYOGENIC ENGINEERING, New Jersey: D. Van Nostrand Co., Inc., 1959.

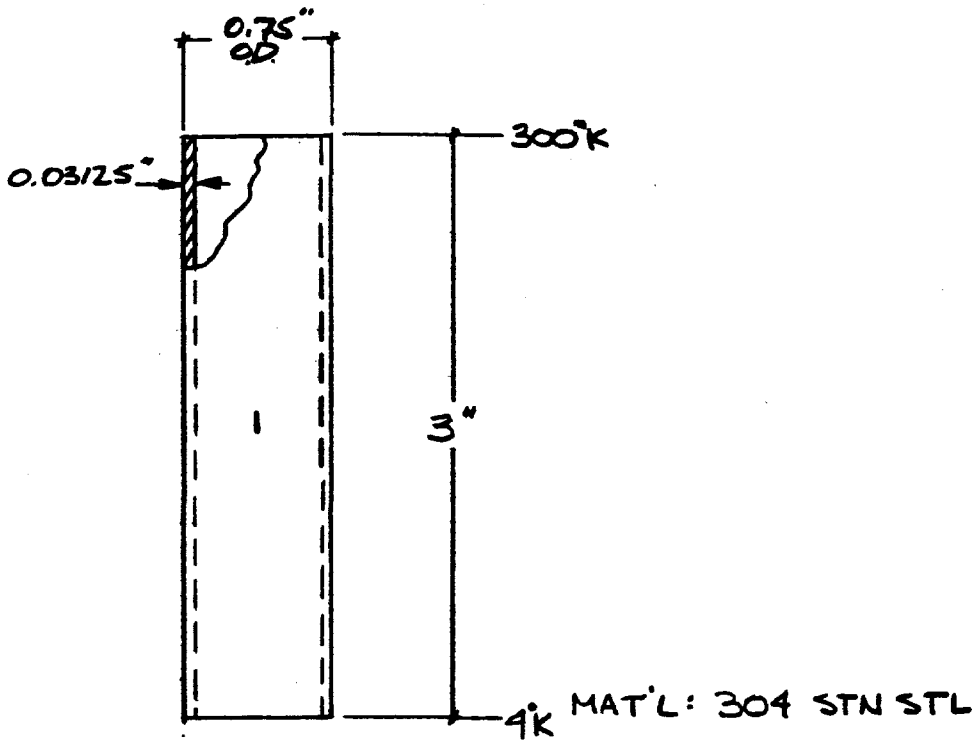
4. Stewart, Richard B., et al, A COMPENDIUM OF THE PROPERTIES OF MATERIALS AT LOW TEMPERATURE (PHASE II), National Bureau of Standards, 1961.

Appendix A


*** THERMAL CONDUCTIVITY *
K (W/CM K)
.150
.100
.050
0.000
0.0
50.0
100.0
150.0
200.0
TEMP (K)


*** THERMAL CONDUCTIVITY *
K (W/CM K)
0.0000
.0020
.0040
.0060
.0080
.0100
0.0
50.0
100.0
150.0
200.0
TEMP (K)

Appendix B



$$Q = \frac{A}{L}\int_4^{300} K\,dT$$

$$Q = \frac{0.0706\ in^2}{3\ in} \times 2.54\ \frac{cm}{in} \times 30.6\ \frac{W}{cm}$$

$$Q = 1.828\ W$$

"CRYOSPT" yields $Q = 1.836\ W$ (0.438% Δ)

```
g,cryospt/un=93380
/cryospt

                    *** CRYOSPT ***

THIS PROGRAM CALCULATES THE HEAT FLOW DOWN A SUPPORT
COLUMN CONSISTING OF AN ASSORTMENT OF DIFFERENT MATERIALS.

SECTION #1 IS AT THE 300K POINT (X=0)...SECTION #N IS AT
THE 4K POINT (X=L).

YOU MAY ALSO SPECIFY A STRAPPING POINT TO AN 80 K SHIELD.
THIS MUST OCCUR BETWEEN TWO OF THE DEFINED SECTIONS.

CONTACT T. NICOL (EXT. 3441) WITH QUESTIONS OR COMMENTS...
                          .
                          .
                          .
[RETURN] TO CONTINUE...
?


*** SECTION # 1 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN   2. G10   3. COP   4. MNG   5. BRS
 6. ALM   7. STL   8. TFE   9. GLS

 1. RECTANGULAR   2. ROUND (SOLID)   3. ROUND (TUBE)

? 1,3

ENTER LENGTH (HEAT FLOW AXIS), O.D., AND WALL THKNS...(IN.)...
? 3,.75,.03125


*** SECTION # 2 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN   2. G10   3. COP   4. MNG   5. BRS
 6. ALM   7. STL   8. TFE   9. GLS

 1. RECTANGULAR   2. ROUND (SOLID)   3. ROUND (TUBE)
```

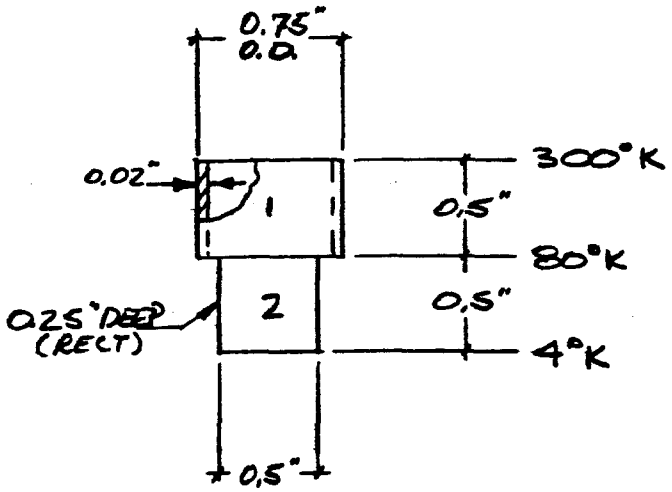
*** CRYOSTAT SUPPORT ***
*** SUMMARY OF RESULTS ***

SEC #	MAT'L	LENGTH (IN)	LENGTH (CM)	X-SECT (IN^2)	X-SECT (CM^2)	DELT (K)
1	STN	3.000	7.620	.071	.455	295.3

NO 80K STRAP SPECIFIED

HEAT FLOW INTO 4K : 1.836 WATTS

[RETURN] TO CONTINUE...



MAT'L 1 (300-80) : 304 STL

MAT'L 2 (80-4) : G-10

$$Q_{300\to 80} = \frac{A_1}{L_1}\int_{80}^{300} K\,dT \qquad \text{(K for STN STL)}$$

$$Q_{300\to 80} = \frac{0.0459\,in^2}{0.5\,in} \times 2.54\,\frac{cm}{in} \times 27.12\,\frac{W}{cm}$$

$Q_{300\to 80} = 6.319$ W "CRYOSPT" yields Q = 6.338 W (0.3

$$Q_{80\to 4} = \frac{A_2}{L_2}\int_{4}^{80} K\,dT \qquad \text{(K for G-10)}$$

$$Q_{80-4} = \frac{0.125\,in^2}{0.5\,in} \times 2.54\,\frac{cm}{in} \times 0.1701\,\frac{W}{cm}$$

$Q_{80-4} = 0.108$ W "CRYOSPT" yields Q = 0.107 W (0.934

```
ENTER RE-RUN OPTION...[RETURN] TO QUIT...

1. RE-RUN USING OLD DATA (CHANGE 80K STRAP LOCATION)
2. ENTER NEW DATA BY SECTION #
3. START OVER WITH NEW GEOMETRY

? 3


*** SECTION # 1 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

? 1,3

ENTER LENGTH (HEAT FLOW AXIS), O.D., AND WALL THKNS...(IN.)...
? .5,.75,.02


*** SECTION # 2 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

? 2,1

ENTER LENGTH (HEAT FLOW AXIS), WIDTH, AND DEPTH...(IN.)...
? .5,.5,.25


*** SECTION # 3 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

?

ENTER THE SECTION # ABOVE THE 80K STRAP...
[RETURN] IF NO 80K POINT IS TO BE SPECIFIED...
? 1
```

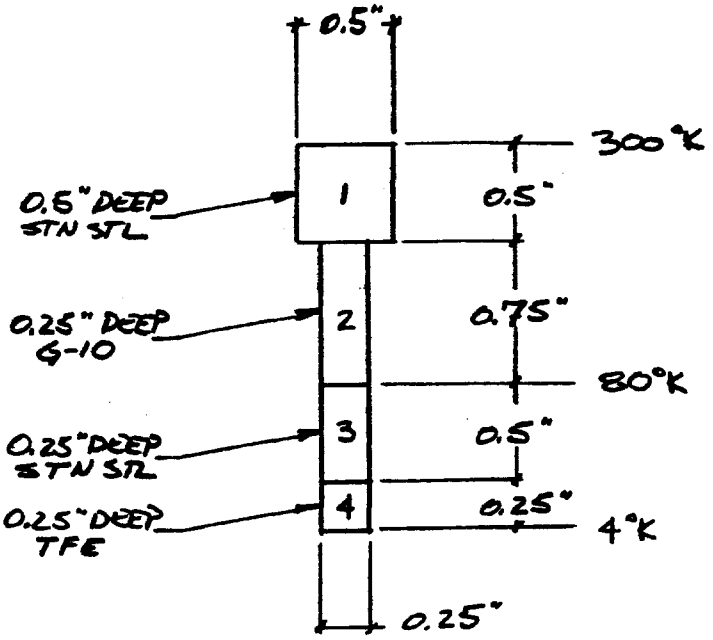
*** CRYOSTAT SUPPORT ***
*** SUMMARY OF RESULTS ***

SEC #	MAT'L	LENGTH (IN)	LENGTH (CM)	X-SECT (IN^2)	X-SECT (CM^2)	DELT (K)
1	STN	.500	1.270	.046	.296	220.4
2	G10	.500	1.270	.125	.806	75.6

80K STRAP BETWEEN SECTIONS 1 AND 2

HEAT FLOW INTO 80K : 6.338 WATTS
HEAT FLOW INTO 4K : .107 WATTS

[RETURN] TO CONTINUE...



ALL RECTANGULAR SECTIONS

```
ENTER RE-RUN OPTION...[RETURN] TO QUIT...

1. RE-RUN USING OLD DATA (CHANGE 80K STRAP LOCATION)
2. ENTER NEW DATA BY SECTION #
3. START OVER WITH NEW GEOMETRY

? 3


*** SECTION # 1 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

? 1,1

ENTER LENGTH (HEAT FLOW AXIS), WIDTH, AND DEPTH...(IN.)...
? .5,.5,.5


*** SECTION # 2 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

? 2,1

ENTER LENGTH (HEAT FLOW AXIS), WIDTH, AND DEPTH...(IN.)...
? .75,.25,.25


*** SECTION # 3 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
 6. ALM    7. STL    8. TFE    9. GLS

 1. RECTANGULAR    2. ROUND (SOLID)    3. ROUND (TUBE)

? 1,1

ENTER LENGTH (HEAT FLOW AXIS), WIDTH, AND DEPTH...(IN.)...
? .5,.25,.25


*** SECTION # 4 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN    2. G10    3. COP    4. MNG    5. BRS
```

```
? 8,1

ENTER LENGTH (HEAT FLOW AXIS), WIDTH, AND DEPTH...(IN.)...
? .25,.25,.25


*** SECTION # 5 ***

ENTER MATERIAL # , GEOMETRY #...[RETURN] WHEN FINISHED...

 1. STN   2. G10   3. COP   4. MNG   5. BRS
 6. ALM   7. STL   8. TFE   9. GLS

 1. RECTANGULAR   2. ROUND (SOLID)   3. ROUND (TUBE)

?

ENTER THE SECTION # ABOVE THE 80K STRAP...
[RETURN] IF NO 80K POINT IS TO BE SPECIFIED...
? 2
```

*** CRYOSTAT SUPPORT ***
*** SUMMARY OF RESULTS ***

SEC #	MAT'L	LENGTH (IN)	LENGTH (CM)	X-SECT (IN^2)	X-SECT (CM^2)	DELT (K)
1	STN	.500	1.270	.250	1.613	1.0
2	G10	.750	1.905	.063	.403	219.1
3	STN	.500	1.270	.063	.403	3.2
4	TFE	.250	.635	.063	.403	72.9

80K STRAP BETWEEN SECTIONS 2 AND 3

HEAT FLOW INTO 80K : .191 WATTS
HEAT FLOW INTO 4K : .082 WATTS

[RETURN] TO CONTINUE...

```
ENTER RE-RUN OPTION...[RETURN] TO QUIT...

1. RE-RUN USING OLD DATA (CHANGE 80K STRAP LOCATION)
2. ENTER NEW DATA BY SECTION #
3. START OVER WITH NEW GEOMETRY

?
REVERT. *** SUPPORT ANALYSIS TERMINATED ***
```