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VERY HIGH ENERGY SYNCHROTRON RADIATION AT FERMILAB

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ABSTRACT

Synchrotron radiation with energy up to 1 GeV will be produced by an electron beam at the new wide band neutral beam under construction in the Proton Laboratory at Fermilab. The spectrum of the radiation is estimated, and some experiments using it are discussed.

I. Introduction

Synchrotron radiation with energy up to 1 GeV will be produced by a new high energy electron beam under construction at Fermilab as part of the development of a new wide band charged and neutral beam in Proton East.[1] The energy of the electron beam is designed to be between 450 and 800 GeV with a flux of about 10^7 electrons per 10^{12} incident 1 TeV protons in a 20 second spill. Three 50 kG-m magnets 2.4 m long used to dump the secondary electron beam after a bremsstrahlung radiator provides a radius of curvature of about 1000 m for a 600 GeV electron.

The purpose of this note is to discuss the energy and angular distributions of synchrotron radiation in order to assess the usefulness of the synchrotron radiation for nuclear physics or for using the low energy part of the spectrum for atomic or solid state physics experiments. Therefore, the properties of synchrotron radiation are reviewed, and then the radiation from a real electron is briefly discussed. The photon spectrum and polarization are then compared with other sources of radiation.

II. Energy and Angular Distributions

In this section, we calculate the energy spectrum and angular distribution of synchrotron radiation from a single ultrarelativistic electron.[2] For a particle in circular motion with radius of curvature r, the critical energy is given by:

$$E_c = \frac{3\hbar c\gamma^3}{r} \qquad (2.1)$$

In practical units,

$$E_c(\text{MeV}) = \frac{5.92\text{x}10^{-13}\gamma^3}{r(\text{m})} \qquad (2.2)$$

so that a 600 GeV electron with a radius of curvature of 1000 m has a critical energy of about 1 GeV. There are several definitions of critical energy at large in the literature, and this one follows Jackson's notation. Most of the radiation occurs with photon energy less than the critical energy and half the total power is radiated above half the critical energy defined in this way. Thus, half the total radiated power is above 500 MeV in the above example.

The energy radiated per unit photon energy per unit solid angle by an electron in a circular orbit of radius r is given by:

$$\frac{d^2I}{dEd\Omega} = \frac{\alpha}{3\pi^2}\left(\frac{Er}{\hbar c}\right)^2\left[\frac{1}{\gamma^2} + \theta^2\right]\left[K^2_{2/3}(\xi) + \frac{\theta^2}{\frac{1}{\gamma^2}+\theta^2}K^2_{1/3}(\xi)\right] \qquad (2.3)$$

where E is the photon energy, α is the fine structure constant, θ is the angle of the observation point above the plane of the electron trajectory, $K_{2/3}(\xi)$ and $K_{1/3}(\xi)$ are modified Bessel functions, and

$$\xi = \frac{E_r}{3\hbar c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2} = \frac{E}{E_c}(1+\gamma^2\theta^2)^{3/2}. \qquad (2.4)$$

Figure 1 illustrates the geometry. The number of photons per unit photon energy per unit solid angle is just

$$\frac{d^2N}{dEd\Omega} = \frac{1}{E}\ \frac{d^2I}{dEd\Omega} \tag{2.5}$$

In practical units, the above expressions become

$$\frac{d^2N}{dEd\Omega} = 6.33\text{x}10^{21}E(\text{MeV})r^2(\text{m}^2)\left(\frac{1}{\gamma^2} + \theta^2\right)\left[K^2_{2/3}(\xi) + \frac{\theta^2}{\frac{1}{\gamma^2}+\theta^2}\ K^2_{1/3}(\xi)\right] \tag{2.6}$$

$$\xi = 1.69\text{x}10^{12}\ E(\text{MeV})r(\text{m})\left(\frac{1}{\gamma^2} + \theta^2\right)^{3/2} \tag{2.7}$$

Some comment must be made about the approximations used in arriving at the photon energy spectrum (2.3). The calculation of the fields involve an integration over all time so that some approximation must be made to carry out the integrations. Although the acceleration may be different from zero only for a short time, extending the limits of integration to all times only results in a significant error for energies of order $\hbar c/r$, which is very small compared with the critical energy, so the approximation is justified. This is also why $dI/d\Omega$ is not dependent on ϕ, the azimuthal angle, even though the radiation is in fact beamed into a narrow pencil in the direction of the electron velocity.

The photon energy spectrum extends from the lowest energies to near the critical energy, and varies as $E^{-2/3}$ for energies far below the critical energy. The radiation is collimated in a narrow cone around the direction of motion with a critical angle (the 1/e point), given by

$$\theta_c = \frac{1}{\gamma}\left(\frac{E_c}{3E}\right)^{1/2} \tag{2.8}$$

so that near the critical energy, most of the radiation is in a cone with half angle $1/\gamma$ or about 1 μr for a 600 GeV electron. Lower energy photons are radiated into a larger solid angle.

Many useful approximations to the energy and angular distributions have been calculated. Integration over the latitude θ for photon energies $E<<E_c$ gives a photon spectrum into azimuthal angle ϕ of

$$\frac{d^2N}{dEd\phi} = \frac{3.25}{2\pi} \alpha \frac{1}{(\hbar c)^{1/3}} r^{1/3} E^{-2/3} \quad (E<<E_c) \tag{2.9}$$

so that

$$\frac{d^2N}{dEd\phi} \cdot \frac{N_e}{\Delta t} = 0.0648\ r^{1/3}(m^{1/3})E^{-2.3}(MeV^{-2/3})\frac{N_e}{\Delta t}(sec) \tag{2.10}$$

which gives about 10^4 photons/sec-MeV-mr at an energy of 50 MeV.

The energy radiated per unit photon energy interval peaks at a photon energy of the same order as the critical energy. (Note that the number of photons per unit energy interval has no such peak, but that the number of photons in a constant fraction of energy bandwidth, dE/E, peaks at the same energy.) The peak in the radiated power is of order $\alpha\hbar\gamma$.

III. Properties of the Radiation in Time

The duration of the radiation pulse is very short because the photons are beamed in a very narrow cone in the direction of the electron motion, which rapidly sweeps across an observer. The radiation from a single particle occurs in bursts with a duration of about

$$\delta t = \frac{1}{2\gamma^3} \frac{r}{c} \tag{3.1}$$

or about 10^{-22} sec for r=1000 m and $\gamma=10^6$. If the electrons in the beam are well separated in time , then the radiation consists of very short flashes repeated as electrons go by. For 10^7 particles distributed over a 20 second spill, beam particles are separated by 2 μs on the average.

IV. Polarization

The two terms in equation 2.3 give the intensities for the radiation with the electric vector polarized in the plane of the electron motion and perpendicular to it. Therefore, the linear polarization is given by

$$P_L = \frac{I_{\parallel} - I_{\perp}}{I_{\parallel} + I_{\perp}} = \frac{K^2_{2/3}(\xi) - \frac{\theta^2}{\theta^2 + \frac{1}{\gamma^2}} K^2_{1/3}(\xi)}{K^2_{2/3}(\xi) + \frac{\theta^2}{\theta^2 + \frac{1}{\gamma^2}} K^2_{1/3}(\xi)} \tag{4.1}$$

The circular polarization is given by

$$P_c = \frac{\frac{\theta^2}{\theta^2 + \frac{1}{\gamma^2}} K_{2/3}(\xi) K_{1/3}(\xi)}{K^2_{2/3}(\xi) + \frac{\theta^2}{\theta^2 + \frac{1}{\gamma^2}} K^2_{1/3}(\xi)} \quad (4.2)$$

Figure 3 illustrates the polarization for a 600 GeV electron beam and a radius of curvature of 1000 m for energies from 10 KeV to 1 GeV.

V. Considerations for a Real Electron Beam

The discussion of synchrotron radiation from a single electron has shown that the synchrotron radiation at energies of the order the critical energy are confined to an angular region of order $1/\gamma$ from the plane in which the electrons are bent. For 600 GeV electrons, this angle is of order 1 μr, which is small compared with the angular divergence of the electron beam. Therefore, the size and divergence of the photon beam are determined by the size and divergence of the electron beam projected to the observation point of the photons. Lower energy photons are less well collimated with the electron beam, according to equation 2.8, but even for 100 KeV photons, the critical angle is only of order 100 μr.

VI. Wigglers and Enhancement of Synchrotron Radiation

Instead of using the synchrotron radiation from bending the electrons into the beam dump, it may be desirable to tailor an magnetic field which would enhance the synchrotron radiation in the desired region of the photon spectrum. The possibilities include using a weak bending field to produce lower energy photons or wiggler magnets to produce lower energy synchrotron radiation or a more monochromatic photon spectrum.

Weak magnetic fields present several problems. The separation of the electron beam is very small because the the small magnetic fields bend the the electron through an angle of about

$$\theta_{EL} = \frac{L}{2r} \qquad (6.1)$$

where L is the magnet length and the critical angle is only about 1 μr. For the smallest magnetic fields, of the same order as the Earth's magnetic field, about 1 G, the bending radius is about $2x10^7$ m for a 600 GeV electron. The critical energy is about 50 KeV, which could be useful for solid state and atomic physics purposes, but there is a substantial flux of much higher energy radiation very nearby. Furthermore, since the peak intensity is proportional to γ, the peak energy radiated is about 100 times large than radiation from an electron in a 6 GeV storage ring, but that is not enough to make up for the deficit in the number of electrons passing an observation point per unit time, which is about 10^{11} particles every 2.5 μs, or about 4×10^{16} electrons per second in CESR, compared to about 10^6 particles per second.

VII. Comparison with Other Sources of Radiation

Photons in the energy range of several hundred MeV are not a scarce commodity. Other sources of radiation are nuclear transitions, bremsstrahlung, backscattered Compton photons from laser light fired at an electron beam, and nuclear reactions. Nuclear sources which emit photons in the energy range of a few hundred MeV are available. A strength of 100 Curies corresponds to 3.7×10^{12} disintegrations per second with decay products emitted isotropically. A collimator covering 10^{-6} steradian intercepts about 3.7×10^6 photons per second. The backscattered polarized photon beam at the storage ring Adone at Frascati has a photon energy adjustable between 5 and 80 MeV and an intensity between 10^5 and 10^7 photons per second.[3]

Wigglers are magnets with fields that vary in space and are designed to modify the spectrum of synchrotron radiation or increase its flux.[4] In a storage ring, the wiggler magnets do not disturb the stable orbits, a consideration largely irrelevant to the Fermilab electron beam. The wiggler field could be larger or smaller than the bending magnetic field, and the flux of synchrotron radiation could be increased by superposing radiation from several bends.

If the number of magnetic field oscillations is large, the magnet is called an interference wiggler and can produce almost monochromatic radiation. Unfortunately, for a magnetic field with a period of about 10 cm, the range of energies that could be produced are just a few MeV, which would probably be lost in the background for a small wiggler.

VIII. Backgrounds

The synchrotron radiation is emitted in an extremely noisy and hostile environment which could make it hard to detect. The beam dump and the high energy bremsstrahlung photon beam are separated by only three inches at 600 GeV, and any detector for the synchrotron radiation must be between them. Halo from the electron beam and electron interaction with nearby material produces a background of photons. Furthermore, synchrotron radiation in the fringe field of the bending magnet makes a background of lower energy synchrotron radiation.

IX. Conclusion

Synchrotron radiation with energy up to 1 GeV will be available at Fermilab from the electron beam at the wide band photon beam in Proton East. The energy, polarization, and angular distributions from single electrons have been calculated, and the effects of a realistic electron beam have been discussed. A comparison with other sources of radiation has been made, and methods for enhancing the radiation has been briefly discussed.

Consideration is being given to using the radiation for nuclear physics, or at least measuring the photon flux from the very high energy synchrotron radiation. Possible uses that will be considered include pumping gamma ray lasers (GRASERS),[5] and photonuclear reactions. Future notes will describe these possible applications.

I would like to thank Joel Butler for discussion of this problem.

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4. H. Winick and S. Doniach eds., Synchrotron Radiation Research, New York: Plenum, 1980.

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Figure Captions

Figure 1. Geometry of the electron beam. The radius of curvature is r, the angle above the plane of the electron trajectory is θ, and the azimuthal angle is ϕ.

Figure 2. Photon energy spectrum from 600 GeV electrons with a radius of curvature of 1000 m. The vertical axis is the number of photons per MeV per mr. To find the photon flux, multiply by the number of electrons per second crossing an observation point.

Figure 3a. Linear and circular polarization of synchrotron photons as a function of θ, the angle above the electron trajectory for photon energies from 0.01 MeV to 1.0 MeV. The electron beam energy was 600 GeV, and the radius of curvature was 1000 m.

Figure 3b. Linear and circular polarization for photon energies 10.0 MeV to 1000.0 MeV.

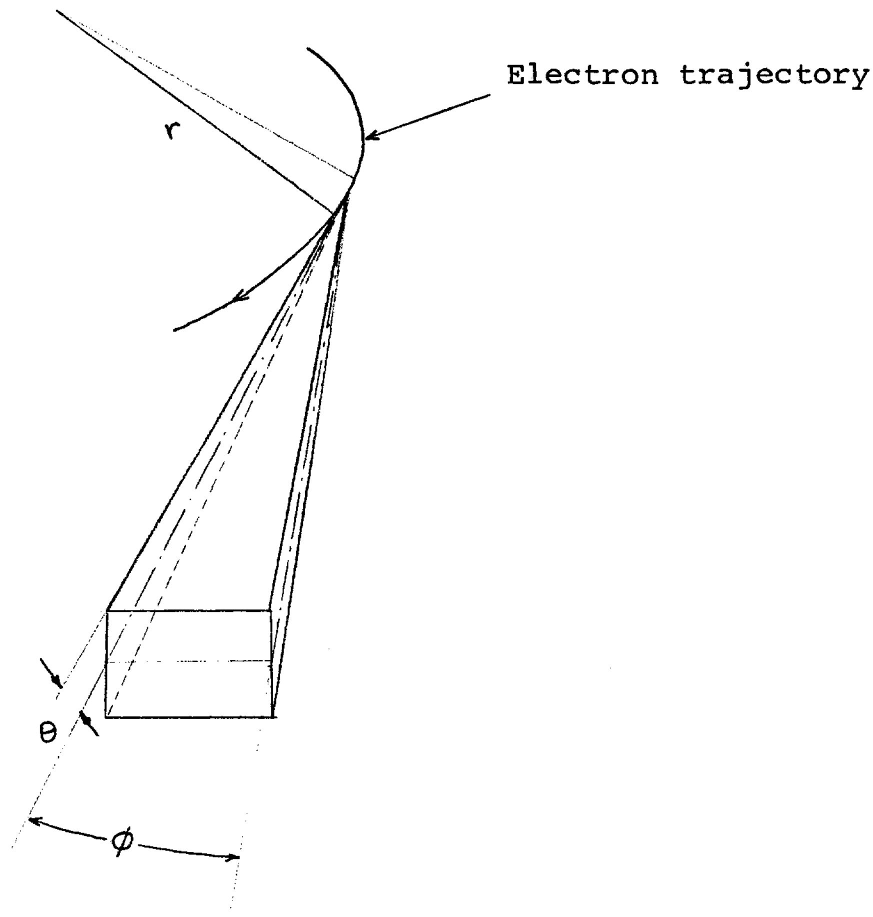


Figure 1.

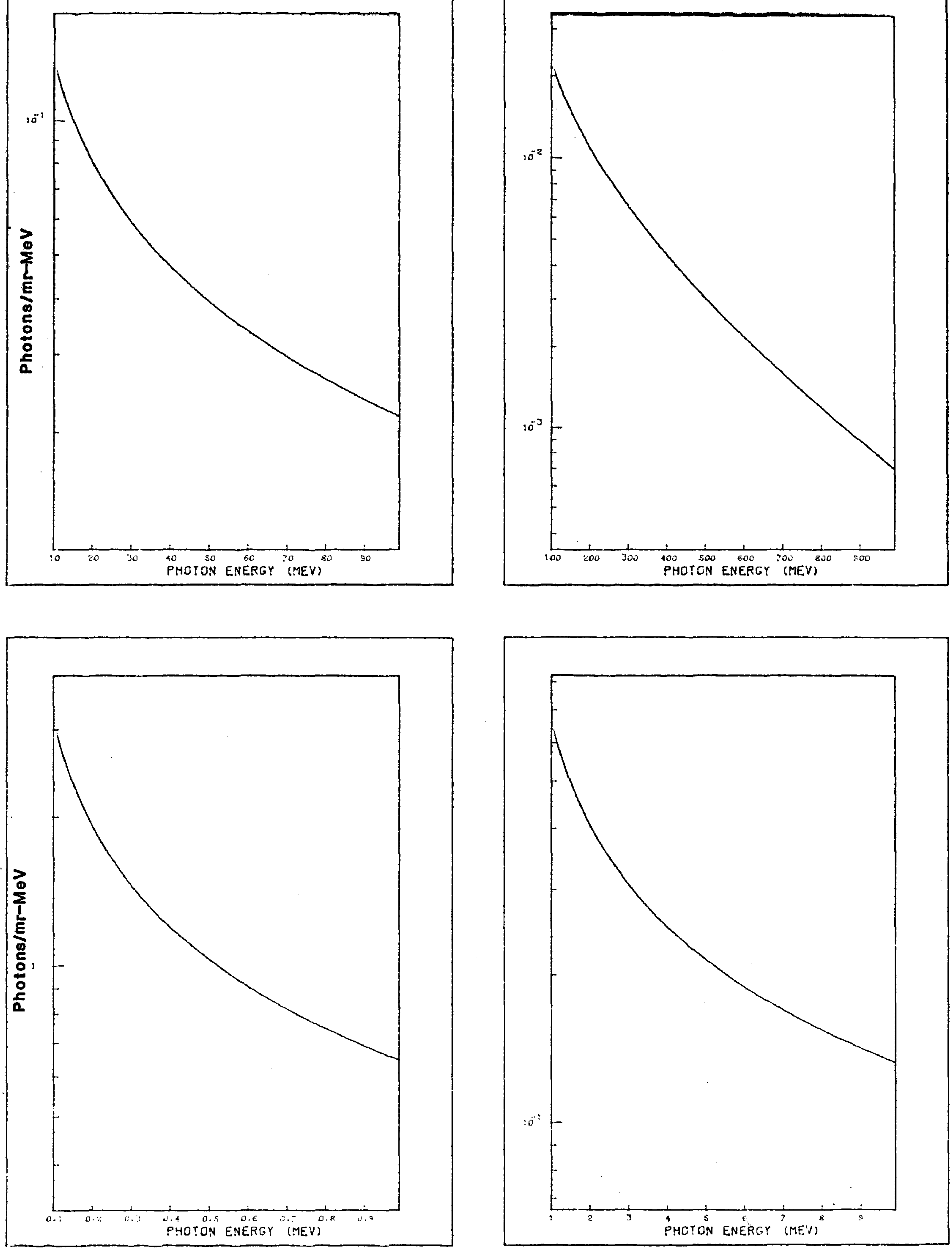
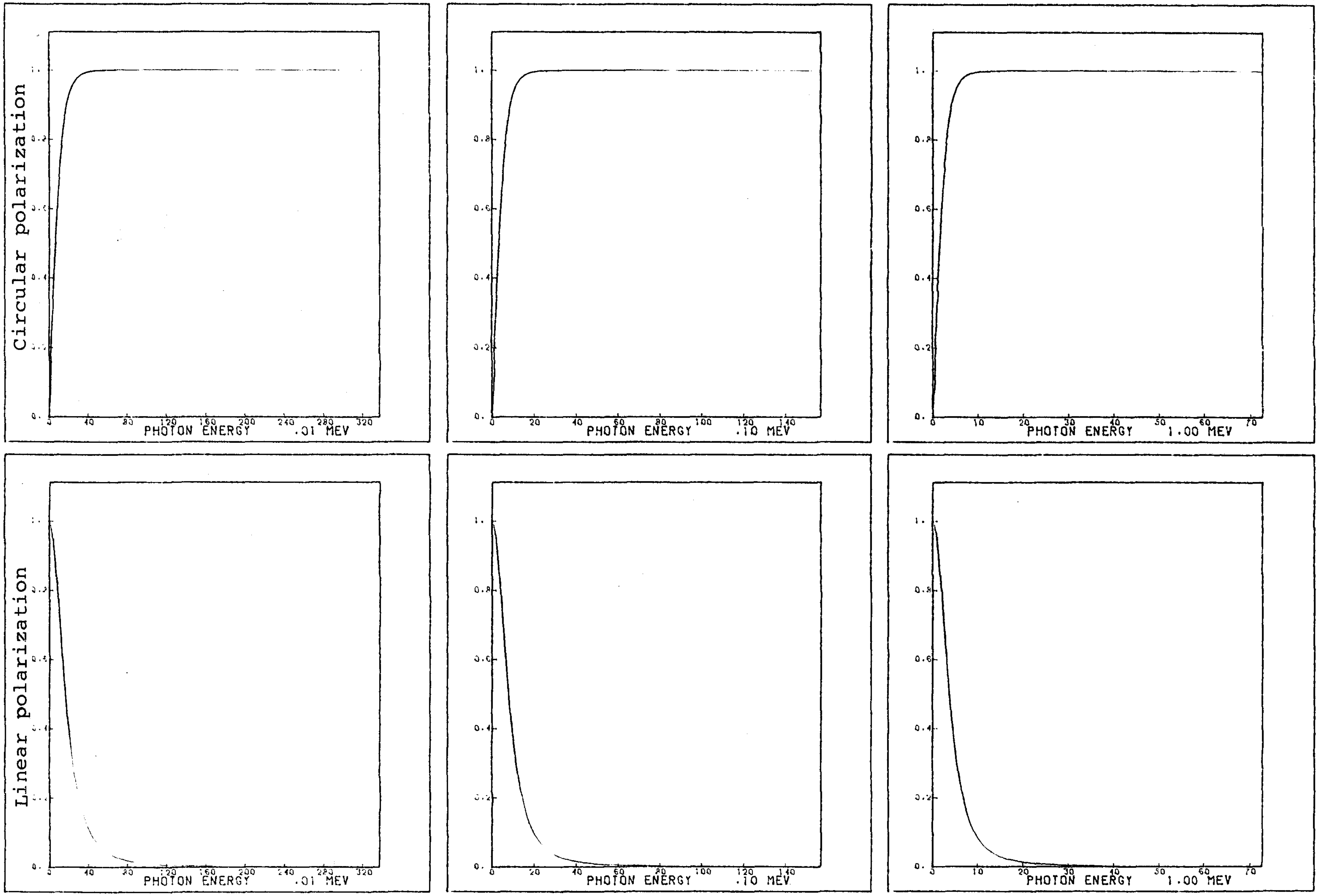
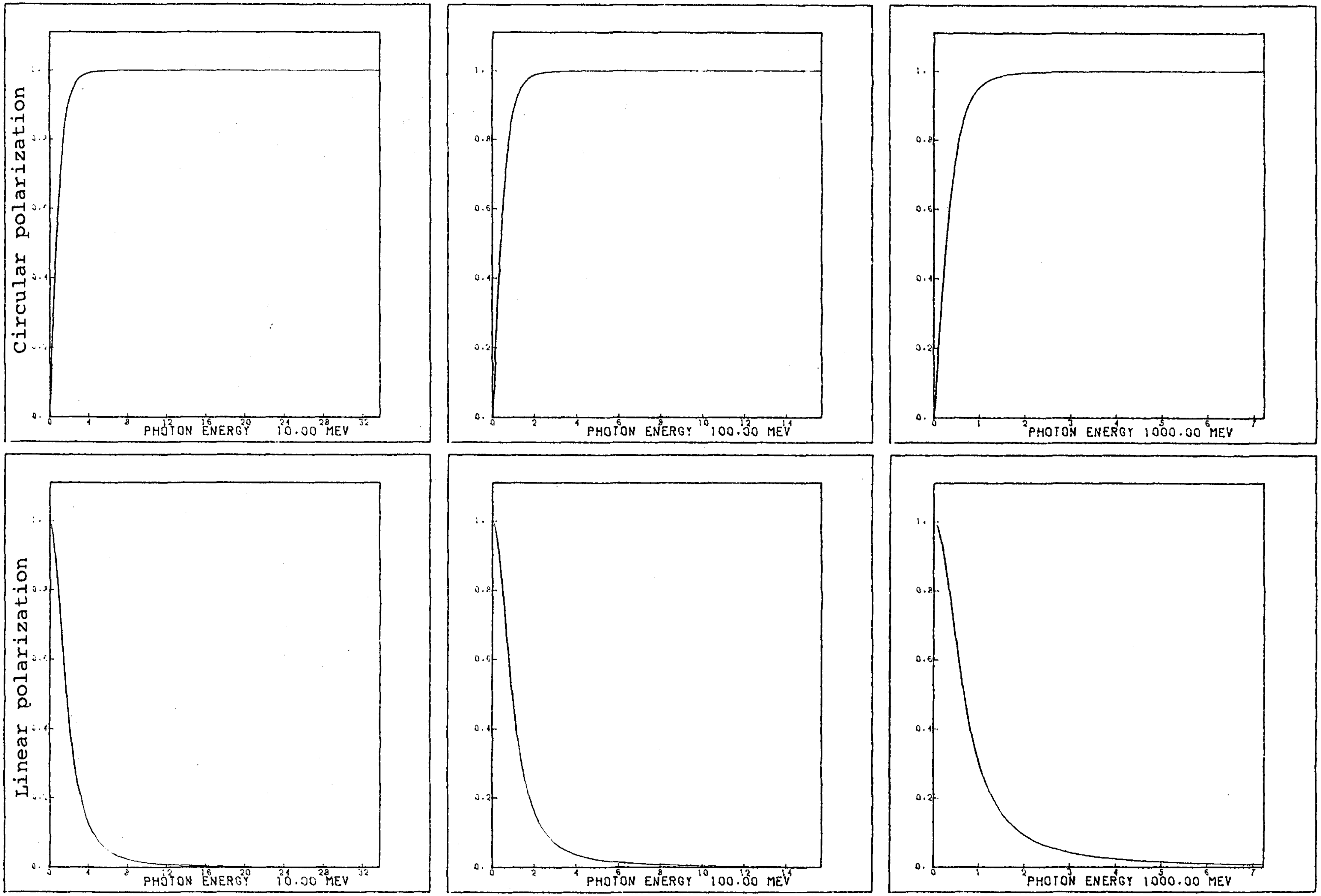

Photons/mr-MeV
10 20 30 40 50 60 70 80 90
PHOTON ENERGY (MEV)
100 200 300 400 500 600 700 800 900
PHOTON ENERGY (MEV)
Photons/mr-MeV
PHOTON ENERGY (MEV)
1 2 3 4 5 6 7 8 9
PHOTON ENERGY (MEV)

Figure 2.



Theta (microradians)

Figure 3a



Theta (microradians)

Figure 3b.