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TINKERING AT THE MAIN RING LATTICE

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I. Whys and Wherefores

This is a story of, at best, an ambiguous success in trying to modify the main ring lattice so that it will be better suited for a good cause. It must be admitted that, contrary to claims made often at special occasions, our main ring is not as robust as we wish. Its successful operation has been largely dependent on the delicate and painstaking tuning by our operators who perfected their art in the past ten years. When one contemplates any change of lattice in a machine like this, the reason for the change must be truly compelling. I hope we all agree that the copious production of usable antiprotons ($\bar{p}$'s) using the proton beam from the main ring and the lossless injection of cooled $\bar{p}$'s into the main ring do indeed justify such a change.

According to the latest design report of the $\bar{p}$ source,[1] the scenario involving the main ring goes like this: At 125 GeV/c*, the proton beam is squeezed in the longitudinal phase so that the bunch width is less than a nanosecond and the momentum spread is as large as ±0.2%. The transverse emittance for 95% of the beam is expected to be 0.2π mm-mr at this momentum. The beam is kicked horizontally by the existing kicker at C48 and extracted vertically at F17 by means of two Lambertson magnets. After the longitudinal and transverse cooling in the debuncher-accumulator complex, the $\bar{p}$ beam at 8.89 GeV/c will be injected back to the main ring at F17, reversing

* Momentum instead of the traditional kinetic energy will be used throughout this report.

the process for the proton extraction. The expected quality of $\bar{p}$'s is ±0.0225% in ($\Delta p/p$) and ∿2π mm-mr for the transverse emittance.

	momentum	($\Delta p/p$)	emittance
protons	125 GeV/c	±0.2%	0.2π mm-mr
antiprotons	8.89 GeV/c	±0.0225%	2π mm-mr

The large momentum spread in the proton beam is an unavoidable consequence of the effort to increase the longitudinal density of $\bar{p}$'s coming from the target. If the main ring were found to accomodate a larger momentum spread, a possibility which cannot be totally excluded at present, the temptation to squeeze the beam further would be hard to resist (although this would certainly mean more rf cavities). As for the transverse emittance of $\bar{p}$'s, it may be possible to do a better cooling than the present design but the emittance cannot be much smaller than 2π mm-mr. For one thing, there is the increased heating in the longitudinal direction via the intrabeam scattering.*

The choice of C48-F17 for the kicker-Lambertson system is a very good one from many points of view. The powerful kicker at C48 is already available at no extra cost. This kicker will be used for the beam transfer from the main ring to the superconducting ring and Mike Harrison has already installed three bump magnets at C26, C32 and D38 to counteract the large excursion of the kicked beam.[2] In addition, we have a local bump system (D46-E17) which is essential in maneuvering the beam, protons to be extracted from or $\bar{p}$'s injected at F17, around the Lambertson at EØ. The available free space at F17 for the Lambertson magnets is of course an obvious advantage. The problem facing us arises from the fact that F17, being a horizontally focusing station, has the maximum beta and also happens to be one of eighteen places in the main ring where the momentum dispersion X_p takes its maximum value. The total beam size in the radial direction can be expressed as the quadratic sum of two contributions, the betatron amplitude and the dispersion,

* Sandro Ruggiero, private communication.

$$\text{beam size} = \pm\sqrt{\beta_h\varepsilon_h + [X_p(\Delta p/p)]^2}$$

where β_h is the horizontal beta function and $\pi\varepsilon_h$ is the horizontal beam emittance. If the particle distribution is Gaussian in the (x,x') space and uniform within $\pm(\Delta p/p)$, some 92% of the beam will be inside this beam size. If the distribution is Gaussian in $(\Delta p/p)$ as well, the percentage is of course higher. Assuming the entrance to the first Lambertson to be at 2m downstream from F17, one finds for the ideal main ring operated at $\nu_h = \nu_v = 19.4$,

$$\beta_h = 92.08\text{m}, \quad X_p = 5.45\text{m}$$

<u>beam size</u>	protons at 125 GeV/c:	±11.7mm,
	$\bar{p}$'s at 8.89 GeV/c:	±13.6mm.

Note that, for protons, the momentum dispersion contributes more than 90% of the total beam size while it is practically all betatron amplitude for $\bar{p}$'s. If we are to ease the situation here, the main ring lattice must be modified such that both β_h and X_p decrease. There is an additional feature in the C48-F17 arrangement which makes the situation even worse. When the beam is moved inward at F17, it has a rather large positive angle because of the strong horizontal focusing at F17. For the shift of -40mm, the angle is +0.79mrad so that the kicked beam moves outward by ∿8mm at the end of the downstream Lambertson. It has been pointed out to me by Carlos Hojvat, who designed the system originally, that any type of closed-orbit bump is of no help since both the kicked beam and the circulating beam will be affected by it. Such a bump can change the position and angle of both beams together relative to the geometrical center line of aperture.* An obvious solution is to roll the first (upstream) Lambertson to cancel the positive angle and make the beam parallel to the circulating beam in the second (downstream) Lambertson. Figs. 1A and 1B show schematically how two Lambertsons can be arranged for the optimum beam clearance. Note that the dotted line

* According to the design report, ref. 1 (p. 2, Chapter 3), four magnets are already placed at F15, F17, F18 and F22 for this purpose.

in Fig. 1A for the upstream Lambertson indicates the rolled wall when the beam is bent upward. This particular system is simply an example to show the general features and it is not meant to be the design of the actual system. Parameters used to draw Figs. 1A and 1B are:

Lambertsons	effective length:	5.2m
	bend angle:	16mrad
	opening angle of the field-free slot:	$\pm 40^{\circ}$
	gap:	1.5"
	field at 125 GeV/c:	12.8kG

Optimists would say there are ample clearances at all points along the Lambertsons and we should not tamper with the main ring lattice. What follows then is strictly for pessimists who should like to see a little more shoulder room here and there.

II. Ways and Means

At the outset, we impose a number of conditions so that the proposed modification is practical and reasonable. However, not all of these conditions are absolutely essential and the final design is inevitably a compromise of several often conflicting requirements.

1. no change in the basic geometry of the ring. This excludes, for example, a bypass with large horizontal bends. Because of this restriction, a meaningful amount of reduction in the dispersion must be achieved with quadrupoles only.
2. the localized change in both X_p and β together with the minimum perturbation in the vertical direction everywhere in the ring. A single quadrupole is obviously out of question and quadrupoles must be placed at locations where β_h is much larger than β_v.
3. no appreciable change in tunes. A small change (of the order of 0.01) may not be dangerous for the operation. This condition may not be important if one can introduce a tune bump.

4. minimum number of quadrupoles and power supplies. The existing elements in the ring should not be disturbed too much although it may be unavoidable to move some correction magnets to other locations. Quadrupoles should be powered in series whenever it is possible.

With these conditions in mind, our plan is then to find the best locations for two pairs of focusing-defocusing quadrupoles of the same strength, one pair to reduce X_p but not β_h and the other pair to reduce β_h without at the same time unduely increasing X_p. In order to avoid possible misunderstandings, it should be mentioned here that one can reduce β_h and X_p at F17 simultaneously by means of a single pair of quadrupoles. For example, a horizontally focusing quadrupole at E44 and defocusing quadrupole at F42, each with the strength $(B'\ell/B\rho) = 0.0075$ will reduce β_h from 92.1m to 50.4m and X_p from 5.45m to 2.60m at F17. However, there are several unpleasant features in this system and it cannot be seriously recommended unless other spaces are not available for two pairs of quadrupoles.*

Beta bump

Some years ago, Tom Collins taught me how to make a localized beta bump with a pair of quadrupoles. Since his method is an interesting one but cannot be found in any readily available report (as far as I know), it is explained fully in the appendix. From this, one finds that the pair of quadrupoles should be placed such that the phase advance from one to the other is $(2n\pi)$. If the localization of β is the only requirement, the phase advance can be $(2n+1)\pi$ as well as $(2n\pi)$. This choice is however disastrous in that the perturbation of the dispersion outside the beta bump is usually substantial. The explanation of this point is given below where a localized dispersion bump is discussed. Once the locations are decided, the strength parameter defined as

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* Actually, E44 is at present occupied by the beam postion monitor for the main ring radial feedback system.

$$k = \beta_h(B'\ell/B\rho), \qquad (k>0 \text{ focusing}) \qquad (1)$$

which should be equal in magnitude and opposite in sign, determines the amplitude of beta variation between two quadrupoles. Since the value of β_h is very nearly the same at all regular horizontal stations (95m - 100m), two elements can be excited in series with one power supply. If one asks that the perturbed beta is to be no more than twice the unperturbed one within the bump, one takes $|k| \simeq 0.7$, see (A.22). The minimum value of β(perturbed)/β(original) is then 0.5, the best reduction factor, and this happens at the phase advance of 35°(mod π) from the first quadrupole if it is horizontally focusing, and at 145°(mod π) if defocusing.

F17, where β_h is to be reduced, must necessarily be inside the bump but the kicker at C48 can be either inside or outside the bump.

1. C48 inside the beta bump. The first quadrupole is upstream of C48 and the other quadrupole is downstream of F17. The bump extends over a significant fraction of the entire ring. The perturbation in β covers the entire distance from the kicker to the Lambertsons. The advantage is that the transfer matrix from the kicker to the Lambertson is unchanged and the kicker-Lambertson relation remains optimum.

2. C48 outside the beta bump. The (12) element of the transfer matrix is

$$\sqrt{\beta_h(C48)\,\beta_h(F17)}\,(\text{sin of the phase advance}).$$

Since β_h(C48) is not changed much (localization of β), the reduction of β(F17) diminishes the effectivness of the kicker at C48. In order to get the same amount of displacement at the Lambertson, the kick must be more and the excursion of the kicked beam between C48 and F17 is correspondingly larger. The beam angle at F17 also becomes larger. On the other hand, the beta bump could in principle be confined to a very small fraction of the ring.

Finally, to minimize the disturbance in the dispersion outside the beta bump, two quadrupole locations should have small and not too different values of the unperturbed dispersion.

Dispersion bump

Dispersion is simply the closed orbit for a unit value of $(\Delta p/p)$ and any dispersion bumps can be regarded as an extension of the familiar closed-orbit bump. The only significant difference is that the necessary kick is provided not by steering dipoles but by the shifted beam position in quadrupoles, the shift being the original dispersion. For $\Delta \equiv X_p$(perturbed) - X_{po}(unperturbed), one finds

$$\frac{d^2\Delta}{ds^2} + (K + \delta K)\Delta = -(\delta K)X_{po} \tag{2}$$

where $K \equiv B'/B\rho$ is the original quadrupole focusing in the ring and δK is the additional contribution from the bump quadrupoles. Note that δK is nonzero only at these quadrupole locations. For an ideal bump with two equal-strength quadrupoles, values of β at the bump quadrupole locations are unchanged. Furthermore, the tune remains the same and, consequently, so does the phase advance between quadrupoles. We are then faced with the problem of a simple local closed-orbit bump in the original linear lattice with completely fixed kick parameters

$$\beta_h(B\ell/B\rho) \equiv \beta_h(B'\ell/B\rho)X_{po}. \tag{3}$$

Since X_{po} is always positive in the main ring, one gets the maximum amount of reduction (maximum $|\Delta|$ with $\Delta<0$) when the phase advance from the first quadrupole to F17 is 90°(mod 2π) and this quadrupole is focusing. For the opposite polarity, the phase advance should be 270°. One naturally selects a place where X_{po} is large so that, for a given quadrupole strength parameter k defined by Eq. (1), the reduction in X_p is efficient. This is important in preventing a large increase in β at F17. From (A.21), one finds

$$\beta(\text{perturbed})/\beta(\text{original}) = 1 + k^2$$

for $\phi = 90^\circ$ or 270°.

Unlike beta bumps, there is really no difference in the (12) element of the transfer matrix whether C48 is inside or outside the dispersion bump. The phase advance from C48 to F17 is close to 90°(mod 2π) and the phase advance from the first quadrupole to F17 (if the quadrupole is downstream of C48) is also 90°(mod π). The beam kicked at C48 goes through the center of the quadrupole and there is no effect coming from that quadrupole.

III. Recommended System (for an intrepid* soul only)

In the main ring, one is seldom lucky enough to find two open spaces of the ideal phase distance available for installation of some elements. Often he may have to engage in a negotiation with several people to pry open a precious foot or two of ministraights. Since this problem is not of purely technical nature, we will ignore the question of space-availability here and will come back to it in the next section. Several possibilities for the dispersion bump and for the beta bump have been studied. For each case, lattice functions of the ideal main ring with two bump quadrupoles (treated as thin lenses) have been obtained at all stations in the horizontal as well as in the vertical directions.

dispersion bump (large X_{po} locations desirable)

1. E17 - F26 phase distance 356°, E17 to F17 = 84°, X_{po} = 5.45m & 4.43m
2. E28 - F26 phase distance 377°, E28 to F17 = 105°, X_{po} = 5.74m & 4.43m
3. E44 - F26 phase distance 331°, E44 to F17 = 59°, X_{po} = 5.24m & 4.43m
4. F15 - F26 phase distance 339°, F15 to F17 = 67°, X_{po} = 3.78m & 4.43m.

The first of these bumps is the longest and the last one is the shortest in its length. The phase distance is almost perfect for the first choice

* "... absence of fear ... resolute self-posession ... the sense of invulnerability to fear in any situation" - The American Heritage Dictionary of the English Language, New College Edition.

and rather bad for the last two, giving nontrivial perturbation in β outside the bump. As far as the localization of the dispersion is concerned, the first system is the best although the second is not bad either.

my preference E17 - F26 (focusing - defocusing)

$B'\ell = \pm 25$ kG at 125 GeV/c

At F17, entrance to the first Lambertson,

$$X_p = 2.59\text{m}, \quad \beta_h = 112.7\text{m}, \quad \beta_v = 34.0\text{m}$$

With the kick angle of -0.436 mr at C48,

beam center shift at F17 = -40mm,
beam direction at F17 = +0.790 mr.

See Figs. 2A&B.

beta bump (small and equal X_{po} desirable at quadrupole locations)

A. C48 kicker inside the bump

A1. B34 - F22 phase distance 357°, B34 to F17 = 221°, X_{po} = 2.59m & 2.16m

A2. C46 - F24 phase distance 360°, C46 to F17 = 158°, X_{po} = 3.18m & 2.39m

B. C48 kicker outside the bump

B1. E32 - F42 phase distance 357°, E32 to F17 = 38°, X_{po} = 4.69m & 5.57m (too large!)

B2. F11 - F22 phase distance 351°, F11 to F17 = 215°, X_{po} = 2.57m & 2.16m.

The bump A1 may be too long to feel comfortable. B2 is very attractive because of its length but the kicker-Lambertson relation is considerably deteriorated by F11 quadrupole. B1 is not acceptable because of the large values of X_{po} at the quadrupole locations.

<u>my preference</u> C46 - F24 (defocusing - focusing)

$B'\ell = \pm 3.0$ kG at 8.89 GeV/c

At F17, entrance to the first Lambertson,

$\beta_h = 42.7\text{m}, \quad \beta_v = 38.8\text{m}, \quad X_p = 6.39\text{m}$

With the kick angle of -0.421 mr at C48,

beam center shift at F17 = -40 mm,
beam direction at F17 = +0.787 mr.

See Figs. 2A&B.

IV. IF's, BUT's & THEREFORE

If one simply superimpose Fig.2 over Fig.1, the advantage of having two bumps is obvious. The picture looks especially good for the proton beam at 125 GeV/c, encouraging the temptation to go for still higher values of $(\Delta p/p)$. For $\bar{p}$'s, it is difficult to squeeze the beam size substantially more than the present design; the maximum value of β_h may already be dangerously large. If the performance shown in Fig.2 is not good enough, one may be forced to increase the Lambertson gap beyond 1.5". It is of course better to have the emittance smaller than 2π mm-mr but one should not be too demanding in that direction. Rather, the beta bump should be regarded as a necessary device to cover the uncertainties in the performance of betatron cooling system.

The inevitable question one must answer before taking this scheme more seriously is:

> "Is it possible to transport the kicked beam between the kicker and the Lambertsons, and at the same time, to have a stable circulating beam in the presence of perturbed beta functions and dispersions?"

The answer to the first part of the question is relatively straightforward. It has been demonstrated by Mike Harrison and his collaborators that the kicked beam at 150 GeV/c can be transported from the kicker at C48 to the transfer Lambertson at EØ when the closed orbit is properly modified by means of bump magnets at C26, C32 and D38. A local bump created by magnets at D46 and E17 enables the beam to avoid the EØ Lambertson for our case. When the dispersion bump E17-F26 is on for the proton beam at 125 GeV/c, the combination of the beam excursion and the large perturbed value of X_p creates a few dangerous spots between E17 and F17:

	center of the kicked beam	beam size
E19	-38.5 mm	± 5.0 mm
E26	40.5	15.6
E32	-31.3	6.2
E38	39.0	13.9
E46	-40.3	5.2

In order to counteract the large beam excursion, one must have another orbit bump, for example between E17 and E49. For ±20mm amplitude, the required dipole field at each end with $\beta_h \simeq 70$ m is $B\ell = 1.2$ kG-m. With the beta bump C46-F24 for $\bar{p}$'s at 8.89 GeV/c, the beam excursion and the perturbed beta are in general out of phase and there are no worrisome spots. Besides, the existing steering dipoles with $B\ell =$ 0.2 kG-m are strong enough to create any desired deformation of the closed orbit.

With the second part of the question, "is it possible to have a stable circulating beam in the presence of perturbed beta functions and dispersion?", we are in a very murky area. I would venture to say that our operators should be able to keep the situation under control for the proton beam at 125 GeV/c provided that the momentum spread stays within $\sim$ ±0.2%. The main ring (and I understand other proton synchrotrons also) is relatively forgiving of a large momentum spread of the beam when the emittance is small. As for the stability of $\bar{p}$'s at 8.89 GeV/c with 2π mm-mr, the answer depends on so many factors, both known and unknown, that any honest and intelligent discussions are beyond the scope of this note and (let's face it) beyond my faculty.

Certainly various experts of the laboratory must combine their wisdom and knowledge to come out with a convincing reply to the question.

And, finally, the question of space. It is agreed that all position detectors and steering dipoles are "untouchables". One might include chromaticity-correcting sextupoles in this category. All other elements presently existing in ministraights and medium straights can be negotiable, that is, they can be taken out of the ring outright or can be moved to other locations. With this presumptuous statement, let us look at each station involved in the proposed system:

E17 This is a busy place but space is there. The correction octupole can go and the skew (harmonic) sextupole can move to another place if necessary.

F26 Open.

C46 Abort magnet must go out.

F24 Extraction quadrupole F24Q should be out. Let's not talk about experiments with the beam extracted from the main ring.

E49 We may have a problem here. There are 2 GHz toroid for bunch display, coaxial directional coupler for bunch spreader in addition to two steering magnets. If this location is not possible, we may have to move to the downstream end of E48. Even there, we may have to shift E48 pinger to another location. CERN IBS can certainly go out and the RF frequency pickup is negotiable.

THEREFORE, I believe the proposed system should not be abandoned at this time. It certainly deserves more serious studies and explorations by other experts.

References

1. The Fermilab Antiproton Source Design Report, February 1982.
2. Michael Harrison, UPC-157, March 1982.

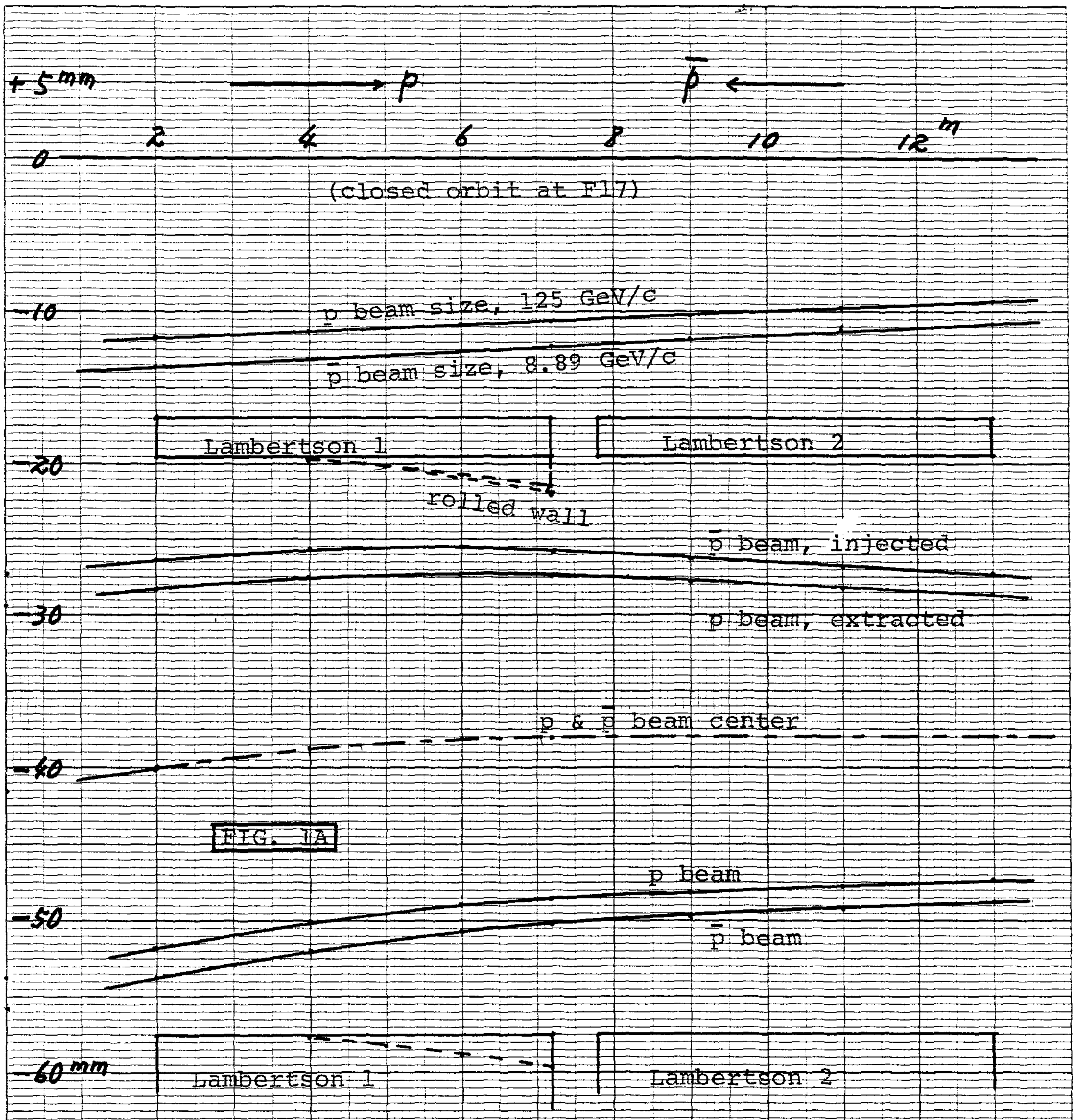

+5 mm
p
p̄
0
2
4
6
8
10
12 m
(closed orbit at F17)
-10
p beam size, 125 GeV/c
p̄ beam size, 8.89 GeV/c
Lambertson 1
Lambertson 2
-20
rolled wall
p̄ beam, injected
-30
p beam, extracted
p & p̄ beam center
-40
p beam
-50
p̄ beam
-60 mm
Lambertson 1
Lambertson 2

FIG. 1A

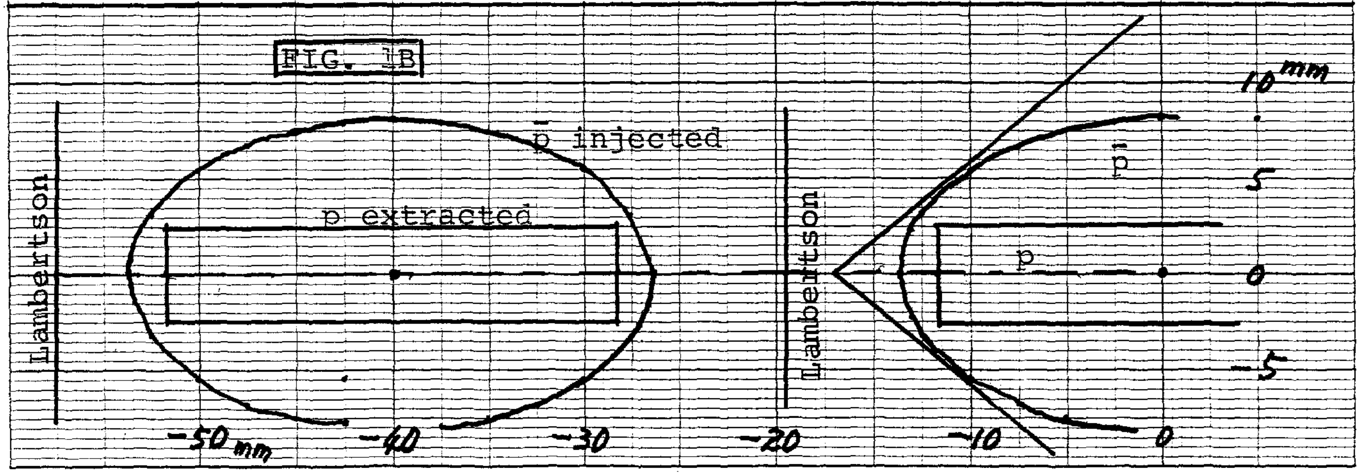

p̄ injected
p extracted
Lambertson
Lambertson
p̄
p
10 mm
5
0
-5
-50 mm
-40
-30
-20
-10
0

FIG. 1B

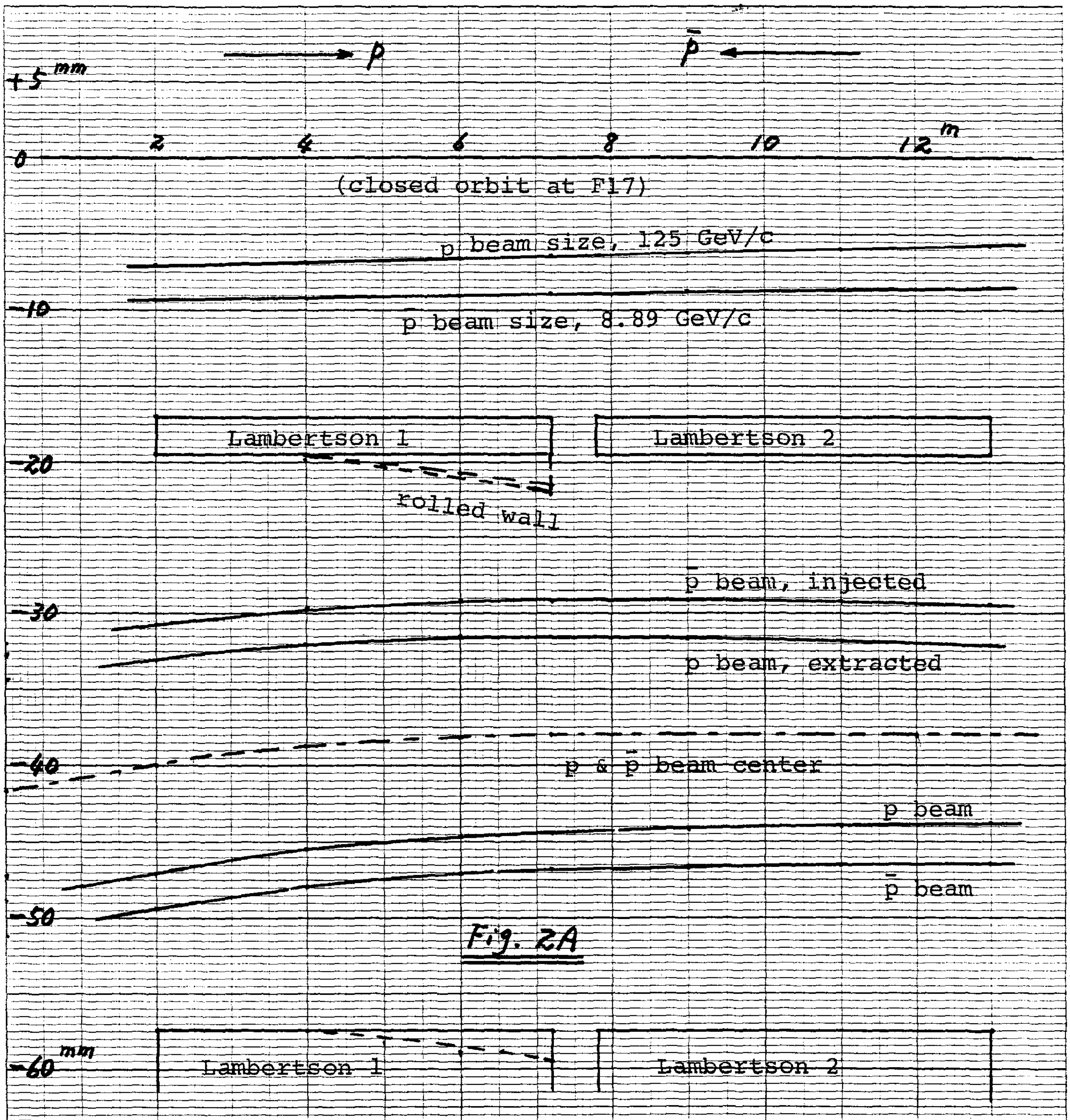

p
$\bar{p}$
+5 mm
0
2
4
6
8
10
12 m
(closed orbit at F17)
p beam size, 125 GeV/c
-10
p beam size, 8.89 GeV/c
Lambertson 1
Lambertson 2
-20
rolled wall
$\bar{p}$ beam, injected
-30
p beam, extracted
-40
p & $\bar{p}$ beam center
p beam
$\bar{p}$ beam
-50
-60 mm
Lambertson 1
Lambertson 2

Fig. 2A

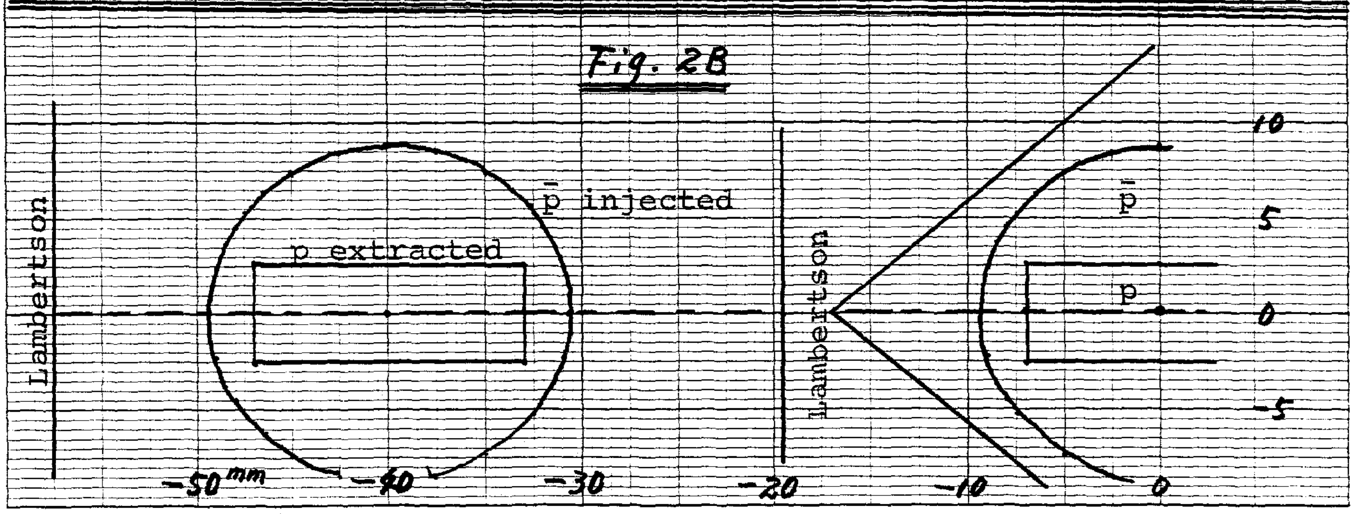

Lambertson
$\bar{p}$ injected
p extracted
Lambertson
$\bar{p}$
p
10
5
0
-5
-50 mm
-40
-30
-20
-10
0

Fig. 2B

APPENDIX <u>Localized Beta Bump</u> (from the note by T. Collins)

The aim is to change the beta function between two points in a ring, from $\phi=0$ to $\phi=\phi_1$ without introducing any change outside this region. The angle ϕ is the phase advance of the betatron oscillation in the direction of our interest (i.e., either horizontal or vertical) measured from the location of the first bump quadrupole. The second quadrupole is placed at $\phi=\phi_1$. It is of course possible to use more than two quadrupoles to create a localized beta bump but the essential features can be explained with only two. In the real application, one naturally tries to minimize the effect on the other direction and, in most cases, to limit the perturbation on the dispersion outside the beta bump region. Inside the bump, the perturbation is unavoidable unless the unperturbed dispersion is zero at the quadrupole locations.

Assume that the betatron oscillation parameters ϕ, $\beta_o(\phi)$, $\alpha_o(\phi)$ and $\gamma_o(\phi)$ are all known everywhere. The subscript "o" indicates that the quantity is the original unperturbed one. It is convenient to use the normalized coordinates

$$\xi(\phi) \equiv x(\phi)/\sqrt{\beta_o(\phi)} \ , \tag{A. 1}$$

$$\eta(\phi) \equiv \sqrt{\beta_o(\phi)} \quad x'(\phi) + [\alpha_o(\phi)/\sqrt{\beta_o(\phi)}]x(\phi) \tag{A. 2}$$

where $x'(\phi) = dx/ds$, s being the distance along the closed orbit. In (ξ,η) space, the transformation from $\phi=\phi_a$ to ϕ_b is a simple rotation

$$\begin{vmatrix} \xi \\ \eta \end{vmatrix}_b = \begin{vmatrix} \underline{c} & \underline{s} \\ -\underline{s} & \underline{c} \end{vmatrix} \begin{vmatrix} \xi \\ \eta \end{vmatrix}_a \tag{A. 3}$$

with $\underline{c} = \cos(\phi_b - \phi_a)$ and $\underline{s} = \sin(\phi_b - \phi_a)$. If the beam is represented by an ellipse in (x, x') space,

$$\gamma_o(\phi)x^2(\phi) + 2\alpha_o(\phi)x(\phi)x'(\phi) + \beta_o(\phi)x'^2(\phi) = W, \tag{A. 4}$$

this ellipse corresponds to a circle in (ξ,η) space,

$$\xi^2(\phi) + \eta^2(\phi) = W. \tag{A. 5}$$

In the absence of nonlinear field, the beam emittance πW is independent of ϕ. Now introduce a local beta bump, that is, modify $(\beta_o, \alpha_o, \gamma_o)$ between $\phi=0$ and $\phi=\phi_1$ by placing quadrupoles at these two locations. Since the machine is still linear, the beam is an ellipse in (x, x') space but with perturbed betatron parameters,

$$\gamma(\phi)x^2(\phi) + 2\alpha(\phi)x(\phi)x'(\phi) + \beta(\phi)x'^2(\phi) = W. \tag{A. 6}$$

The phase ϕ is also modified but the same symbol is used here since it is not important to distinguish the perturbed phase from the unperturbed one. In (ξ,η) space, the ellipse (A. 6) is no longer a circle since the normalization, (A. 1 & 2), is done with the unperturbed parameters (β_o, α_o). Instead, we have another ellipse

$$\gamma_N(\phi)\xi^2(\phi) + 2\alpha_N(\phi)\xi(\phi)\eta(\phi) + \beta_N(\phi)\eta^2(\phi) = W \tag{A. 7}$$

where $\gamma_N\beta_N - \alpha_N{}^2 = 1$. It is easy to find the relations

$$\beta_N \equiv \beta/\beta_o\ , \tag{A. 8}$$

$$\alpha_N \equiv \alpha - [\beta/\beta_o]\alpha_o\ , \tag{A. 9}$$

$$\gamma_N \equiv \gamma\beta_o - 2\alpha\alpha_o + (\beta/\beta_o)\alpha_o{}^2. \tag{A.10}$$

Outside the bump, we should have $\beta = \beta_o$, etc. so that

$$\gamma_N = 1, \qquad \beta_N = 1, \qquad \alpha_N = 0. \tag{A.11}$$

These are the conditions for the beta bump to be localized.

Consider the action of a quadrupole with the gradient B' and the length ℓ in the thin-lens approximation,

$$x \to x, \quad x' \to x' - (B'\ell/B\rho)x \tag{A.12}$$

or

$$\xi \to \xi \quad \eta \to \eta - \beta_o(B'\ell/B\rho)\xi \tag{A.13}$$

where β_o is the unperturbed β at the quadrupole location. In our case with only two quadrupoles, this is the same as β. Introducing the parameter k,

$$k \equiv \beta_o(B'\ell/B\rho); \quad \begin{array}{ll} k>0 & \text{focusing,} \\ k<0 & \text{defocusing} \end{array} \tag{A.14}$$

we can express the action of the quadrupole in (ξ,η) space as

$$\begin{vmatrix} 1 & 0 \\ -k & 1 \end{vmatrix} \tag{A.15}$$

We place a quadrupole with the normalized strength k_o at $\phi=0$ and the other with the strength k_1 at $\phi=\phi_1$. The transformation from $\phi=0$ to ϕ_1 is, in (ξ,η) space,

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -k_1 & 1 \end{vmatrix} \begin{vmatrix} \underline{c} & \underline{s} \\ -\underline{s} & \underline{c} \end{vmatrix} \begin{vmatrix} 1 & 0 \\ -k_o & 1 \end{vmatrix} \tag{A.16}$$

where

$$\underline{c} = \cos(\phi_1) \quad \text{and} \quad \underline{s} = \sin(\phi_1).$$

At $\phi=0$, $\gamma_N = \beta_N = 1$ and $\alpha_N = 0$. At $\phi=\phi_1$, $(\beta_N,\alpha_N,\gamma_N)$ can be calculated from the matrix elements m_{11}, etc. See, for example, CERN "yellow handbook", p. 16, [7.100].

$$\begin{aligned} \beta_N(\phi_1) &= m_{11}^2\beta_N(0) - 2m_{11}m_{12}\alpha_N(0) + m_{12}^2\gamma_N(0) \\ &= 1 - 2k_o\,\underline{s}\,\underline{c} + k_o^{\,2}\,\underline{s}^2, \end{aligned} \tag{A.17}$$

$$\begin{aligned} \alpha_N(\phi_1) &= -m_{11}m_{21}\beta_N(0) - m_{12}m_{22}\gamma_N(0) + (1+2m_{12}m_{21})\alpha_N(0) \\ &= (k_o+k_1)\underline{c}^2 + (k_1-k_o+k_o^{\,2}k_1)\underline{s}^2 - (k_o^{\,2}+2k_ok_1)\underline{s}\,\underline{c}. \end{aligned} \tag{A.18}$$

For the bump to be localized, we must have $\beta_N(\phi_1) = 1$ so that

$$k_o\,\underline{s}^2 = 2\,\underline{s}\,\underline{c} \tag{A.19}$$

(A) $\underline{s} = 0, \quad \phi_1 = n\pi$

From the condition $\alpha_N(\phi_1) = 0$,

$$k_o + k_1 = 0 \tag{A.20}$$

When the phase advance is $n\pi$, we must use a pair of focusing-defocusing quadrupoles with the equal strength parameter $|k|$. However, the value of $|k|$ is arbitrary. Inside the bump $0 < \phi < \phi_1$,

$$\beta_N(\phi) \equiv \beta(\phi)/\beta_o(\phi) = 1 + k_o^2 \sin^2(\phi) - k_o \sin(2\phi). \tag{A.21}$$

The maximum and the minimum values of β_N are

$$\beta_{N,\substack{max\\min}} = (|k|/2 \pm \sqrt{1 + k^2/4}\ )^2 \tag{A.22}$$

The locations where β takes its maximum or minimum value are given by

$$\phi_{max} = \phi_o + \pi/2 \qquad (\text{mod. } n\pi),$$

$$\phi_{min} = \phi_o \qquad (\text{mod. } n\pi) \tag{A.23}$$

The angle ϕ_o is obtained from the relation

$$\tan(2\phi_o) = 2/k_o \quad \underline{\text{and}} \quad \cos(2\phi_o) > 0 \tag{A.24}$$

From (A.22), we see the relation $\beta_{N,max} = 1/\beta_{N,min}$. Remember that in the real (x, x') space,

$$\beta(\phi) = \beta_o(\phi)\beta_N(\phi) \ .$$

(B) $\underline{s} \neq 0, \ \phi_1 \neq n\pi.$

From $\beta_N(\phi_1) = 1$, we find

$$k_o = 2/\tan(\phi_1) \tag{A.25}$$

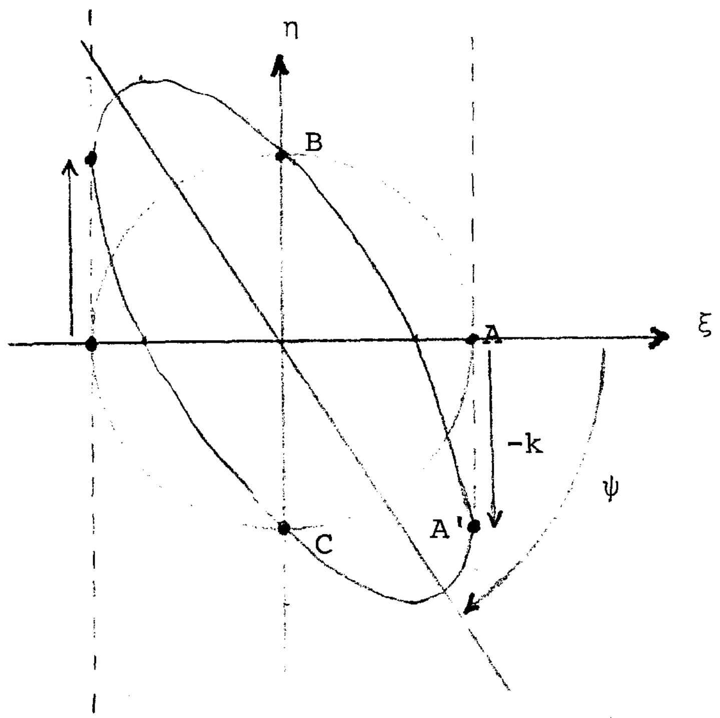
For $\alpha_N(\phi_1) = 0$, we must have [see (A.18)]

$$k_1 = k_o \tag{A.26}$$

In this case, two quadrupoles are identical in their strength parameter k (including the sign) and the value of k is uniquely determined from the phase advance ϕ_1.

The second choice, (B), is a very special application. It is used when two locations with the phase advance of $(n\pi)$ cannot be found in the ring. Unlike (A) where we use a pair of focusing-defocusing quadrupoles with the same strength parameter at the distance of $n\pi$, the choice (B) changes the tune which is certainly a nuisance. Moreover, the dispersion is not localized within the beta bump area. The choice (A) does not always guarantee a localized dispersion. It is necessary to take $\phi_1 = (2n\pi)$ and the unperturbed dispersions at $\phi=0$ and $\phi=\phi_1$ equal.

It is instructive to demonstrate graphically what is happening in the normalized (ξ,η) space. We start with a circle of unit radius.



At $\phi=0$, the circle is transformed to an ellipse by the action of the quadrupole $k_o = k$ (>0, focussing). The point A moves by $-k$ to point A' while B and C are unmoved. The parameters of the ellipse are

$$\beta_N = 1, \quad \alpha_N = k, \quad \gamma_N = 1 + k^2$$

and the direction of the major axis is

$$\tan(2\psi) = 2\alpha/(\gamma - \beta) = 2/k$$

See CERN yellow handbook, p. 19. The ellipse then starts rotating clockwise. The maximum size of the ellipse in ξ-direction is $\sqrt{\beta_N}$ which takes its minimum value and then its maximum value and, after 180° rotation, the situation is back to the original one. The defocussing quadrupole $k_1 = -k$ at $\phi = \phi_1$ then transforms the ellipse back to the circle. This is case (A).
Obviously, the rotation can be any integer multiple of 180° and the amount of move $-k$ is arbitrary. If we stop the rotation when the ellipse is tangential to two vertical dotted lines for the first time, we can come back to the circle by using another focussing quadrupole $k_1 = k$. There is a definite relation between the quadrupole strength k and the phase advance ϕ_1 and fixing one determines the other uniquely (with mod π). This is case (B).
For a combination of many quadrupoles and rotations, the situation is somewhat more complicated but the essential features are the same.

Fermilab

ADDENDUM to TM-1127, "Tinkering at the Main Ring Lattice"

1. According to Stan Pruss, it may be difficult to find a space at E49 for a bump magnet. It is therefore proposed here that the closed-orbit bump will be created by four magnets; two of them are already installed by Mike Harrison. With the dispersion bump E17-F26 on, the bump can be produced by

C26	(Harrison's dipole)	-0.2820 mrad
C32	(Harrison's dipole)	-0.3004 mrad
E48	(to be installed)	+0.1917 mrad
F16	(to be installed)	-0.0776 mrad

The requirement for the integrated field value is modest. The excursion of the kicked beam is maximum ±40mm of which ±20mm is canceled by the bump. The maximum beam size is ±16mm.

2. For $\bar{p}$'s at 8.89 GeV/c, the maximum beam size is ±23mm when the beta bump C46-F24 is on. The closed-orbit bump and the excursion of the kicked beam are almost identical to the proton case. However, the orbit excursion and the beta function oscillation are most of the time out of phase and the beam occupies the range -35mm to +35mm which is not much different from the range occupied by the proton beam at 125 GeV/c. Nevertheless, the much larger betatron oscillation amplitude for this case is a serious drawback for the stable operation.