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RF Stacking and Tail Cooling in the Antiproton Accumulator

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Contents

Abstract

We find criteria on the cooling system to minimize effects of RF stacking on the cooling tail.

1. Introduction

A scheme /1/ has been recently proposed to produce a high-intensity antiproton beam in a stochastic cooling accumulator. According to this scheme, the coasting beam of $\overline{p}$'s extracted from the Debuncher ring will occupy the injection orbit of the accumulator ring with a total momentum spread of about 0.2%. This beam is to be bunched adiabatically. Then the buckets are transformed to moving(decelerating) buckets, slowly enough to obtain as large a capture efficiency as possible. The amount of deceleration depends on the physical distance which must be cleared by the bunch. After deceleration is completed, the RF is turned off slowly to allow the beam to debunch adiabatically. The beam is now ready for stochastic cooling. The debunched beam is soon moved to down in the stream by the tail cooling system in order to make room for the next stacked beam.

Although effects of RF stacking on the already stacked beam have been pointed out by many people, we do not know quantitatively much about this problem. This is a reason that the design of the tail cooling system has been not yet determined in detail. In the present paper, we understand the nature of the above effect and obtain some criteria for designing the tail cooling system.

There are in general two effects that RF stacking gives an already stacked beam, which shall be called " tail " because the main part of an accmulated beam has the common name of " core ". The larger is phase-displacement acceleration and the other is so-called dilution coming from non-adiabatic change of RF parameters, say, $V_{rf}(t)$ and $f_{rf}(t)$, which is understood to be enhanced by non-linearity of RF stacking dynamics.

Phase-displacement acceleration is simply related to the phenomenon of phase-flow as a consequence of Liouville's theorem. Such phase flow is determined only by the nature of a dynamical system. It would, in principle, be possible to determine phase-flow from Boltzmann's equation

$$\frac{\partial f}{\partial t} + [H , f] = 0, \tag{1}$$

where f is the distribution function, H is the time-dependent Hamiltonian, including non-linear terms for the present case, and [,] is the Poisson bracket. This, however, has considerable mathematical difficulties. In the present report, we therefore investigate the structure of the dynamical system which describes RF stacking with a help of numerical experiments. We note that a collection of phase points in the phase space behaves as an incompressible gas. From this incompressible gas analogy, it is easy to suppose that phase-flow close to the RF bucket is most active. Thus we shall particularly concentrate our attention on the phase space region where the RF bucket annihilates, near the separatrix.

In the third section, we discuss dynamics of tail cooling with a help of the Fokker-Planck equation, which has become a common technique in a study of stochastic cooling. From the tail cooling point of view, the effects of RF stacking can be regarded as deformation of the tail which occurs within a limited period (that is, 100 msec), tail cutting due to phase-displacement and non-uniform dilution in the tail due to non-adiabatic diffusion. An easy method for estimating beam-loss due to phase-displacement is presented and used to obtain the criterion which should be imposed on the cooling system.

2. RF Stacking & Its Effects on Cooling Tail

2-a Stacking Process

The stacking process consists of the following four phases:

1. Adiabatic Capture. The injected coasting beam is adiabatically captured in the RF bucket, whose phase space area is slowly increasing until it becomes a little larger than the phase space area occupied by the beam. The period which adiabatic capture requires is chosen from general considerations about the adiabaticity of a harmonic oscillator /2/ and is chosen equal to $3\tau_s$, where τ_s is the period of the synchrotron oscillation at the final stage of this step. (See Fig.1-a)

2. Alteration (Stationary Bucket $\rightarrow$ Moving Bucket). An adiabatic alteration of the RF parameters, say, V_{rf} and f_{rf}, is performed in order to convert a " stationary bucket " into a " moving bucket ", capable of changing the mean energy of the particles trapped inside.(See Fig.1-b)

3. Deceleration. A period of deceleration follows, employing a moving bucket of constant area and constant synchronous phase angle ϕ_s. (See Fig.1-c)

4. Adiabatic Debunching. The RF is turned off adiabatically after it has been transformed again into a stationary bucket of the same area from a moving bucket. (See Fig.1-d)

The above process is performed by manipulating both parameters of the RF, voltage V and frequency f_{rf}. In Fig.2, the RF voltage V and the synchronous phase angle ϕ_s relevant to f_{rf}

are shown as functions of time. In addition, other RF parameters /3/ are listed in the same figure. Since the general nature of phase space dynamics for RF stacking has been discussed in detail in the previous report/4/, we will not examine it extensively except for the present problems of interest.

Results of numerical simulations already shown in Fig.1-a ~ Fig.1-d, also give the energy distribution of an injected and released beam on the stacking orbit. We obtain a final full energy-spread of 0.21% and a final capture efficiency of 98%.

2-b Phase-Displacement

As a consequence of Liouville's theorem, the phase area transported downward in the energy plane by a decelerating RF bucket must be accompanied by the upward trasport of an equal phase area in the region outside the bucket, a property known as phase-displacement, which is seen in Fig.3. This phenomenon gives rise to a serious problem in the previous pulse , which has been moved towards the cooling core by the tail cooling system during one cycle. That is, following phase flow due to the mentioned mechanism, a fraction of the previous pulse is transported upward from the region where a new pulse is deposited. Then the fraction transported upward must be understood to be lost out of the region where the tail cooling system is effective, since it is removed more and more upward in the energy plane every cycle due to the unique direction of motion of a decelerating bucket.

From the tail cooling point of view, we are not very interested in the detailed nature on the phase space (E, Φ) of phase-displacement, because stochastic momentum cooling is usually discussed on the energy plane alone after averaging out the phase variable. Therefore, in order to estimate quantitatively the effects of phase-displacement on the tail, we consider the behaviour of phase points which are located in a small energy bin at the initial time. We divide the energy region of interest into many small bins. One-thousand phase points are distributed in each bin and simulated for an entire RF stacking. After one stacking cycle, these points will be located mainly in higher bins as a result of phase-displacement. Hence, such a procedure yields a kind of a transfer matrix which characterizes quantitatively over-all effects of RF stacking on the tail. This transfer matrix is involved in tail cooling calculations later. We also note that the critical region for phase-displacement exists, as seen in table 1. Roughly speaking, the fact indicates a criterion which must be imposed on the tail cooling system so as to minimize particle-loss. If all parts of the stacked pulse are moved toward the core beyond this critical regions, our anxiety about beam-loss should vanish.

2-c Non-Adiabatic Diffusion

RF stacking must be performed within a limited period of one cycle time in order to retain enough time for tail and core cooling. Such a situation leads to non-adiabatic diffusion particularly in the tail region of interest. It is a kind of dilution and its qualitative nature is explained schematically in Fig.4.

Even the end of the tail is far from the synchronous stable point of a stationary or moving bucket, and it is therefore apparently impossible to linearize the motion of a particle in the tail region. Thus it is just the non-adiabatic behavior of a highly the non-linear system that we should investigate. For the moment, we consider the mathematical structure of such a system.

We may write the dynamics of RF stacking in the difference equations

$$E_{n+1} = E_n + eV(n)\sin\phi_n, \tag{2-1-a}$$

$$\phi_{n+1} = \phi_n + \omega_{rf}(n)\frac{2\pi}{\omega_0}\left[1 + \eta\,\frac{E_{n+1}-E_0}{\beta_0^2 E_0}\right], \tag{2-1-b}$$

where (E_n, ϕ_n) are the energy and the electrical phase angle with which a particle enters the cavity at the time of transit, $(V(n), \omega_{rf}(n))$ are the RF voltage and the RF angular frequency programmed as functions of time or step n, ω_0 is the angular rotation frequency of a particle with the fixed energy E_0, β_0 is the relativistic beta corresponding to the energy E_0, and η is $1/\gamma_T^2 - 1/\gamma_0^2$. If changes of the variable and the parameter

$$\epsilon_{n+1} = (E_{n+1} - E_0)/\beta_0^2 E_0, \tag{2-2}$$

$$\omega_{rf}(n) = h\omega_0\,[\,1 + f(n)\,] \qquad f(n) \ll 1, \tag{2-3}$$

where h is the harmonic number, are made, we have the transformed difference equations

$$\epsilon_{n+1} = \epsilon_n + \frac{eV(n)}{\beta_0^2 E_0} \sin\phi_n , \tag{2-4-a}$$

$$\phi_{n+1} = \phi_n + 2\pi h \eta \epsilon_{n+1} + 2\pi h f(n) (1 + \eta \epsilon_{n+1}) . \tag{2-4-b}$$

To write down the difference equations in the form of exact differential ones we may use a δ-function:

$$\dot{\epsilon} \simeq \frac{\epsilon_{n+1} - \epsilon_n}{T(t)} = \frac{eV(t)\cdot 2\pi}{\beta_0^2 E_0 T(t)} \sin\phi \, \delta_{2\pi}(\tau) , \tag{2-5-a}$$

$$\dot{\phi} = \frac{2\pi h}{T(t)} [\eta \epsilon + f(t)(1 + \eta \epsilon)] , \tag{2-5-b}$$

where T(t) is the time period corresponding to one iteration of the mapping and the rotation period of the synchronous particle. Here $\tau = \Omega(t)t + \tau_0$; $\Omega(t) = 2\pi / T(t)$ and the δ-function of period 2π is given by the Fourier expansion

$$\delta_{2\pi}(\tau) = \frac{1}{2\pi} \left(1 + 2 \sum_{n=1}^{\infty} \cos n\tau \right) . \tag{2-6}$$

Thus the dynamics of RF stacking is described by the Hamiltonian

$$H(\phi, \epsilon ; t) = \frac{2\pi h}{T(t)} \left\{ \eta \frac{\epsilon^2}{2} + f(t) \left[\epsilon + \eta \frac{\epsilon^2}{2} \right] \right\} + \frac{2\pi e V(t)}{\beta_0^2 E_0 T(t)} \cos\phi \, \delta_{2\pi}(\tau) . \tag{2-7}$$

Futhermore a change of the time scale

$$t^* = 2\pi h \int \frac{dt}{T(t)} , \tag{2-8}$$

yields the transformed Hamiltonian

$$H^*(\phi, \epsilon ; t^*) = \eta \frac{\epsilon^2}{2} + f(t^*)\left(\epsilon + \eta \frac{\epsilon^2}{2}\right) + \frac{eV(t^*)}{\beta_0^2 E_0 h} \cos\phi \, \delta_{2\pi}(\tau) . \tag{2-9}$$

It is trivial to obtain the usual Hamiltonian for stationary stacking from (2-9). After neglecting rapidly oscillating terms in (2-9), we have the so-called averaged Hamiltonian

$$\langle H^* \rangle = \eta \frac{\epsilon^2}{2} + f(t^*)\,\epsilon + \lambda(t^*) \cos\phi , \tag{2-10}$$

with

$$\lambda(t^*) = \frac{eV(t^*)}{\beta_0^2 E_0 \cdot 2\pi h} , \tag{2-11}$$

where we also neglect the third term in (2-9) for the reason that it is small compared with the first and second term. Canonical equations derived from (2-10) are written in the form

$$\dot{\phi} = \frac{\partial \langle H^* \rangle}{\partial \epsilon} = \eta \epsilon + f(t^*) , \tag{2-12-a}$$

$$\dot{\epsilon} = -\frac{\partial \langle H^* \rangle}{\partial \phi} = \lambda(t^*) \sin\phi . \tag{2-12-b}$$

Thus, the solutions (ϵ_s , ϕ_s) for the synchronous point are

$$\epsilon_s(t^*) = -\frac{1}{\eta} f(t^*), \qquad (2\text{-}13\text{-}a)$$

$$\phi_s(t^*) = \sin^{-1}\left[-\frac{\dot{f}(t^*)}{\eta\lambda(t^*)}\right]. \qquad (2\text{-}13\text{-}b)$$

We understand that stationary deceleration of RF stacking can be made under the parameter conditions

$$\dot{f}(t^*) = \text{const},$$

$$\lambda(t^*) = \text{const or } V(t^*) = \text{const}.$$

It is difficult to discuss analytically the adiabaticity of the dynamical system (2-10) under slow changes of its parameters, that is, $f(t^*)$ and $V(t^*)$. So we consider a sequence of infinite phase points, whose behaviors are determined by the Hamiltonian (2-9). We assume that phase points comprising the sequence have the same initial condition

$$\epsilon = \epsilon_0 \qquad \text{at} \qquad t^* = 0.$$

We can regard a energy spread among these phase points after the whole change of f and V, as a measure of non-adiabatic diffusion. The energy spread ΔE obtained by numerical calculations is shown as a function of energy E in Fig.5.

We can see the remarkable fine structure in the ΔE-E curve and the existence of the critical region which is consistent with the phase displacement mentioned in the previous subsection. This fine structure seems to us to be directly relevant to the problem of stable or unstable solutions for the non-linear Mathiew equation

$$\frac{d^2 w}{dt^2} + (\alpha - 2\beta^2 \cos 2t) \cos w = 0,$$

because $V(t^*)$ is usually assumed to change following a cosine-like function of time. However, since the fine structure itself is not so important for the present discussion, we shall not examine it extensively. Meanwhile absolute values of diffusion in the tail region do not seem to be negligible whenever there are gradients in the particle distribution. Thus these effects will be involved in cooling calculations as periodic blow-up of the tail.

3. Tail Cooling

3-a Estimation of Beam Loss due to Phase Displacement

Stochastic cooling in the energy plane is usually discussed by the Fokker-Planck equation /5/

$$\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial E}\left[F(E)\psi\right] + \frac{\partial}{\partial E}\left\{\left[D_1(E) + D_2(E)\psi\right]\frac{\partial \psi}{\partial E}\right\}, \qquad (3\text{-}1)$$

where ψ is the density distribution function, and F, D_1 and D_2 are cooling parameters of the system which are usually called the drift coefficient and the diffusion coefficient. Solving (3-1), we can know in principle the time evolution of the particle distribution. In particular, we shall consider the time evolution of the stacked pulse over one injection cycle T_c.

For the sake of simplicity, we assume:

1) F and $(D_1 + D_2\psi)$ are constant in the region of interest,

$$F = \bar{F} \ ,$$

$$(D_1 + D_2\psi) = \bar{D} \ .$$

2) the distribution of a RF stacked beam is Gaussian at t=0,

$$\psi(E, t=0) = \frac{N_0}{\sqrt{2\pi}\,\sigma_0}\, e^{-(E-m_0)^2/2\sigma_0^2} \ ,$$

where N_0 is the number of particles per injection, σ_0 is the standard deviation, and m_0 is the position of the initial distribution center. These assumptions lead to the linear

Fokker-Planck equation

$$\frac{\partial \psi}{\partial t} = -\bar{F}\frac{\partial \psi}{\partial E} + \bar{D}\frac{\partial^2 \psi}{\partial E^2} . \qquad (3\text{-}2)$$

Then we have the exact solution of (3-2)

$$\psi(E,t) = \frac{N_0}{\sqrt{2\pi}\,\sigma} e^{-(E-m)^2/2\sigma^2} , \qquad (3\text{-}3)$$

with $m = m_0 + \bar{F}t,$

$\sigma^2 = \sigma_0^2 + 2\bar{D}t.$

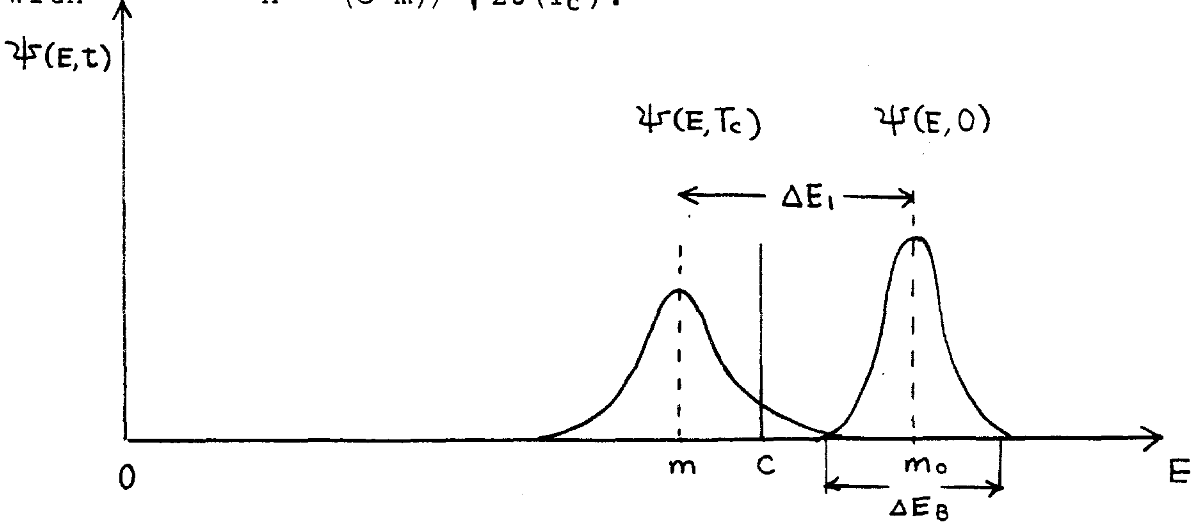
The solution (3-3) apparently represents the parallel displacement of the distribution associated with increasing of the standard deviation. Now, using the solution (3-3), let us calculate the number of particles ,say ΔN, which after one cycle is still left in the upper stream above the critical region mentioned in the previous section. That is,

$$\Delta N = \int_C^\infty \psi(E, T_c)\, dE , \qquad (3\text{-}4)$$

where we denote the critical region by C. Thus, after change of the integration variable, we define the beam-loss ratio r due to phase displacement in the form of the Gauss error function

$$r \equiv \frac{\Delta N}{N_0} = \frac{1}{\sqrt{\pi}} \int_{\frac{C-m}{\sqrt{2}\,\sigma}}^{\infty} e^{-x^2}\, dx = E_{rfc}(n)/\sqrt{\pi} , \qquad (3\text{-}5)$$

with $\quad n = (C-m)/\sqrt{2}\sigma(T_c)$.



It is obvious, from (3-5), that the larger $\overline{F}$ and the smaller $\overline{D}$, the smaller r.

A real situation is generally different from the assumptions made early in this subsection. However, if we have a reasonable way for the coefficients of F and $(D_1 + D_2\psi)$ to be approximated by some constant values, that is, $\overline{F}$ and $\overline{D}$, the loss estimation by (3-5) may be still fruitfull.

3-b Effective Cooling Parameters $\overline{F}$ and $\overline{D}$

This subsection will discuss how we should choose the effective values of $\overline{F}$ and $\overline{D}$. Before we proceed, we shall make the important assumptions:

1) The present cooling system has a perfectly exponential gain, that is,

$$F(E) = \hat{F} \exp\left(\frac{E - m_0}{E_d}\right), \tag{3-6}$$

where E is the distance from the center of the core, E_d is the so called characteristic energy /1/ , and $\hat{F}$ is the value of F(E) at the center of a stacked pulse.

2) The standard deviation σ_0 of the distribution is related to the finite full energy spread ΔE_B of particles which are deposited on the stacking orbit by

$$\sqrt{6\pi}\,\sigma_0 = \Delta E_B . \qquad (3\text{-}7)$$

Then it is trivial to estimate the displacement of the distribution peak over one injection cycle, say, ΔE_1. The behavior of the peak is governed by the differential equation

$$\frac{dE}{dt} = F(E) , \qquad (3\text{-}8)$$

with the initial condition

$$E(0) = m_0 .$$

Substituting (3-6) into (3-8) and integrating both sides, we have

$$\Delta E_1 = -E_d \ln \left[1 - \frac{T_c \hat{F}}{E_d} \right]. \qquad (3\text{-}9)$$

Thus we know the central position between the front and rear peak over one injection cycle,

$$\bar{E} = m_0 + \Delta E_1/2 . \qquad (3\text{-}10)$$

Now, we define the approximated cooling parameters $\bar{F}$ and $\bar{D}$ as follows

$$\overline{F} = F(\overline{E}), \tag{3-11-a}$$

$$\overline{D} = D_2(\overline{E})\overline{\psi}_0, \tag{3-11-b}$$

with $$\overline{\psi}_0 = N_0 / 2\Delta E_B .$$

Here we neglect D_1 and assume that the distribution after one injection cycle does not change significantly except for a the displacement of its peak. In fact, for the present case, $D_2\psi$ is about five times larger than D_1 at the peak, as seen later.

In order to ascertain the accuracy of the above approximation for several examples, the estimation of beam-loss by using (3-11-a) and (3-11-b) is compared with the result when Eq.(3-1) is solved numerically. In Table 3, these comparisons are summarized. They show that the loss ratio r by (3-5) is smaller than the exact value by about 10%. Therefore the estimation by using the mentioned $\overline{F}$ and $\overline{D}$ still has practical meaning.

3-c Criterion for Cooling Parameters

The estimate of the previous subsection is not accurate , but would be as a guide line for designing the tail cooling system. Since we have already derived an approximate relationship (3-5) between the beam-loss ratio and the cooling parameters, we furthermore give a general criterion which should be imposed on the tail cooling system in order to minimize beam loss.

If we want the beam-loss ratio r less than some value, say,

r_0

$$r_0 = \mathrm{Erfc}(n_0)/\sqrt{\pi}, \tag{3-12}$$

the approximate cooling parameters must satisfy the equation

$$\frac{c - m_0 - \overline{F}\,T_c}{\sqrt{2}\,\sigma(T_c)} \geqq n_0 \quad \left(n_0 = \mathrm{Erfc}^{-1}(\sqrt{\pi}\, r_0) \right). \tag{3-13}$$

Then, using the more explicit expression of $\overline{D}$

$$\overline{D} = A\,T^2\,\Psi_0\,\overline{F}^2, \tag{3-14}$$

where T is the revolution period and A is defined in Table 2, we have a quadratic inequality for $\overline{F}$

$$\left(T_c^2 - \frac{T_c A T^2 N_0 n_0^2}{\Delta E_B}\right)\overline{F}^2 + 2k\,\Delta E_B\,T_c\,\overline{F} + (\Delta E_B)^2\left(k^2 - \frac{2n_0^2}{6\pi}\right) \geqq 0, \tag{3-15}$$

where we make use of the relation

$$m_0 - c = k\,\Delta E_B. \tag{3-16}$$

It is trivial to obtain the solution of (3-15)

$$\overline{F} \leqq -\frac{\Delta E_B}{T_c}\left[\frac{k + n_0\sqrt{1/3\pi + k^2 a - a n_0^2/3\pi}}{1 - a n_0^2}\right], \tag{3-17}$$

with

$$a = \frac{A\,T^2 N_0}{T_c\,\Delta E_B},$$

$$= \frac{\beta^2 E_0 \Lambda}{4 T_c T |\eta| W^2} \cdot \frac{N_0}{\Delta E_B}. \tag{3-18}$$

Given all the other parameters except n_0, we can write the right-hand side of (3-17) as a function of n_0 alone, which we denote by $\alpha(n_0)$ and is plotted in Fig.6. We can also derive a similar criterion formula for $\hat{F}$. Substitution of (3-10) into (3-11-a) yields

$$\bar{F} = \hat{F} \left[1 - \frac{T_c \hat{F}}{E_d} \right]^{-1/2}. \tag{3-19}$$

Substituting further (3-19) into (3-11) and arranging for $\hat{F}$, we have the quadratic inequality for $\hat{F}$

$$\hat{F}^2 + \frac{T_c}{E_d} \alpha^2(n_0) \hat{F} - \alpha^2(n_0) \geqq 0. \tag{3-20}$$

Then the solution of (3-20) is

$$\hat{F} \leqq \frac{1}{2} |\alpha| \left[- \frac{T_c}{E_d} |\alpha| - \sqrt{ \left(\frac{T_c}{E_d} \alpha \right)^2 + 4 } \right]. \tag{3-21}$$

3-d Example

As an example, we attempt to integrate numerically the non-linear Fokker-Planck equation which represents exactly tail cooling in the proposed accumulator /1/. In this calculation, we assume the RF stacked pulse is characterized by the parameters:

Gaussian distribution,

$\Delta E_B = 18.0$ (MeV),

$N_0 = 10^8$.

and the cooling system by the parameters:

frequency range of 1-2 GHz

$|\eta| = 0.02,$

$T_c = 2.0$ sec,

$T = 1.6\times10^{-6}$ sec.

Figs.7-a∼7-c show coefficients of the Fokker-Planck equation. As the result of numerical integrations, we find that a beam fraction of 13.4% is lost during the next stacking process. Meanwhile the approximate method discussed in the earlier parts of this section gives a beam-loss of 11%. In this estimate by Eq.(3-5), we use the parameter

$$\bar{E} = m_0 - 9.44\ (\mathrm{MeV})$$

,because we have E_d=31(MeV) and $\hat{F}$ =-13(MeV/sec) which can be read by first-order linearization of the gain curve in Fig.7-a.

4. Conclusion

The unavoidable effects of RF stacking on the cooling tail have been discussed in detail. It is still difficult to say that we have a through understanding of the non-adiabatic diffusion appearing in the present RF stacking, but its quantitative effect has been obtained at least for the proposed system /1/ which is directly available for cooling calculations. Unfortunately, in the present paper, long-range effects due to this non-adiabatic diffusion over a whole cooling time , that is, several houres, were not investigated.

Further considerations of non-adiabatic variation in a non-linear system will be given elsewhere /6/.

In addition, a simple method estimating beam-loss has been presented which can serve as a guide line for designing the tail cooling system. A more accurate estimate may also be possible by applying a perturbation technique.

Acknowledgement

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References and Footnotes

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Figure Captions

Fig.6 : Critical curve for $\overline{F}$ versus the parameter n_o or r_o

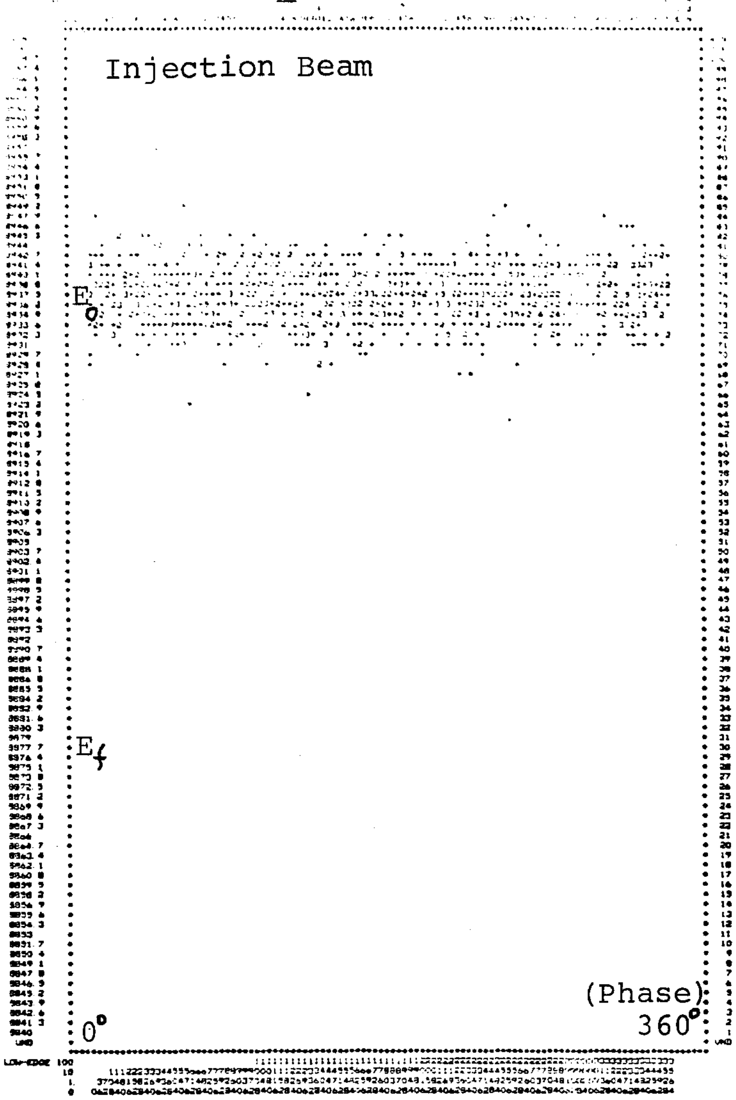
Fig.7-a $\sim$ 7-c : Coefficients of the Fokker Planck equation (3-1). as functions of E.

E is the energy deviation from the cooling core.

Fig.7-d : Distribution of particles

The RF stacked pulse is shown by the dotted line, and the pulse after T_c by the solid line. The shadowed area is the survival after the next stacking process.

Fig.1-a



Distribution of Injected Beam

In the Energy Plane

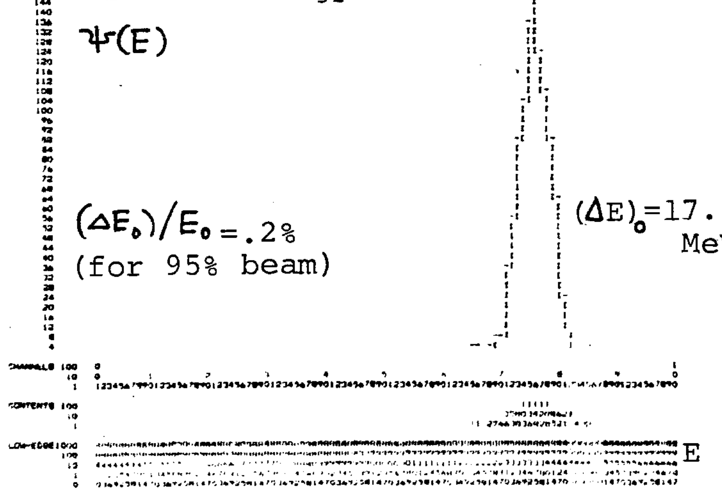


Fig.1-b

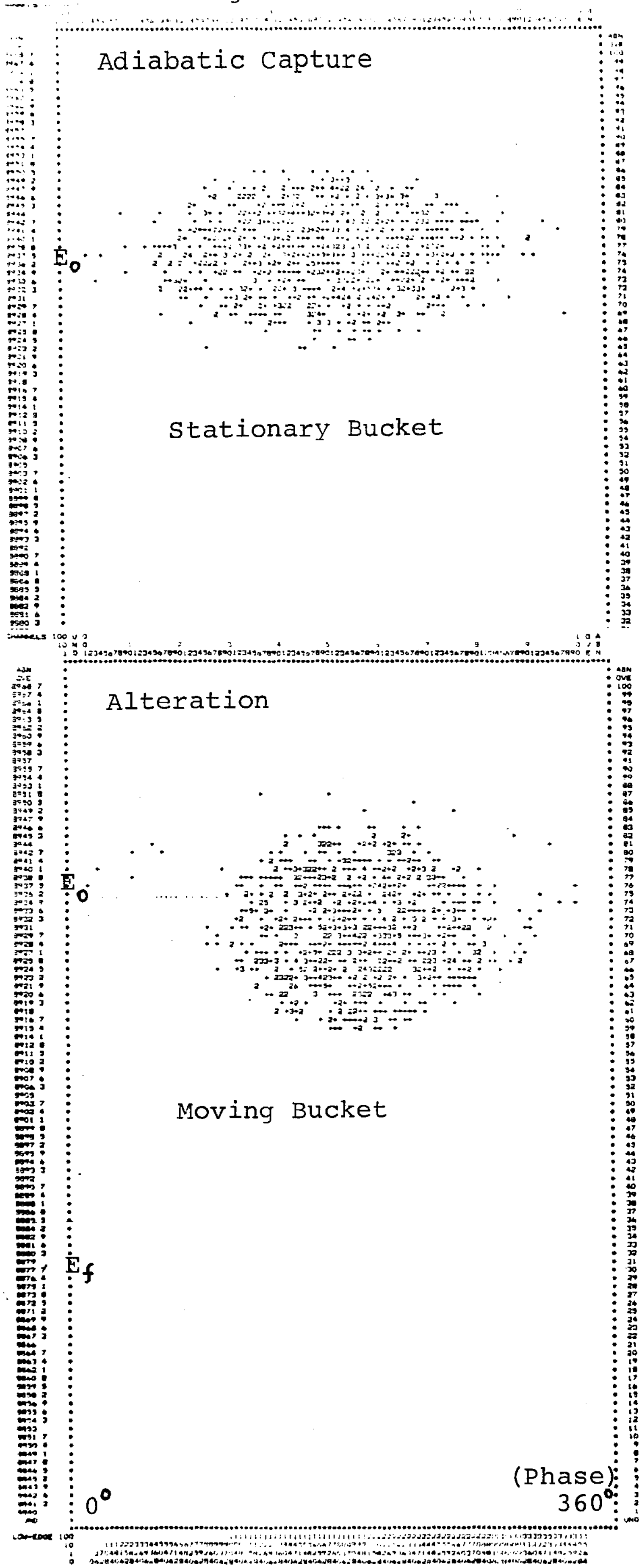


Fig.1-c

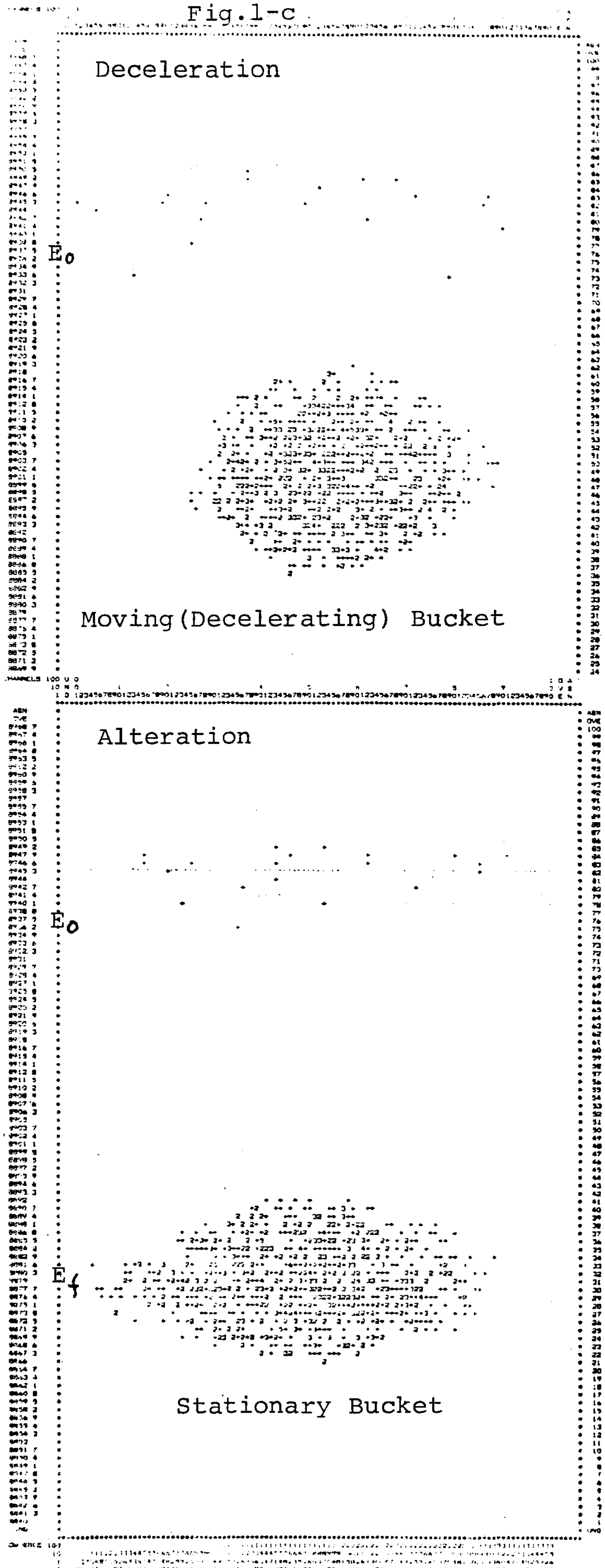
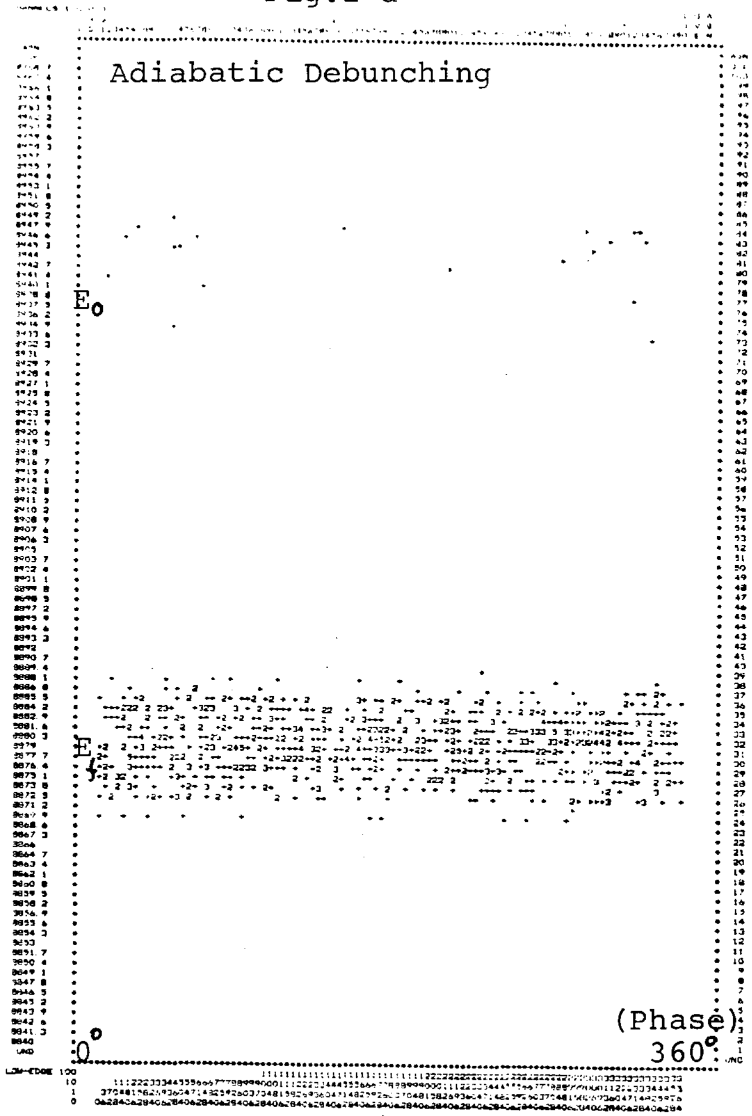


Fig.1-d



Distribution of Stacked Beam in the Energy Plane

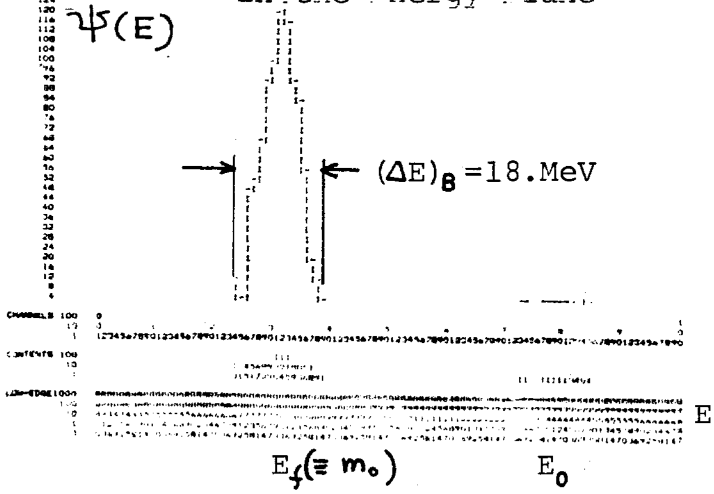


Fig.2

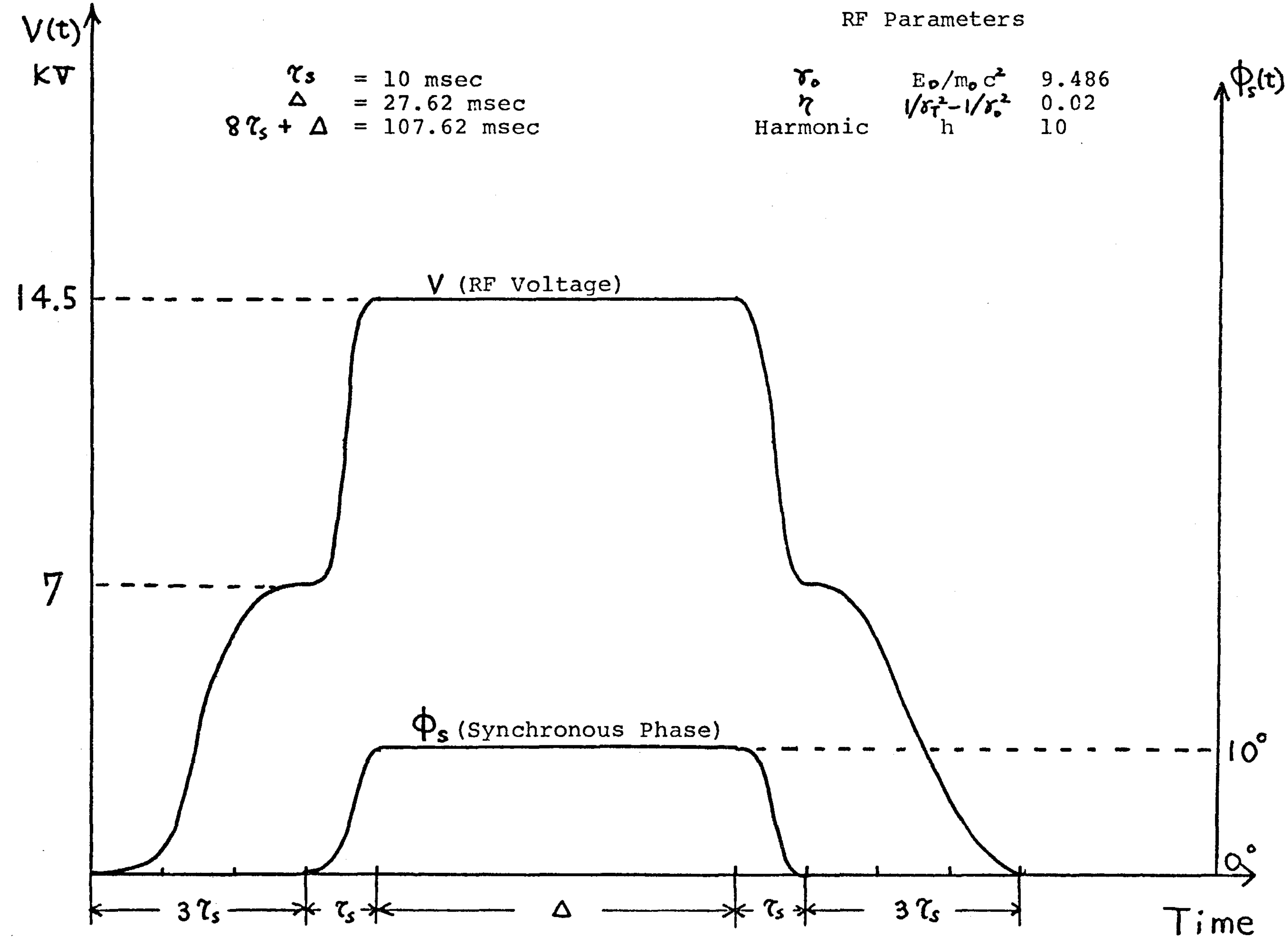

RF Parameters
τ_s = 10 msec
Δ = 27.62 msec
$8\tau_s + \Delta$ = 107.62 msec
γ_0 E_0/m_0c^2 9.486
η $1/\gamma_T^2 - 1/\gamma_0^2$ 0.02
Harmonic h 10
V(t) kV
14.5
7
V (RF Voltage)
ϕ_s (Synchronous Phase)
$\phi_s(t)$
10°
0°
$3\tau_s$
τ_s
Δ
τ_s
$3\tau_s$
Time

Fig.3

Phase-Displacement

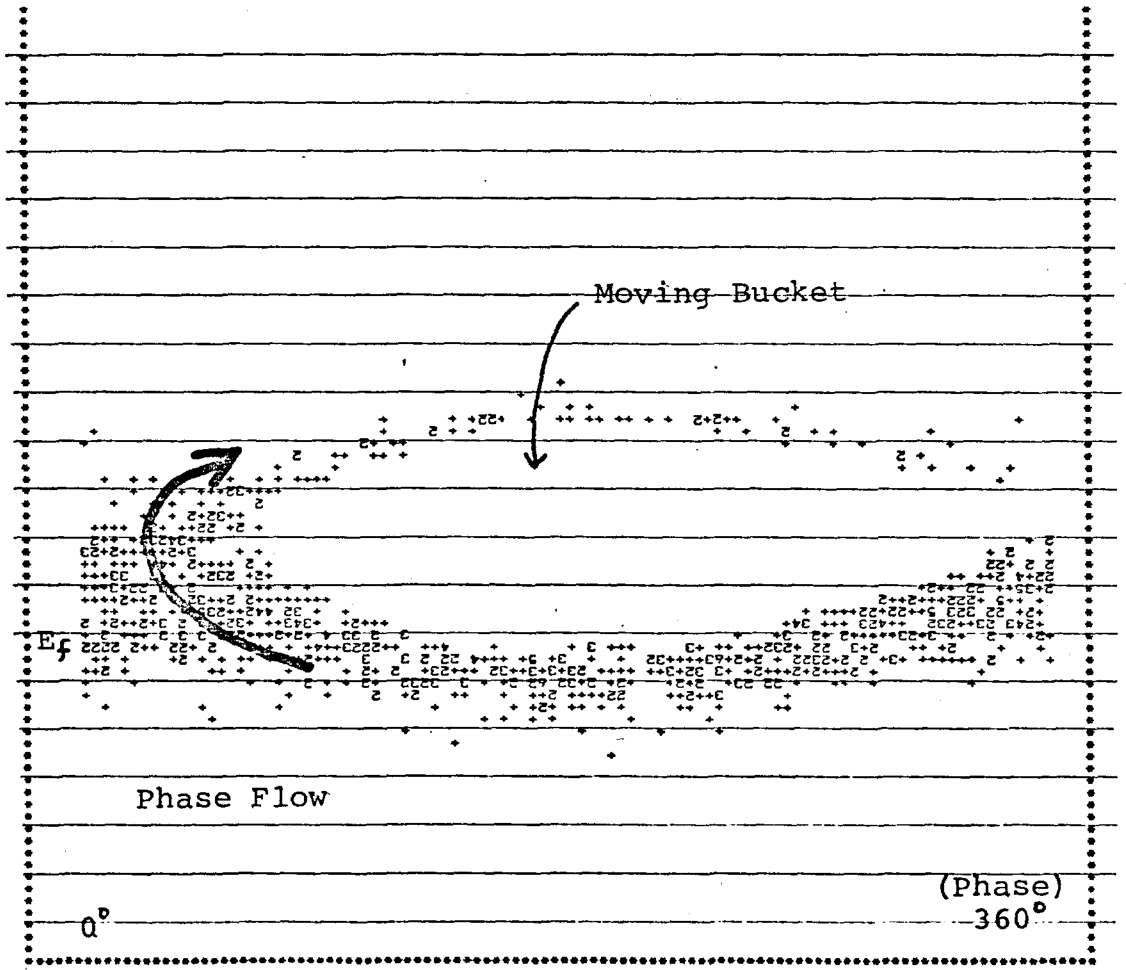
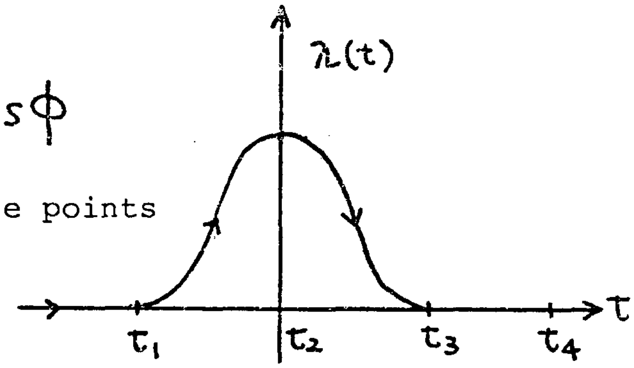


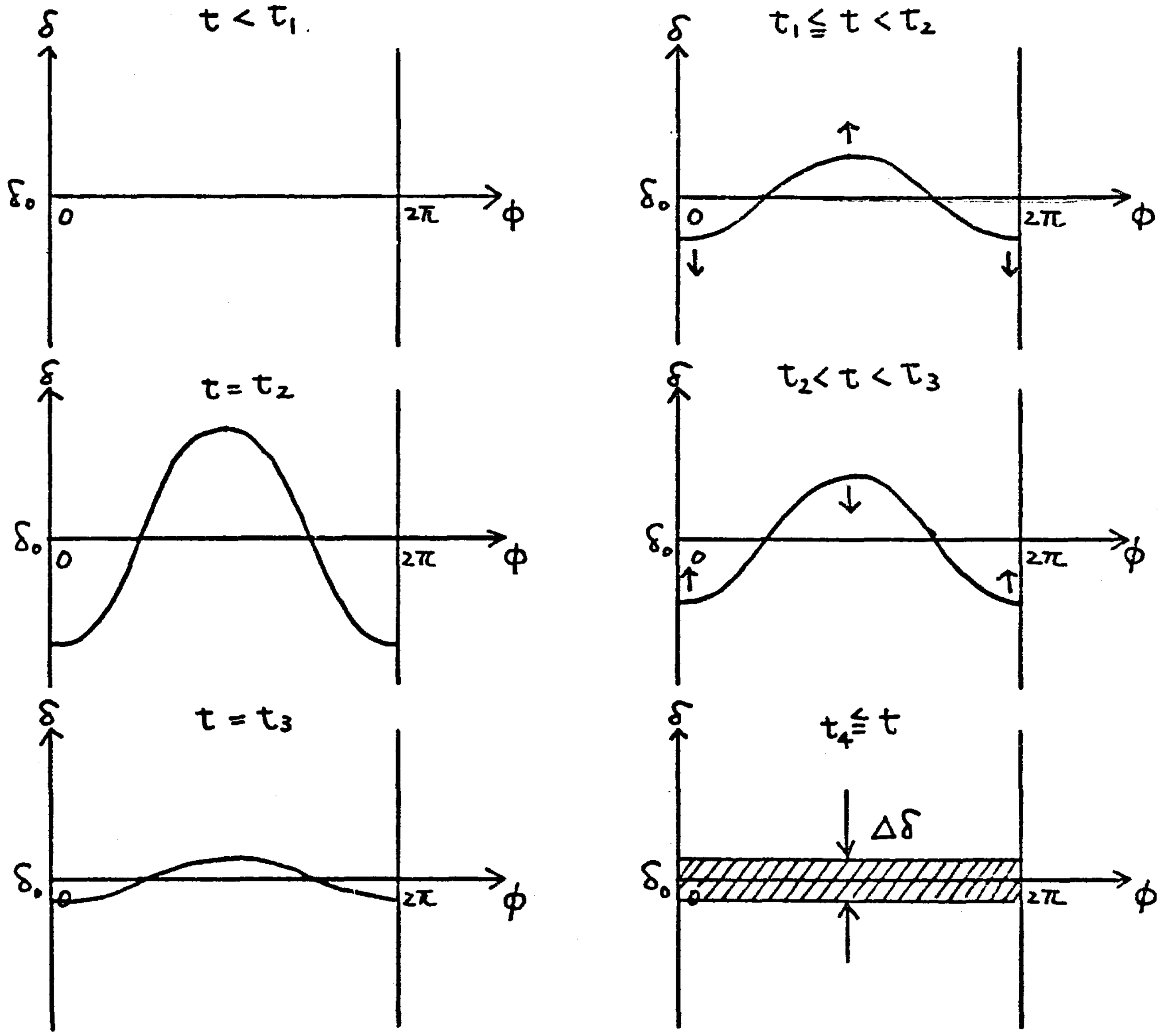
Fig.4
Non-Adiabatic Variation of Non-Linear System (Simple Model)

$$H(\delta, \phi; t) = \frac{1}{2}\delta^2 + \lambda(t)\cos\phi$$

Consider the infinite sequence of phase points with the initial condition:

$$\delta = \delta_0$$





$\Delta\delta$: Effective Spread of Phase Points

Fig.5

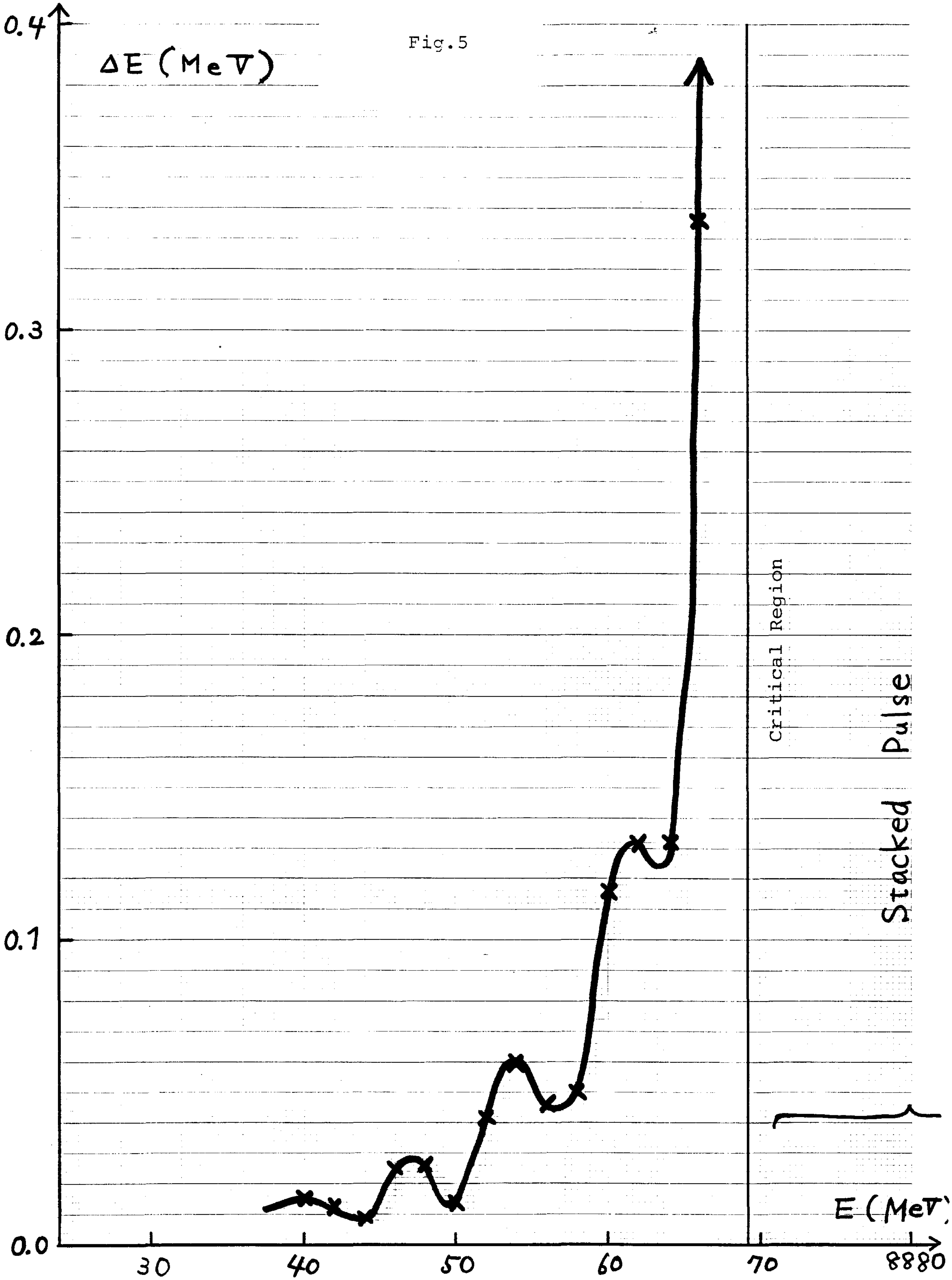
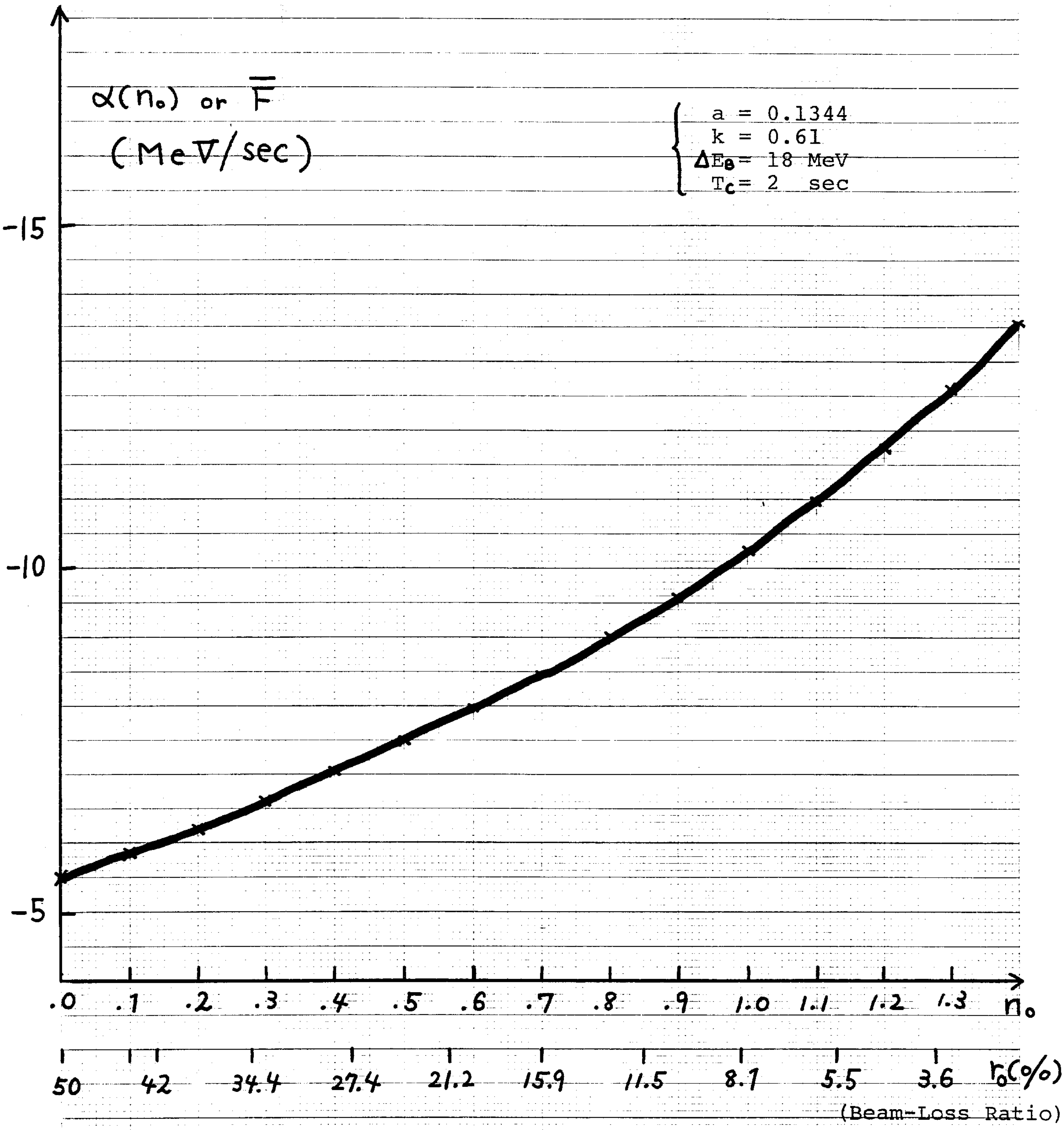

ΔE (MeV)
0.4
0.3
0.2
0.1
0.0
30
40
50
60
70
8880
E (MeV)
Critical Region
Stacked Pulse

Fig.6


α(n₀) or $\overline{F}$
(MeV/sec)
a = 0.1344
k = 0.61
ΔE_B = 18 MeV
T_c = 2 sec
-15
-10
-5
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
n_0
50
42
34.4
27.4
21.2
15.9
11.5
8.1
5.5
3.6
r_0(%)
(Beam-Loss Ratio)

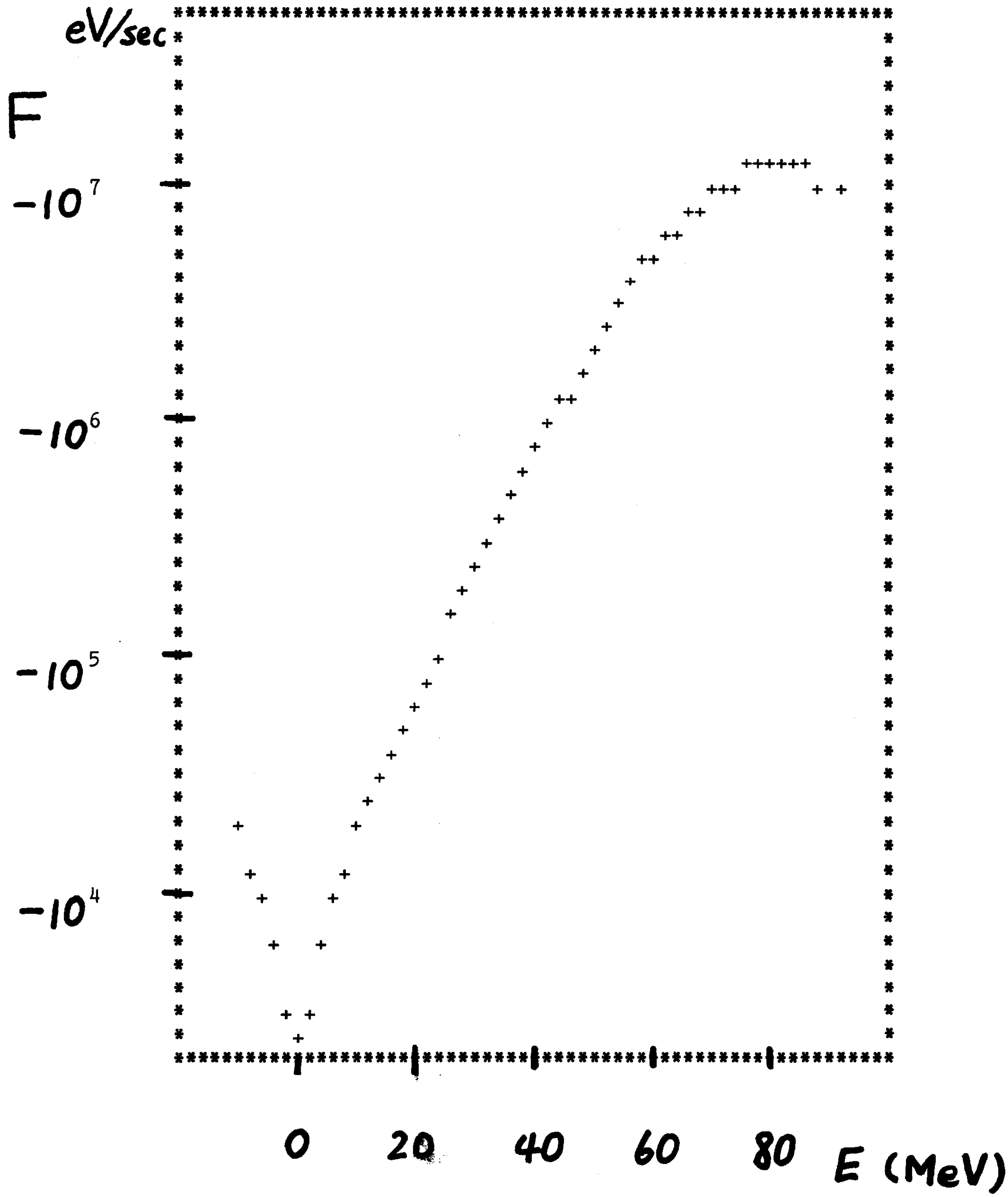


Fig.7-a

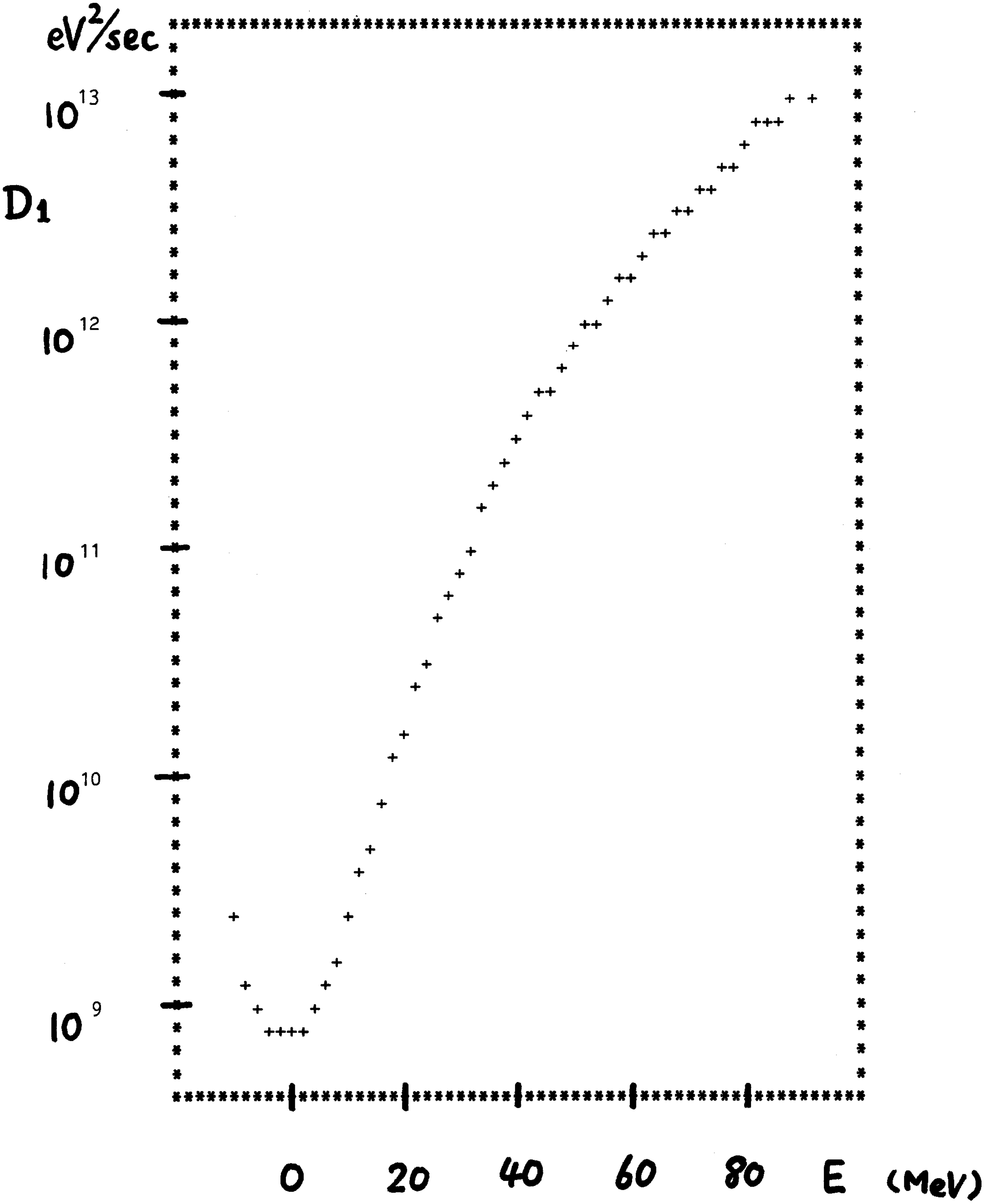


Fig.7-b

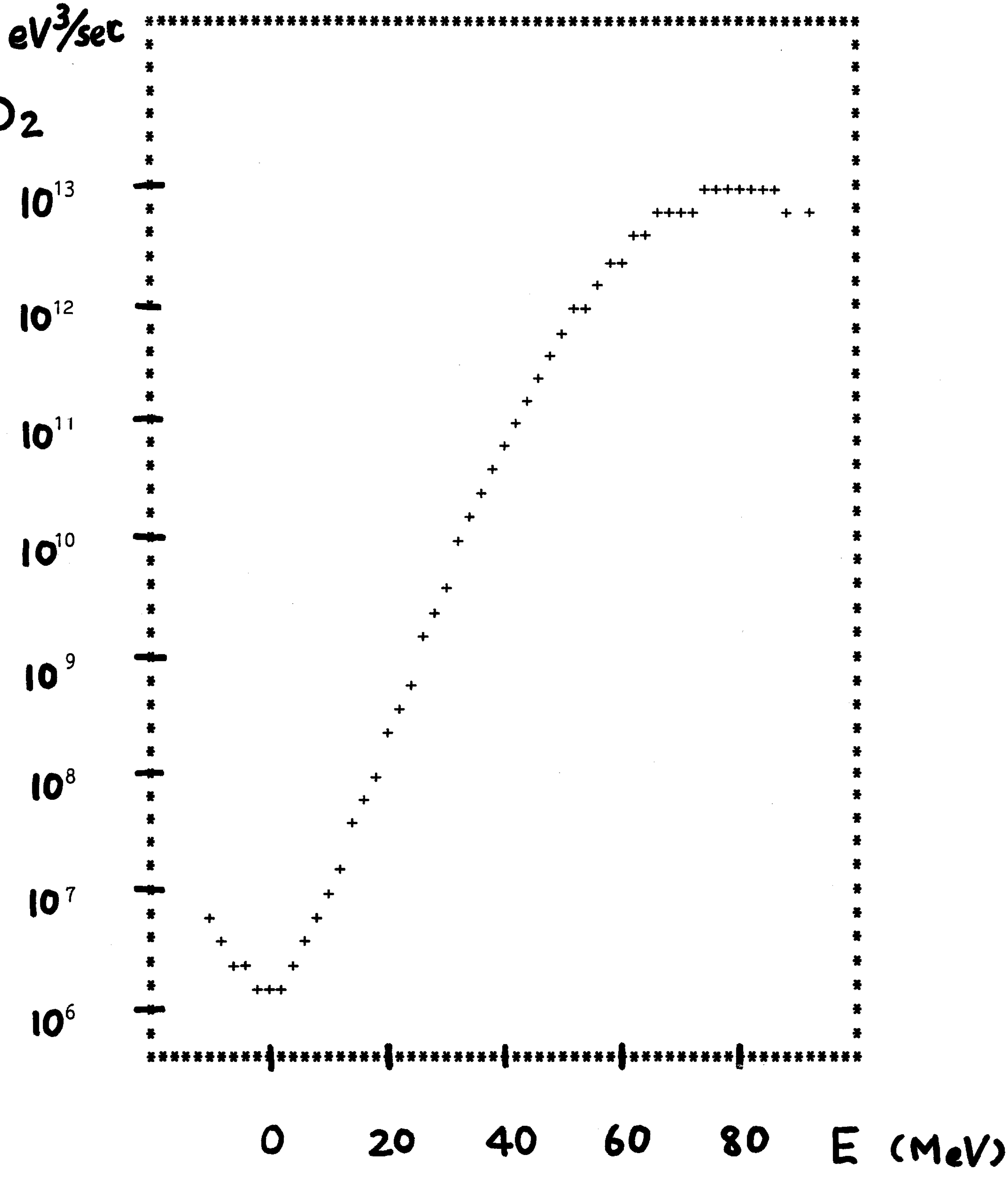


Fig.7-c

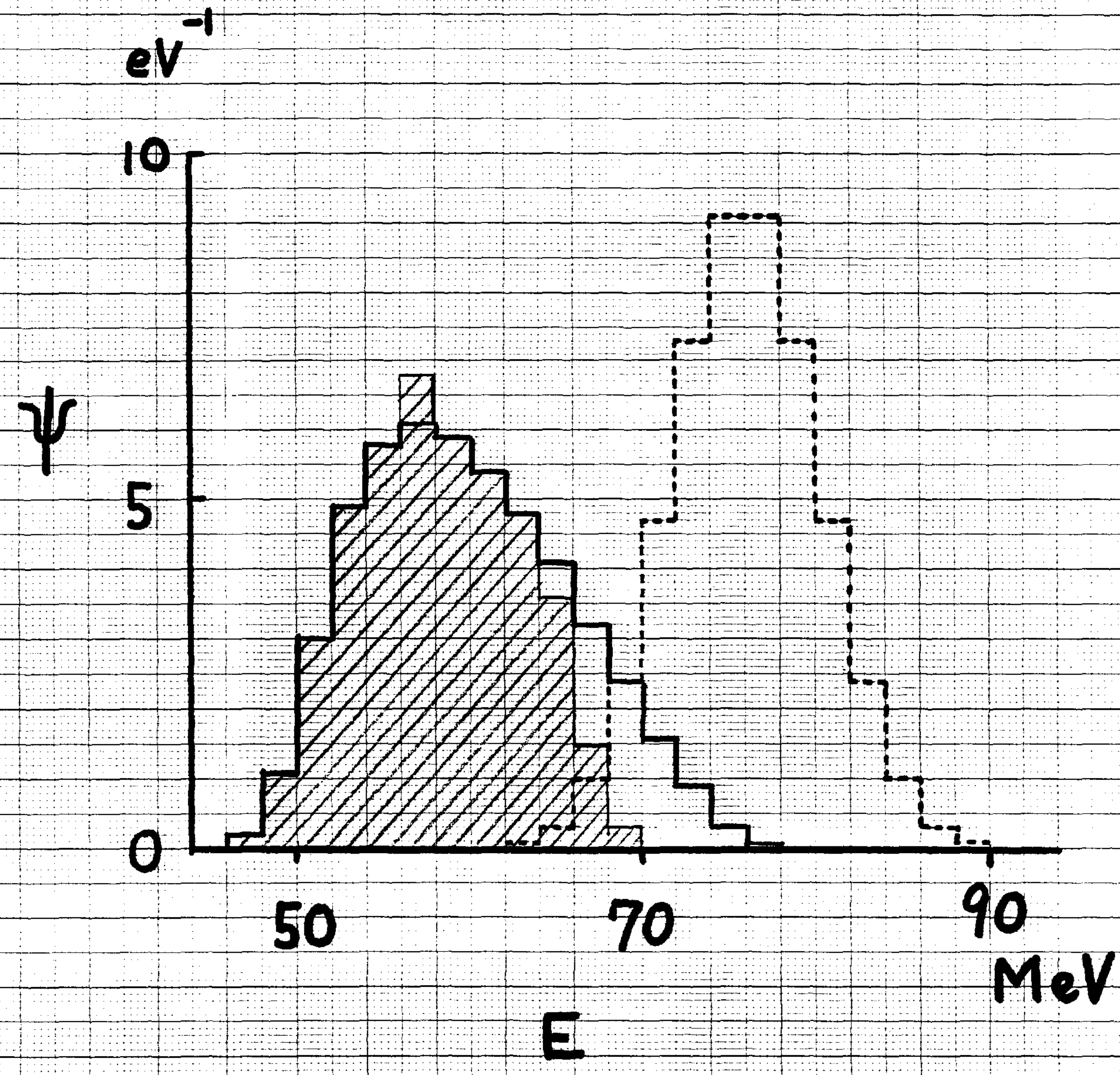


Fig.7-d

Table 1

Transfer Matrix

$$X' = M X$$

M : Infinite Matrix (each element / 10^3)

($M_{\pi\pi}$)	84–82	82–80	80–78	78–76	76–74	74–72	72–70	70–68	68–66	66–64	64–62	62–60	60–58	58–56	56–54
84–82						3	4								
82–80						2	5	1							
80–78						8	4								
78–76						22	18	3							
76–74						57	40	9							
74–72						40	81	31							
72–70						20	128	137							
70–68							183	680	63						
68–66								58	915	27					
66–64									23	970	7				
64–62										7	979	14			
62–60											15	976	9		
60–58												9	986	5	
58–56													4	994	2
56–54														3	995

Table 2

Cooling Parameters of Test Cases

$F = V/T$, T : revolution periood

$D = D_1 + D_2 \psi$

$V = 2\, E_b\, T\, W\, G\, S_p\, S_k$

$D_1 = E_k^2\, W\, G\, S_k^2$

$D_2 = A\, V^2$

$A = \beta^2 E_0\, \Lambda\, /(4\, T^3\, |\eta|\, W^2\,)$

$E_k^2 = f\, k\, \Theta\, e^2\, N_k\, R_k\, /2T$

$W = f_{max} - f_{min}$; bandwidth

$\Lambda = \ln(f_{max}/f_{min})$

$S_p = S_k = S_0 \exp(-E/E^*)$; sensitivity of pick-up and kicker

$E^* = E_d\, /2$ (14 MeV)

G : electric gain

E_0 : total energy

E : energy deviation

$E_b = e\, \sqrt{N_p\, N_k\, R_p\, R_k}\, /T$

β = velocity

f : noise figure

k : Boltzman constant

Θ : temperature of amplifiers

$N_p = N_k = 256$: number of pick-ups and kickers

R_p, R_k : impedence of pick-up and kicker

Table 3

Results of the system given in Table 2

$\|\eta\|$		0.04	0.01	0.02	0.02	0.01
W	(GHz)	1	1	1	2	2
$\hat{F}$	(MeV/sec)	-14.2	-13.9	-12.5	-12.5	-20.0
$\hat{D}_1$	($\times 10^{12}$ eV^2/sec)	10.5	10.0	8.2	4.1	10.4
$\hat{D}_2$	($\times 10^{12}$ eV^3/sec)	4.85	18.5	16.5	4.84	9.57
		ln2	ln2	ln6	ln6	ln2
initial value						
m_0	(MeV)	7	7	6	6	7
σ_0	(MeV)	3.7	3.7	3.7	3.7	3.7
final value						
$m-m_0$	(MeV)	-11	-11	-10	-9.5	-14
σ	(MeV)	4.5	6.3	6.0	4.5	4.6
$r(C,T_C)$		0.86	0.83	0.96	0.99	0.65
Estimation						
$m-m_0$	(MeV)	-10.5	-10.3	-8.2	-8.2	-13.7
σ	(MeV)	4.7	6.7	6.3	4.5	5.3
$r(C,T_C)$		0.78	0.71	0.83	0.90	0.53