

SUPERFISH AND THE EVALUATION OF AXISYMMETRIC  
CAVITIES WITH RF QUADRUPOLE FOCUSING

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March 2, 1982

I. Introduction

SUPERFISH is a computer program which calculates the electromagnetic fields and associated resonance frequencies in axially symmetric rf cavities of otherwise arbitrary shape. The program determines these quantities not only for the fundamental TE mode but also for higher modes. A detailed derivation and description of the algorithm has been published; therefore only a brief summary of the essentials are presented in Sections II and III. In Section IV the TE fields are calculated for a rf quadrupole structure housed inside a cylindrical copper cavity. The RFQ linac is introduced in Section V; and a structure with simple vanes is modeled with the help of SUPERFISH.

Over the past twenty years various computer codes have evaluated parameters for rf cavities, utilizing an overrelaxation method to solve a set of inhomogeneous

linear-field equations (see references 1-2). The convergence rate is small, however, when the diameter to length ratios are large, or when the calculation involves modes higher than the fundamental. In some instances the solution may not converge at all with standard methods for overrelaxation - factor optimization. SUPERFISH, on the other hand, uses a direct, noniterative method to solve a set of inhomogeneous field equations. This allows evaluation of resonance frequencies, fields, and secondary quantities in extreme geometries.

## II. DESCRIPTION OF SUPERFISH

A detailed description of the methods used in SUPERFISH appear in the literature; and therefore a brief summary will be presented. Assume we wish to calculate the electromagnetic quantities of an excited, cylindrical rf cavity. The problem interior is covered by an irregular triangular mesh, with boundaries defined by short, straight mesh lines. RF fields are described by the azimuthal magnetic field strength  $H$ ; and Maxwell's equations are represented by one difference equation for  $H$  at every mesh point on and inside the problem boundary. This homogeneous set of linear equations (linear in the  $H$  field) is transformed into a special orientation by setting  $H=1$  at one

(arbitrarily chosen) point. The difference equation at this particular point is eliminated from the system; and the remaining set of inhomogeneous, linear equations is solved with a non-iterative Gaussian block elimination and back substitution process. A "new" field value, at the H=1 point, is calculated from known fields of neighboring points. H (new) in general differs from 1; and this difference multiplied by an appropriate factor can be interpreted as the current I of magnetic charges necessary to drive the cavity (at the H=1 point and selected frequency) to the field value one. Since the coefficients of the original set of difference equations have frequency dependence, I must depend on  $k(=\omega/c)$  as well.

Resonances in the frequency spectrum are characterized by  $I=0$  ( $I=I(k^2)$ ). In order to locate resonances, the program uses the normalized quantity D and not I directly.

$$D = \frac{2\pi r_L k I}{\int H^2 dv}$$

$r_L$  = distance of removed point from mesh axis.

D simplifies the root finding procedure, because of the following pattern.

every resonance has:  $D=0$ ,  $\frac{dD}{dK^2} = -1$

between every two resonances:  $D=0$  (once)

$$\frac{dD}{dK^2} = +1$$

Typically, it takes 3-6 evaluations of  $D$  to find a resonance frequency. One solution of the difference equation is necessary for each calculated  $D$  (see the flow diagram in figure 1).

### III. THE POISSON GROUP

SUPERFISH is the main code in a series of programs known as the POISSON GROUP (see figure 2). AUTOMESH, LATTICE, and TEKPLOT are the remaining units. The first stage of analysis takes place in AUTOMESH, where the "logical" mesh is constructed, and straight lines, arcs of circles, and segments of hyperbolas are used to regenerate the physical dimensions of the problem. AUTOMESH produces a file (TAPE73) that is read by the next program in the series, LATTICE. This program fills the problem (boundary) with an irregular, triangular mesh, as shown in figure 3. By proper choice of input data, it is possible (in AUTOMESH) to specify the size of the triangle and to have several areas of different density within the problem.

Variations in mesh density directly affect the computational time and TE field distribution. For example, a problem which requires 2000 mesh points takes of order 100. seconds to determine each frequency. Doubling the

number of points lengthens the solution time to nearly seven minutes on the VAX/VMS computer. Output file (TAPE35) is rewritten by SUPERFISH once the resonance calculations are finished; and the output format produces separate graphs of the physical mesh and equipotential field lines. TEKPLOT and GRALIB drive the graphics package on the ADM interactive terminals\*.

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\*Several ADM terminals on the 7<sup>th</sup> floor of Wilson Hall are equipped with a RETRO-GRAPHICS microprocessor which emulates a 4010 terminal with graphics capability.

#### IV. ARBITRARY CROSS SECTIONS

Conventional applications of SUPERFISH use cylindrical coordinates with the Z axis along the beam direction and radius r orthogonal to Z. Recently the code has been modified to evaluate the electromagnetic properties of cavities with arbitrary cross section in the x-y plane. These modifications include making the Z coordinate infinite and constructing devices with different dielectric materials. Figure 4, shows the TE field distribution generated by SUPERFISH for the cross section of a rf quadrupole cavity. The four rods are powered by a combined dc and rf voltage ( $V_1 + V_0 \cos(\omega t)$ ) and arranged so that a

positively charged pair lies in the vertical plane and a negatively charged pair in the horizontal plane.

Employing basic electrodynamics, the potential energy expression is

$$\phi = \frac{e}{r_0^2} (V_1 + V_0 \cos(\omega t)) (x^2 + y^2)$$

where  $r_0$  is the radius from the cavity center to the top (bottom) of rod.

The electric force acting on the charged particle beam is

$$F_x = \frac{-\partial\phi}{\partial x} \quad , \text{ where } e < 0$$

$$= \frac{2ex}{r_0^2} (V_1 + V_0 \cos(\omega t))$$

$$F_y = \frac{-\partial\phi}{\partial y} \quad , \text{ where } e > 0$$

$$= \frac{-2ey}{r_0^2} (V_1 + V_0 \cos(\omega t))$$

$$F_z = 0$$

Applying Newton's second Law of Motion

$$\ddot{X} = \frac{2ex}{mr_0^2} (V_1 + V_0 \cos(\omega t))$$

$$\ddot{Y} = \frac{-2ey}{mr_0^2} (V_1 + V_0 \cos(\omega t))$$

$$\ddot{Z} = 0$$

, therefore the axial velocity of any charged particle is constant and is not affected by the voltages applied to the rods. In addition, charged particles traversing the central region of the cavity ( $r \leq r_0$ ) will experience focusing forces in the vertical plane and defocusing forces in the horizontal plane.

#### V. RFQ DEVELOPMENT

Since 1956, there have been suggestions that electric fields inside linear accelerators could be used for radial focusing as well as acceleration<sup>3-6</sup>. The various proposals were based on non-cylindrically symmetric electrodes which would generate transverse quadrupole fields in specially shaped gaps between drift tubes or waveguides. RF self-focusing proved to be important mechanism at low

velocities because electric fields are velocity independent.

Kapchinaskii and Teplyakov, in 1970 proposed a linac structure in which quadrupole focusing fields were spatially continuous (with harmonic modulation of the vanes) along the Z axis. Then in 1979 an accelerator known as the radio-frequency quadrupole linac (RFQ) was designed by a group at Los Alamos National Laboratory and operated during the Proof-of-Principal-Test<sup>7</sup>. See figure 5, for an example of the RFQ cross section with simple vanes. Simple vanes do not have the harmonic modulations required for beam acceleration.

The RFQ is an efficient low-velocity accelerator capable of producing bright exiting beams. During operation the device continually provides the following functions 1) a DC beam is accepted at 30 Kv and radially matched into advancing stages of the structure, 2) an adiabatically bunched beam has a high capture efficiency (>95%); and 3) the beam is accelerated to 1.0 MeV/meter (for protons). Through proper design it is possible to control the particle distribution in the phase-stable bucket so that nonadiabatic acceleration effects are minimized. Furthermore, the final synchronous phase can be adjusted (e.g.  $\alpha = -30^\circ$ ) so that the beam is suitable for capture by a drift tube linac.

The author has employed SUPERFISH to design the cross section of a hypothetical RFQ with simple vanes. One quadrant in each of four plots is shown in figure 6. Note the resonance frequency dependence on the size of the vanes; where an empty cavity is plotted for reference. Figure 7, shows the SUPERFISH output summary. There are several noteworthy points 1) The power dissipated, energy stored, etc. are normalized to an electric field of 1.0 MeV/m. All SUPERFISH numbers must be adjusted to the operational level of the resonant system. 2) There is only one quadrant of the cavity drawn because of axial symmetry. Therefore the calculated power dissipation must be multiplied by 2. 3) The cavity losses are calculated assuming copper in all cavity walls. 4) Experimenters state that good rf assembly practices only achieve about 85% of the Q (calculated). 5) SUPERFISH divides the cavity boundary into segments and calculates the power dissipation in each segment. This information is very useful when cooling water, calculating temperature gradients; and specifying the length and number of segments along the cavity boundary.

## VI. Conclusion

The computer code, SUPERFISH presently has world wide distribution and was designed to calculate the electromagnetic fields and associated resonance frequencies in axially symmetric cavities. The program has grown in capabilities to include setting a problem up in x-y plane, and using ferrite material or dielectrics in the cavity. The author has applied SUPERFISH to several problems associated with rf quadrupole focusing. First, a cross section with TE fields inside a copper cavity containing four rods, and the x-y plane of a 216 MHZ, one meter in length RFQ. Future studies at Fermilab will examine the possibilities of using the RFQ, as a low-velocity, bright beam, injector.

## VII. Acknowledgements

The author wishes to express special thanks to members of the Linear Accelerator groups at Fermi National Accelerator Laboratory and Los Alamos National Laboratory, especially Drs. Cryil D. Curtis and Jim M. Potter. R.F. Holsinger (Field Effects, Inc.) and D.F. Reid (Los Alamos) were quite helpful during initial operations of SUPERFISH on the CDF VAX/VMS computer.

## REFERENCES

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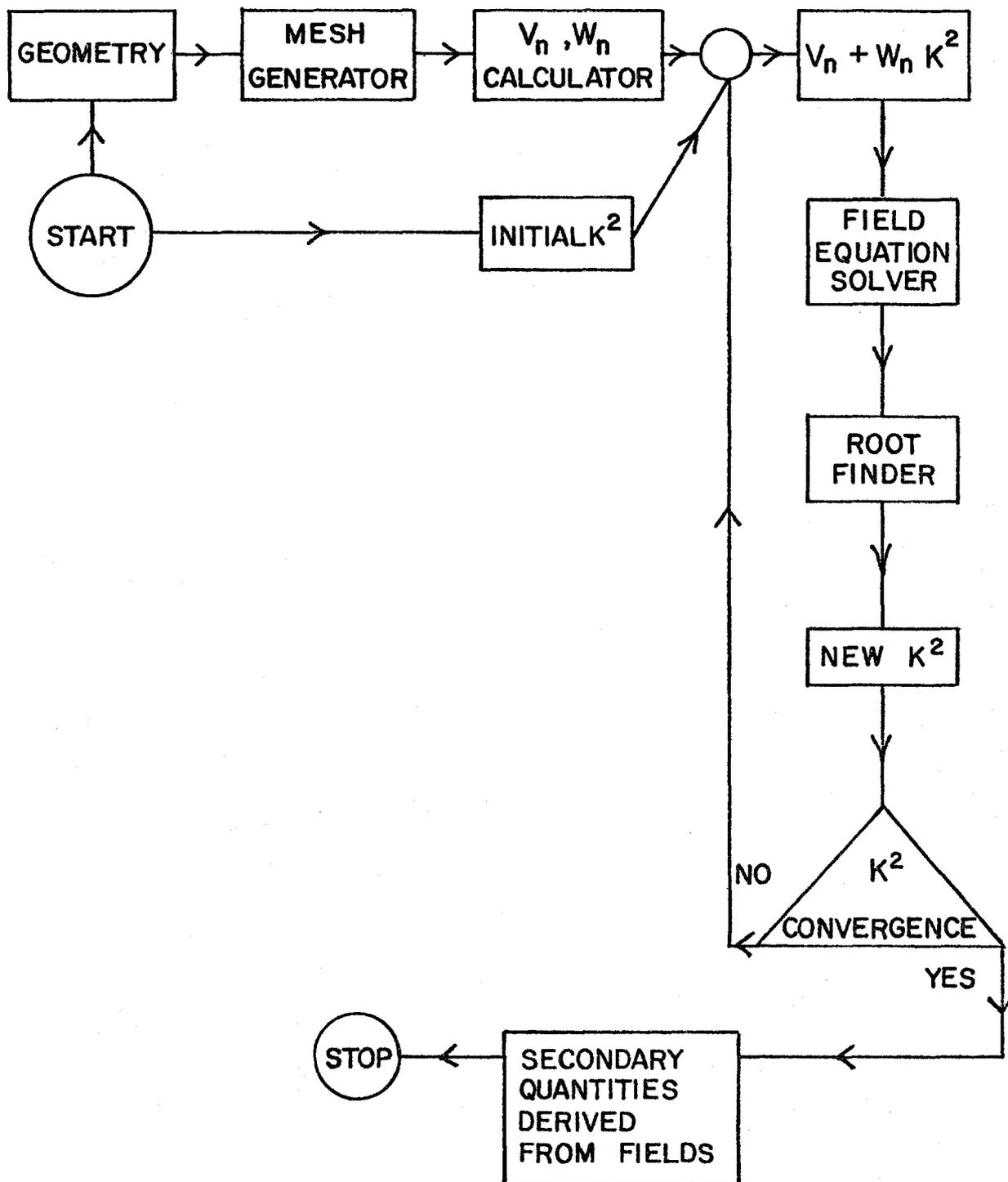


Fig 1: FLOW DIAGRAM OF SUPERFISH

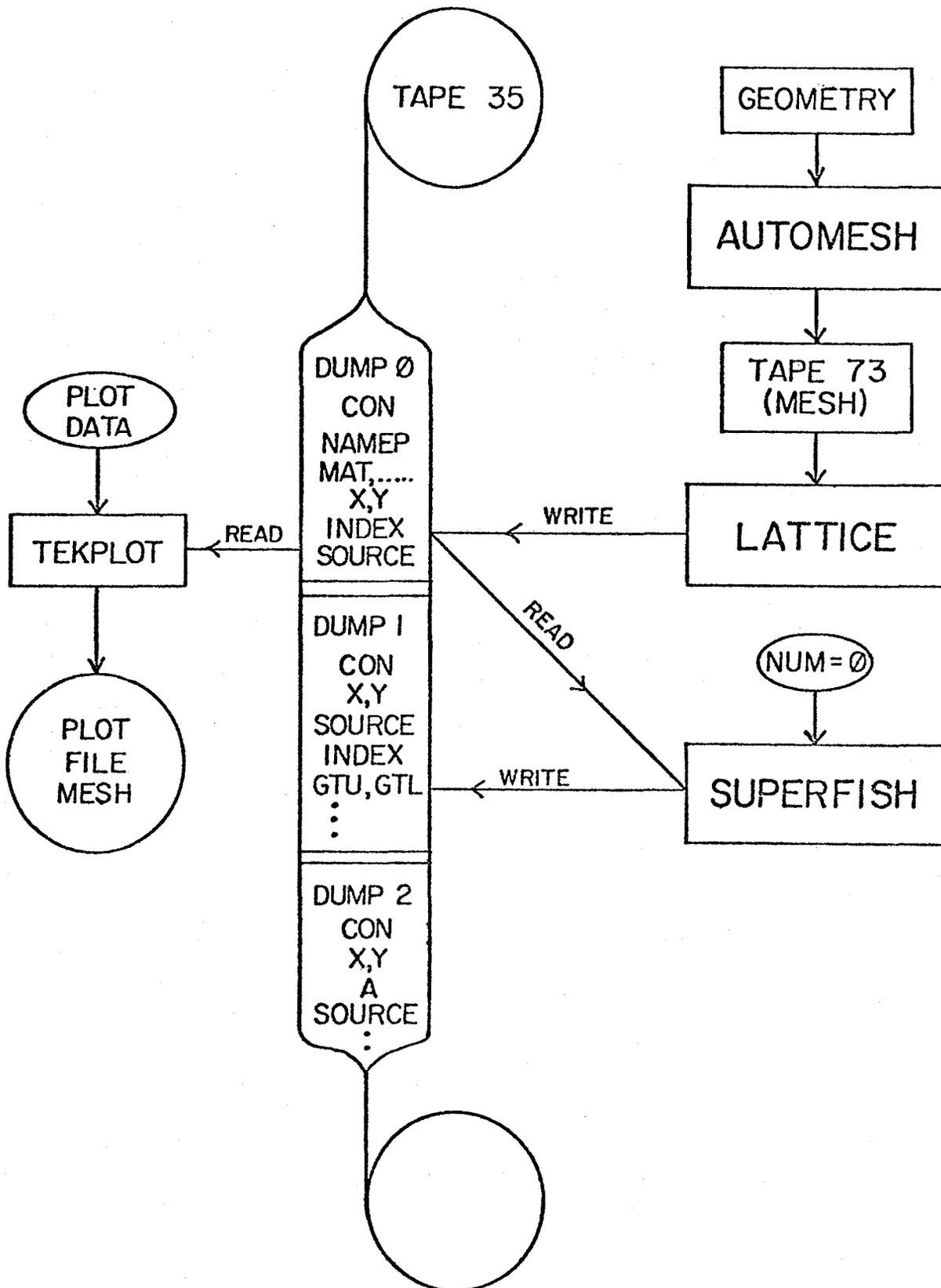


FIGURE 2. COMMUNICATION BETWEEN "POISSON GROUP" PROGRAMS USING "TAPE 35" (DISK OR TAPE FILE).

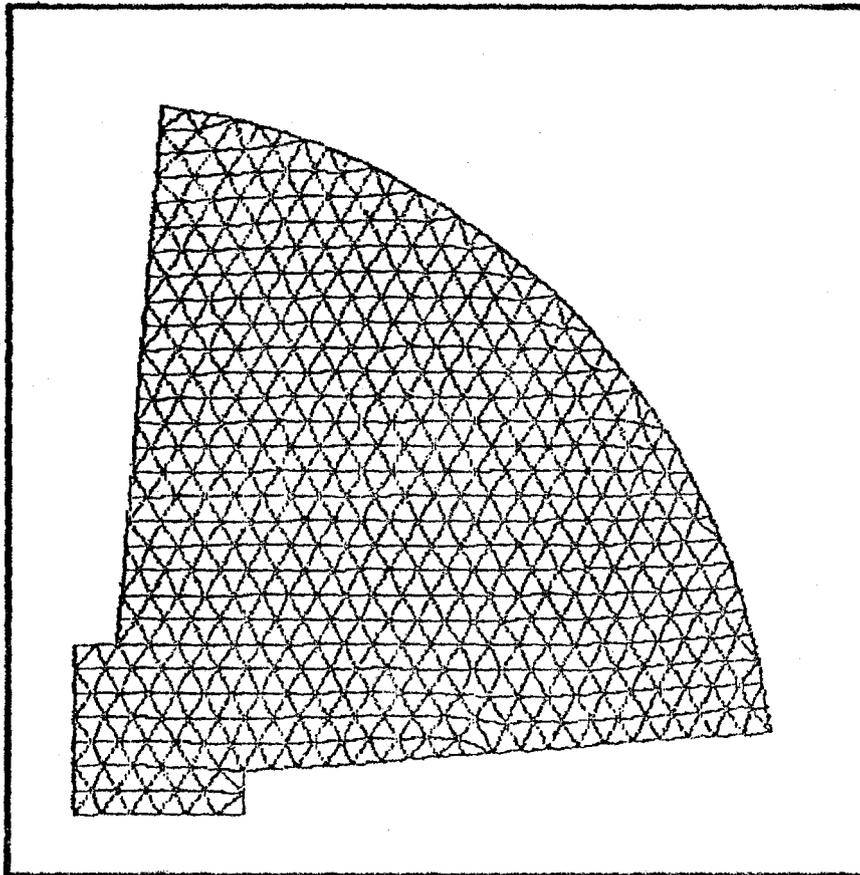


Fig.3: A MESH CONFIGURED FOR  
THE RFQ WITH SIMPLE VANES.

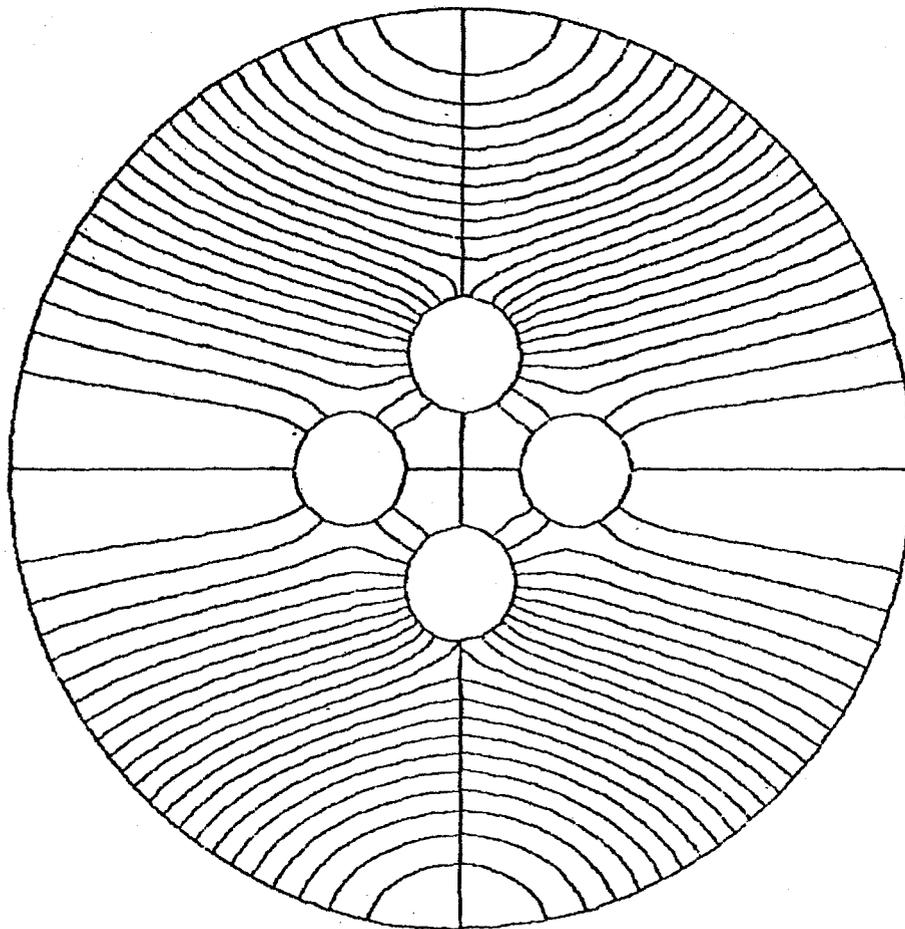


Fig.4: RF QUADRUPOLE FOCUSING  
INSIDE A CYLINDRICAL COPPER  
CAVITY.

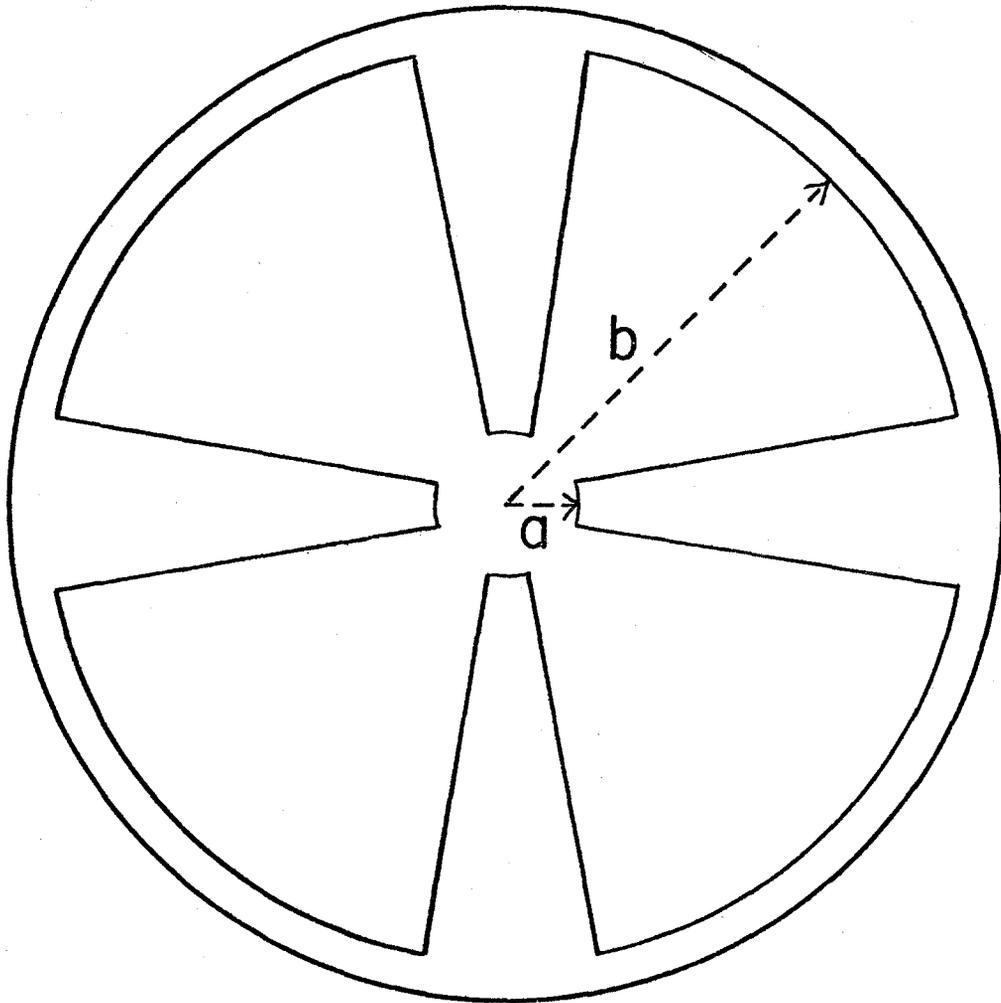
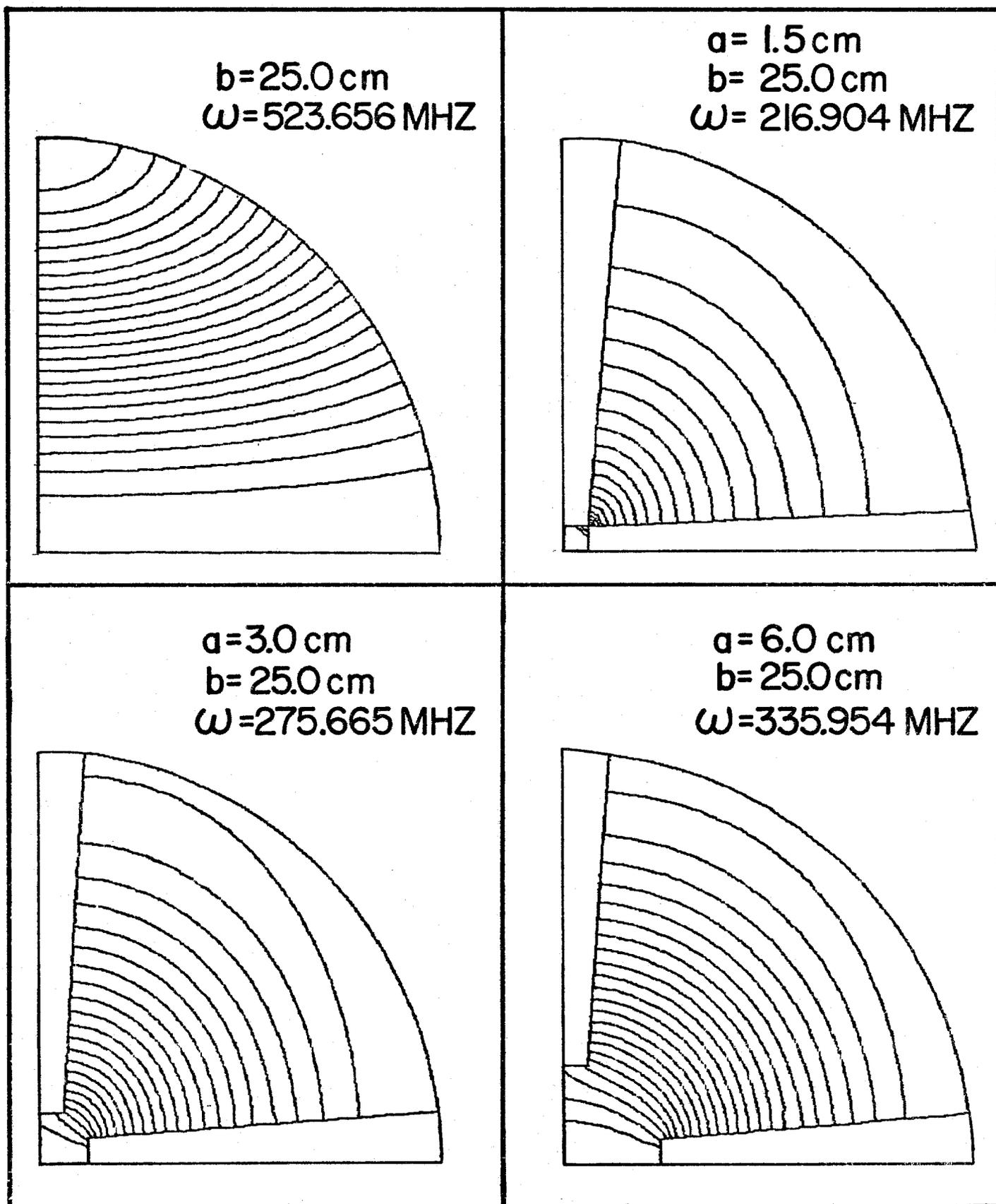


Fig.5: CROSS SECTION OF THE RFQ WITH SIMPLE VANES. (Not drawn to scale,  $a=3.0\text{cm}$ ,  $b=25.0\text{cm}$ ).

FIGURE 6: TE FIELD DISTRIBUTIONS INSIDE VARIOUS RFQ GEOMETRIES.



PROBLEM NAME = RFQ WITH SIMPLE VANES

CAVITY LENGTH = 3.000 CM CAVITY DIAMETER = 0.000 CM

D. T. GAP = 3.000 CM STEM RADIUS = 1.000 CM

FREQUENCY (STARTING VALUE = 201.000) = 216.904 MHZ

BETA = 0.0217 PROTON ENERGY = 0.221 MEV

NORMALIZATION FACTOR (EO=1 MV/M) ASCALE = 283.9

STORED ENERGY (MESH PROBLEM ONLY) = 0.0005 JOULES

POWER DISSIPATION (MESH PROBLEM ONLY) = 34.19 WATTS

T, TP, TPP, S, SP, SPP = 0.000 0.000 0.000 0.000 0.000 0.000

Q = 20299 SHUNT IMPEDANCE = 438.70 MOHM/M

PRODUCT Z\*T\*\*2 ZTT = 0.00 MOHM/M

MAGNETIC FIELD ON OUTER WALL = 75 AMP/M

MAXIMUM ELECTRIC FIELD ON BOUNDARY = 2.601 MV/M

ISEG	ZBEG (CM)	RBEG (CM)	ZEND (CM)	REND (CM)	EMAX (MV/M)	POWER (W)	D-FREQ (DELZ)	D-FREQ (DELR)
2	0.000	1.500	1.500	1.500	1.42364	0.0 WALL	0.0000	1.4176
3	1.500	1.500	3.000	24.819	2.19811	5.2 WALL	4.2699	0.2747
5	24.819	3.000	1.500	1.500	2.60082	29.0 WALL	0.3498	5.4409
6	1.500	1.500	1.500	0.000	2.28064	0.0 WALL	2.9146	0.0000

DUMP NUMBER 1 HAS BEEN WRITTEN.

\$

FIGURE 7: SUPERFISH OUTPUT SUMMARY FOR "RFQ WITH SIMPLE VANES".