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Fermilab

A Description of the Satellite Expander Pressure Trace and the Relationship Between Efficiency and Intake Cutoff

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Introduction

Figure 1 illustrates a typical satellite refrigerator expansion engine pressure trace, a record of cylinder pressure through one cycle. Such a trace can occasionally be seen on CRT's in the Main Control Room or in refrigerator buildings since it is useful for monitoring engine performance. The engine cools the inlet helium gas by expanding it to a lower pressure, thus allowing the gas to do work. This expansion, which occurs from point 3 to point 4 in figure 1, is intentionally ended at a pressure at point 4 somewhat above the exhaust line pressure as shown, so it is followed by "blow down" of the expanded gas from its pressure at point 4 to the exhaust line pressure at point 5 when the exhaust valve opens. One of the characteristics of a "good" trace is a sharp little step from points 4 to 5 since it indicates that the exhaust valve and rings are probably holding cylinder pressure and that there is good throughput.

Assuming that the valves and rings are sealing tightly, the size of the pressure drop from point 4 to point 5 depends on the "intake cutoff," the ratio of the cylinder volume when the intake valve closes to the maximum cylinder volume. The pressure after expansion to point 4 also depends on deviations of the cylinder fluid from ideal gas behavior, significant for helium below 10 ^{o}K at our pressures. We can control the intake cutoff of our engines by varying the collet or tappet clearances or by changing intake cams. (See my TM-1036 for a detailed description of the relationship between intake cutoff and cam action.)

The purpose of this paper is to describe an analysis which quantifies the theoretical effect of incomplete expansion on engine efficiency. An expression for engine performance as a function of cutoff is derived.

This analysis corrects a basic assumption in TM-1036, "A Tabulation of a Few Theoretical Calculations for Satellite Refrigerator Expanders," which was used in predicting the enthalphy of the output from the engine. In that paper enthalphy after expansion was found from $h = h(\rho,s)$, where s = entropy is assumed constant and ρ = density is found from $\rho_{expanded} = \rho_{inlet} \times$ (intake cutoff). This is the enthalpy at point 4 in the expansion process in figure 1. As was stated in TM-1036, the final pressure drop (from points 4 to 5 when the exhaust valve opens) was assumed to be isenthalpic, changing gas temperature like a Joule-Thomson valve would. Thus, the enthalpy at point 4 was taken to be the enthalpy of the exhaust gas.

That assumption about the final pressure release is false, and the results of this analysis are that the loss of efficiency due to incomplete expansion is significantly less than the loss predicted by the method in TM-1036. The final blowdown is better modeled as a "tank-discharge process" rather than isenthalpic expansion.

<u>The Tank-Discharge Process.</u>[1]

Suppose a constant-volume tank initially contains m_i mass of fluid at pressure, P_i, greater than the atmosphere or line outside

1. Mooney, David A., <u>Mechanical Engineering Thermodynamics</u>, (Prentice-Hall, Inc., New York, 1953), pp. 83-88

the tank, and at temperature, T_i. Then a valve on the tank is opened, discharging the gas into the air or the line at pressure P_ℓ. Call the enthalpy of the discharged gas h_ℓ. An energy balance performed on the tank results in the following. The change in energy in the tank is $m_i u_i - m_f u_f$, where u is internal energy and subscript f denotes conditions in the tank after discharge. Neglecting changes in kinetic and potential energy, the energy which flowed out is $(m_i - m_f)\ h_\ell$. Thus, $(m_i - m_f)\ h_\ell = m_i u_i - m_f u_f$.

If the mass left in the tank after discharge, m_f, is negligible compared to the initial mass, m_i, we have $h_\ell = u_i$; the internal energy of the stored gas becomes the enthalpy of the flowing gas.[2] Conversely, in filling a tank the enthalpy of the flowing gas becomes the internal energy of the gas in the tank. This result arises from the fact that the energy of gas flowing across the boundary of our system (the tank) includes the "flow work" term, pv, and is thus enthalpy, while the energy of the gas in the tank is just its internal energy.

For example, suppose helium in a bottle at 300 °K and 20 atm. (u = 950 J/g) discharges into an insulated 1 atmosphere line. The enthalpy of the gas in the 1 atm. line would then be 950 J/g, so the temperature of the gas in the line is approximately 180 °K. The helium cools significantly.

<u>Our Expansion Process</u> (See figure 1)

Our process may be considered to consist of the following components:

1-2. The filling of the dead space with high pressure gas, a "tank-filling" process (which warms the gas) combined with mixing with the gas initially in the dead space at point 1.

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2. Morain, W.A., and Holmes, J.W. (1963). Proceedings of the 1962 Cryogenic Engineering Conference, Vol. 8, Paper D6, p. 229.

2-3. The filling of the cylinder at constant pressure as the piston moves up with the inlet valve open. (We typically see noise in the trace from 2 to 3 which often looks like damped oscillations, probably due to the sudden surge of pressure into the system.)

3-4. Isentropic expansion of a constant mass.

4-5. Release of the partially expanded gas to the exhaust pressure, a "tank-discharge" process.

5-6. The emptying of the cylinder at constant pressure as the piston moves down with the exhaust valve open.

6-1. Isentropic compression of a constant mass into the dead space.

The above model of our expansion process already impicitly contains assumptions that the pressure drops across the intake and exhaust valves are negligible, the valves and seals do not leak, and heat transfers through the head and between the cylinder walls and gas is negligible (an adibatic process).

The effects of blowby (leaking piston rings), a leaking exhaust valve, and non-reversible expansion are addressed for our Satellite expanders in TM-1036 and more generally in analyses of reciprocating cryogenic expanders[3,4,5] and steam engines[6], which are thermodynamically like our expanders.

In this analysis I will focus on the effect of the late intake cutoff, hence the assumption will also be made that dead space is negligible, i.e., processes 1 to 2 and 6 to 1 will be neglected.

<u>The Resulting Equations</u>.

I would like to find an expression for the overall change in enthalpy of the gas, $\Delta h = h_i - h_e$ where h_i is specific enthalpy of the

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3. W.A. Morain and J.W. Holmes, <u>op.cit</u>.
4. G.G. Haselden, <u>Cryogenic Fundamentals</u>, (Academic Press, New York, 1971), pp. 453-467.
5. Barron, Randall, <u>Cryogenic Systems</u>, (McGraw-Hill, 1966), pp. 161-171.
6. Mooney <u>loc.cit</u>.

inlet gas and h_e is exhaust enthalpy.

For the filling of the cylinder, points 2 to 3, $h_2 = h_3 = h_i$. The total energy which entered the cylinder is mh_i, where m is mass of the fluid taken in per cycle.

Gas leaves the cylinder during two processes, 4-5 and 5-6. The energy of the gas leaving the cylinder in process 4-5 is $(m_4-m_5)\ h_5 = m_4u_4 - m_5u_5$ from the analysis of a "tank-discharge" process. Since blowdown is to exhaust conditions, $h_5 = h_e$ and $u_5 = u_e$. Also m_4, the mass of the fluid in the cylinder at point 4, is still m, the total mass taken in. Substituting gives for the tank discharge process 4-5: $(m-m_5)\ h_e = mu_4 - m_5u_e$. Rearranging gives $h_e = u_4 + (m_5/m)(h_e-u_e)$. From the definition of h, $h_e-u_e = P_ev_e$; also $m_5v_e = \Delta V$ = total volume swept out by the piston in the exhaust stroke, process 5-6. Substitution give $h_e = u_4 + P_e\ \Delta V/m$. But the mass, m, taken in per stroke is related to ΔV by $\Delta V/m = v_4$. Thus, $h_e = u_4 + P_ev_4$. Substitution for h_e yields

$$\Delta h = h_i-u_4-P_ev_4. \tag{1}$$

This expression for Δh can be evaluated if the inlet conditions, exhaust pressure, and the cutoff are known. $v_4 = v_i$/cutoff, and u_4 is found from the assumption of isentropic expansion to v_4. Note that if $P_e = P_4$ we have $\Delta h = h_i - h_4$, but if $P_e < P_4$ we have $\Delta h > h_i-h_4$.

Examples

1. Complete isentropic expansion. In this case, since expansion is to exhaust pressure, $P_4 = P_e$ so $\Delta h = h_i-h_4$. h_4 is found from assuming isentropic expansion to P_4.

For inlet helium at 30 ^{o}K, 20 atm., h_i = 166.9 J/g, u_i = 101.9 J/g, s_i = 13.03 J/g ^{o}K. Assume the exhaust pressure to be 1.2 atm. h_4 = h (P = 1.2 atm., s = 13.03 J/g ^{o}K) = 62.5 J/g. Thus, Δh ideal (complete isentropic expansion) = 104.4 J/g.

2. Intake cutoff = 0.38. (A typical dry engine.) Using equations (1) with the inlet conditions as given in example (1) and the exhaust line pressure, P_e, equal to 1.2 atm. I find Δh = 166.9 J/g - 68.6 J/g = 98.3 J/g. Compared to complete isentropic expansion to the 1.2 atm. exhaust pressure the best possible efficiency for this engine is $\frac{98.3\ J/g}{104.4\ J/g} = 0.94$.

3. Intake cutoff = 0.7. (A typical wet engine.) For inlet helium at 6 ^{o}K and 20 atm. isentropic expansion is to about 1.8 atm., so expansion is complete: The engine efficiency is not limited by the cutoff. But for warmer inlet conditions, such as during cooldown, this cutoff is extremely large.

So consider inlet helium at 30 ^{o}K and 20 atm. as in the previous examples. Use P_e = 1.8 atm. in equation (1). Then Δh = 166.9 J/g - 88.4 J/g = 78.5 J/g. The pressure before the exhaust valve opens, P_4, is about 10 atm. in this case.

For isentropic expansion to 1.8 atm. we would have Δh = 166.9 J/g - 71.1 J/g = 95.8 J/g. Therefore the best possible efficiency of our wet engines with "warm" helium is $\frac{78.5\ J/g}{95.8\ J/g} = 0.79$.

4. Intake cutoff = 1. (Effectively a 180^{o} cam.) This is a "square wave" engine; point 4 in figure 1 would be at 20 atm. Therefore $u_4 = u_i$ and equation (1) reduces to $\Delta h = v_i\ (P_i - P_e)$. For inlet conditions of 30 ^{o}K and 20 atm. and exhaust line pressure, P_e, equal to 1.2 atm. we have Δh = 61.1 J/g. Compared to esentropic expansion to 1.2 atm. the efficiency is $\frac{61.1\ J/g}{104.4\ J/g} = 0.59$.

In this case assuming $h_e = h_4$ would result in $\Delta h = 0$: one would predict that the engine would do no work. Since the entire intake stroke is at high pressure and the exhaust stroke at low pressure such a result would not make sense. This was a motivation for the above analysis.

See figure 2 for a plot of theoretical efficiency versus cutoff for the inlet conditions of the above examples. The upper curve in figure 2 is the best possible efficiency; the lower curve is the same curve translated down so that it passes through efficiency = 0.70 for cutoff = 0.38, a good dry engine.

Figure 3 and 4, taken from TM-1036, are tables of cutoffs for various collet and tappet clearances for the intake cams and timing of our dry engines. Thus, figures 3 and 4 with figure 2 provide a theoretical estimate of the effect of collet or tappet clearance on engine efficiency.

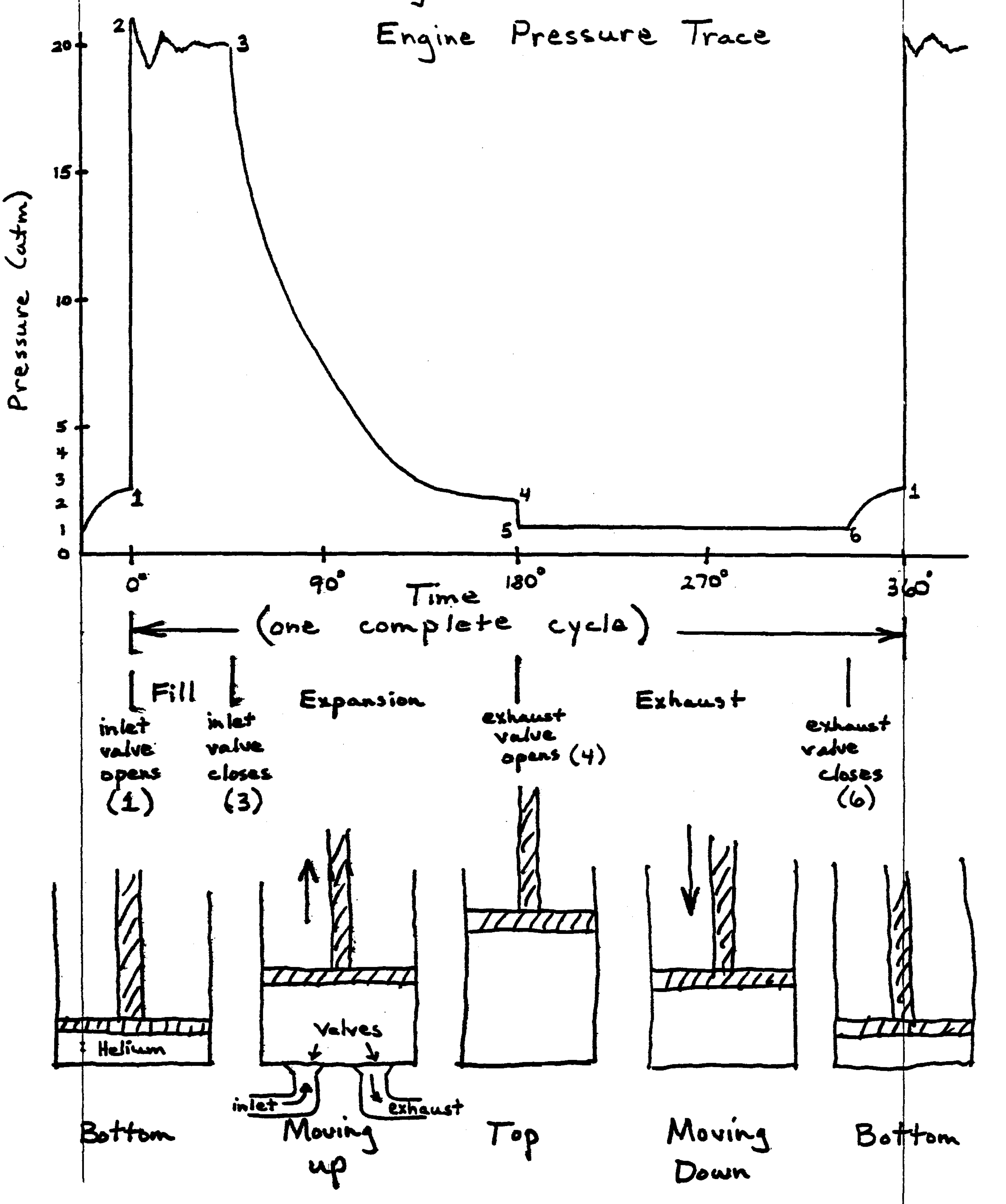
Figure 5 is the table of theoretical calculations from TM-1036 with column 6 revised to be Δh as calculated from equation (1) and column 9 revised accordingly.

Conclusions

1. Our 0.38 intake cutoff on the dry engines costs us only about 6% in efficiency while increasing the throughput per stroke by about 75% above that for the same engine with a cutoff which theoretically would yield complete isentropic expansion.

2. The best possible theoretical efficiency for our wet engines running with warm gas is 0.79. If other losses are the same as in a good dry engine (efficiency = 0.70), the actual wet engine efficiency with warm gas should be about 0.55.

Figure 1



Engine Pressure Trace
Pressure (atm)
20
15
10
5
4
3
2
1
0
2
3
1
4
5
6
1
0°
90°
180°
270°
360°
Time
(one complete cycle)
Fill
Expansion
Exhaust
inlet valve opens (1)
inlet valve closes (3)
exhaust valve opens (4)
exhaust valve closes (6)
Helium
Valves
inlet
exhaust
Bottom
Moving up
Top
Moving Down
Bottom

Theoretical Efficiency vs. Intake Cutoff Based on Equation (1)
for a Satellite Expander with Inlet Helium
at 20 Atmospheres and > 20°K

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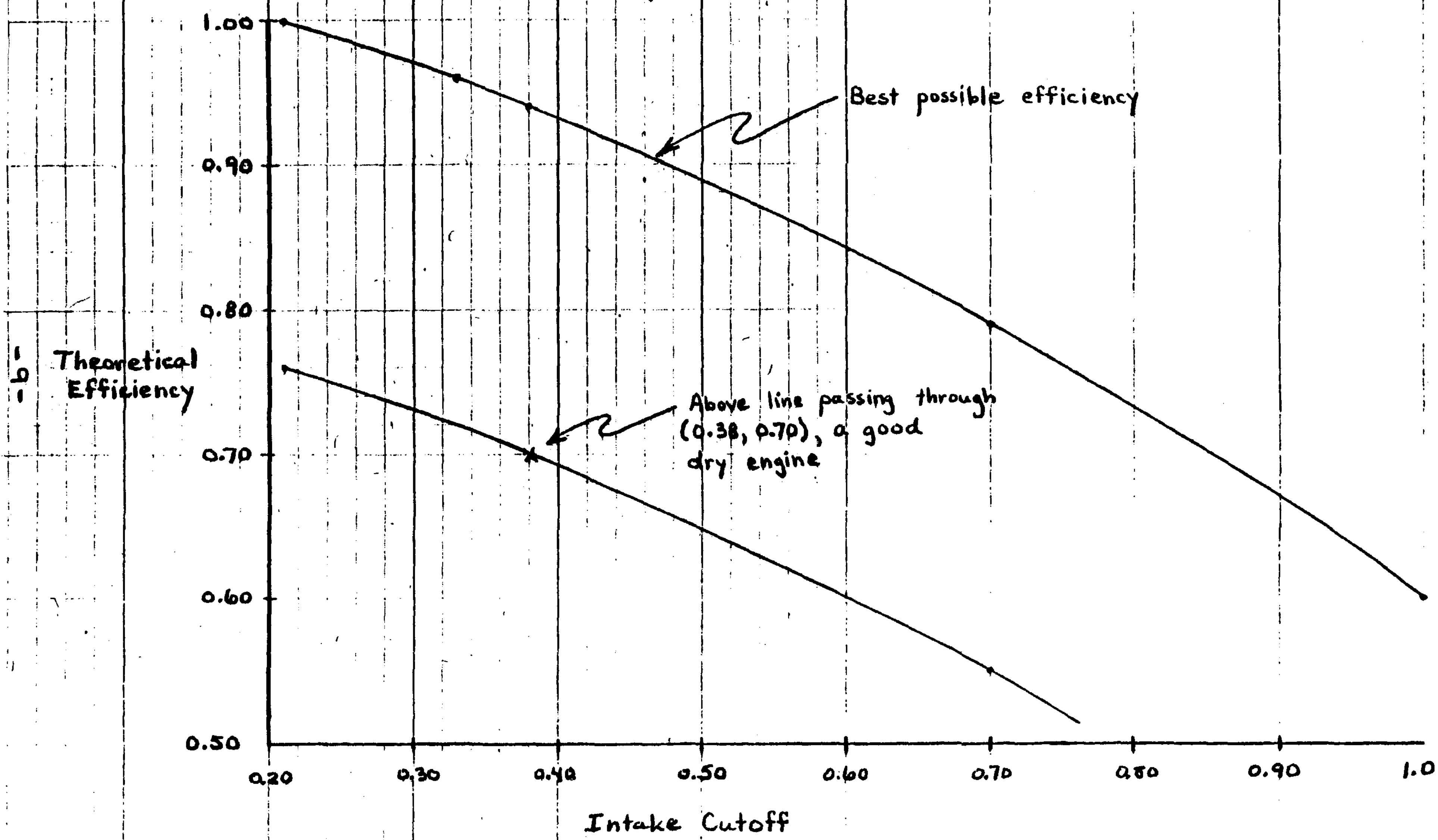

Figure 2

FIGURE 3

Effect of Tappet Clearance on Intake Cutoff for Gardner-Fermi Dry Engine (100° cam)

Tappet Clearance (inches)	Cam Advancement (degrees)	Intake cutoff
0.005	8	0.46
0.010	8	0.44
0.020	8	0.41
0.030	8	0.38
0.040	8	0.37

FIGURE 4

Effect of Collet Clearance on
Intake Cutoff for CTI
Dry Engine

Collet Clearance (inches)	Cam Advancement (degrees)	Intake Cutoff
0.005	10	0.43
0.010	10	0.40
0.020	10	0.38
0.035	10	0.36
0.008	4	0.46
0.035	14	0.33

FIGURE 5 TABLE OF CALCULATIONS

1	2	3	4	5	6*	7	8	9	10	11
Type of Engine	Intake Cam Size	Intake Cutoff	Inlet Temp Assumed (°K)	Cylinder Pressure after Ideal Expansion (atm)	Δh Ideal for Given Cutoff (J/g)	Flow Rate (g/sec)	Engine Speed (RPM)	Ideal Power Out (using column 6) (HP)	Reduction of Δh (and % effect) of 32 W Heat Leak (J/g)	Reduction of Δh due to ½ g/sec Blowby (J/g)
Gardner-Fermi Dry 3.187" Dia piston x 3.00" stroke 1 piston	100° 15° "lost"	0.46	30	5	93	20	213	2.5	1.6 (2%)	1.7 (2%)
						30	320	3.7	1.1 (1%)	1.1 (1%)
	85° 15° "lost"	0.33	30	2.8	100	20	297	2.7	1.6 (2%)	2.1 (2%)
						30	446	4.0	1.1 (1%)	1.4 (1%)
	70° 15° "lost"	0.21	30	1.2	105	20	467	2.8	1.6 (2%)	2.6 (2%)
						30	(701)	4.2	1.1 (1%)	1.8 (2%)
CTI Dry 3.00" Dia piston x 2.00" Stroke 2 pistons in parallel	90° 8 mil collet clearance	0.46	30	5	93	20	181	2.5	1.6 (2%)	1.7 (2%)
						30	271	3.7	1.1 (1%)	1.1 (1%)
	90° 20 mil collet clearance	0.38	30	3.5	98	20	219	2.6	1.6 (2%)	2.0 (2%)
						30	(329)	3.9	1.1 (1%)	1.3 (1%)
	90° 35 mil collet clearance	0.33	30	2.8	100	20	252	2.7	1.6 (2%)	2.1 (2%)
						30	(378)	4.0	1.1 (1%)	1.4 (1%)
CTI Wet 2.00" Dia piston x 2.00" Stroke 2 pistons in parallel	120° 10 mil collet clearance	0.70	8	2.5	15.8	40	125	0.85	0.8 (5%)	0.2 (1%)
						60	188	1.27	0.5 (3%)	0.1 (1%)
		0.70	6	1.0 (approx)	13	40	110	0.70	0.8 (6%)	0.2 (2%)
						60	165	1.05	0.5 (4%)	0.1 (1%)
		0.70	5.6	< 1	~ 13	40	108	0.7	0.8 (6%)	0.2 (2%)
						60	162	1.0	0.5 (4%)	0.1 (1%)

* Column 6 is calculated using $\Delta h = h_i - u_4 - p_e v_4$, where $v_4 = v_i$ /cutoff and u_4 is found from assuming isentropic expansion to v_4. $p_e = 1.2$ atm for the Dry engines $p_e = 1.8$ atm for the Wet engine. I[illegible]t pressure is 20 atm.