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# DETECTOR OF MICROWAVE RADIATION FROM COOLER ELECTRON BEAM

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## Introduction

The cooling ring electron beam (~ 100 KeV) is magnetically confined (B ~ 1 kG). Any transverse temperature of such a beam may therefore be measured by the amplitude of the electrons' gyro motion. In our case the gyro frequency will be ~ 3 GHz which will produce a conviently observable microwave signal from 1) incoherent transverse energy imparted by the $1000^{o}$ C cathode and 2) coherent energy imparted by guide field imperfections.

I discuss the theory of such microwave radiation within a metallic beam guide (as compared to free space Thompson radiation). Next a specific pickup configuration is described. Finally, the limiting sensitivity of such a device is calculated.

## Oscillating Electron in a Guide

Consider the case of an electron spiralling down a cylindrical tube (all specific calculations will be based on our cooling device: R = 7.5 cm, $\beta = 0.57$, $\gamma = 1.21$, $\omega_c/2\pi = 2.8$ GHz). For the cases of interest the gyro radius is minute (~ $5 \times 10^{-3}$ cm).

If the electron were in free space, then a continuous spectrum of radiation would be observed, radiated with dispersion:

$$\omega = \gamma\omega_c \left(1 + \frac{\vec{K}\cdot\vec{V}}{\omega_c}\right)$$

$$\omega_{min/max} = \gamma\omega_c \,(1 \pm \beta) \tag{1}$$

Within a guide, the radiation must additionally satisfy the guide dispersion relations.

$$\omega^2 = c^2K_z^2 + \omega_n^2$$

$$\omega_n = n^{th} \text{ mode cutoff} \tag{2}$$

Since we assume $\vec{v}_e = \beta c\, \hat{e}_z$, (1) and (2) together give a <u>discrete</u> spectrum:

$$\omega = \gamma\omega_c \left(1 \pm \beta\sqrt{1 - \frac{\omega_n^2}{\omega_c^2}}\right) \tag{3}$$

This is depicted in Fig. 1 as the graphical intersection of the Lorentz transform line (1) and the mode dispersion curve (2). It is clear how, as R grows $\frac{\omega_c}{\omega_n} \to \infty$ and the density of accessible modes grows. The descrete spectrum of all modes taken at once "fills in" the continuous free spectrum.

If a guide is chosen such that only a few modes have cutoffs below $\omega_c$, then we expect most of the radiated energy to be coupled to the guide field in narrow bands. By contrast, the doppler shifted radiation of a free space electron spans ~ 4 GHz of which only ~ 5% can be accepted by a typical 100 MHz band with low noise amplifier.

Since we will not have an infinitely long guide, the bands will have finite width. It is to be expected that linewidths will be determined by $1/\tau_e$ and $1/\tau_\omega$.

$\tau_e$ = electron transit time (through guide section L)

$\tau_\omega$ = wave transit time = $L/\nu_{group}$.

To see this in detail consider the current equivalent of one electron's motion:

$$J(\vec{R},t) = e\vec{v}\;\delta(z - \beta ct)\quad \delta(x)\;\delta(y)$$

or

$$J_\perp(z,t) \approx e\vec{v}_\perp\;\delta(z - \beta ct)\;\delta(x)\;\delta(y)$$

The assumptions being: 1) axial motion, 2) neglect of changes in radial position, 3) and neglect axial current.

This moving source may be regarded as an axial line (length L) of transverse current sources, each with a continuous spectrum:

$$J_\perp(p,\omega) = \frac{e}{\beta c}|v_\perp|\left(\hat{e}_r\cos(\gamma\omega_c p) + \hat{e}_\theta\;\sin(\gamma\omega_c p)\right)\;\exp(i\;\omega p)\;\delta(x)\;\delta(y) \tag{5}$$

where $p = z/\beta C$

From this Fourier transform point of view we treat each axial point at each frequency as a <u>fixed</u> source of infinite time duration. The excitation of a guide mode by such a source is elementary:

$$\frac{\omega}{8\pi K_n(\omega)}\underset{\substack{\text{guide}\\\text{cross}\\\text{section}}}{\iint}\vec{E}_n\times(\vec{E}_n^{*}\times\hat{e}_z)\cdot dA = \iiint\vec{E}_n\cdot\vec{J}_\perp(\omega)\,d^3x \tag{6}$$

where n referes to the $n^{th}$ mode, and $K_n(\omega)$ is the mode dispersion relation.

Restricting ourselves to an infinitesmal slice $dz$ as the source and restricting excitations to one polarization $\hat{e}_r$ (only for writing convenience), Eq. 6 becomes:

$$\frac{\omega\, E_{on}^2}{8\pi\, K}\, f_n = E_{on}\, \frac{e}{\beta c}\, |v_\perp|\, \cos(\gamma\omega_c p)\, \exp(i\omega p)\, dz \qquad (7)$$

where $f_n$ is a purely geometrical factor characteristic of the $n^{th}$ mode shape and $E_{on}$ is the peak value of the excited electric field.

The net field at $z = L$ (end of guide) will be the sum of the excitations (7) over the entire guide length weighted with the propagation phase factor for the mode n:

$$E_{on}(\omega,\, z = L) = \frac{e}{\beta c}\, |v_\perp|\, \frac{8\pi K}{\omega f_n} \int_0^L \cos\left(\gamma\omega_c \frac{z}{\beta c}\right) e^{i\omega \frac{z}{\beta c}}\, e^{-iK_n(L-z)}\, dz \qquad (8)$$

$$= \frac{e}{\beta c}\, v_\perp\, \frac{8\pi KL}{\omega f_n}\, I(\omega)$$

with this $P(\omega)$, the spectral power available at $z = L$, may be calculated:

$$P(\omega,\, L) = \frac{c}{8\pi}\, \frac{\omega}{K_n(\omega)} \iint \vec{E}_n(\omega,\, L) \times (\vec{E}_n^{\,*}(\omega,\, L) \times \hat{e}_z)\cdot dA \qquad (9)$$

$$= \frac{c\, \omega}{8\pi\, K_n}\, |E_{on}|^2\, f_n$$

$$= \quad [f_n^{-1} \; (\frac{\lambda}{2\pi})^2 \; \frac{ck_n}{\omega} \cdot (\frac{L\omega}{\beta c})^2] \;\; [\frac{r_o}{c} \; E_\perp] \;\; |I(\omega)|^2 \tag{10}$$

where $\frac{r_o}{c}$ is the classical electron radiation coupling, and $E_-$ is the electron's transverse energy. The first bracket is a geometrical constant of order unity, while $|I(\omega)|^2$ is a grating diffraction pattern modulation of the form:

$$\frac{\sin^2 \Delta L/2}{(\Delta L)^2} \tag{11}$$

where

$$\Delta = \frac{\gamma\omega_c - \omega}{\beta c} + k_n(\omega)$$

The peak of $|I(\omega)|^2$ thus corresponds to $\Delta = 0$ which is the same condition as (3) above. The width of the pattern is:

$$\Delta\nu = (\frac{\partial\Delta}{\partial\omega})^{-1} \frac{1}{2L} \tag{12}$$

As we suspected (12) is determined by $1/\tau_c$ and $1/\tau_\omega$ $(\partial\Delta/\partial\omega \propto 1/\tau_c \pm 1/\tau_\omega)$.

The above analysis is exact, yielding the same result as a rigorous Green's function treatment. of the excitation of a guide by a transiting electron current.[1]

Signal Pickup in Cooling Straight

Our waveguide is a 2.5 m pipe ~ 15 cm in diameter. At one end of this guide pickup loops are to be installed to intercept the excited waves propagating toward it. To choose which end of the guide (up or downstream with respect to the electron stream) we note that (12) gives narrower line widths for $K_n$ negative with respect to $\beta$. This choice also avoids the situation $\partial\Delta/\partial\omega = 0$, which condition is satisfied only when $K_n$ and $v_e$ are of the same sign.

With this choice we find:

$$\frac{\partial\Delta}{\partial\omega} = \frac{1}{|v_e|} + \frac{1}{|v_{group}|}$$

Far from cutoff $v_{group} \simeq c$ $(\omega >> \omega_n)$ yielding:

$$\frac{\partial\Delta}{\partial\omega} \simeq \frac{3}{c}$$

or $$\Delta\nu = \frac{c}{6L} = 20 \text{ MHz}$$

As $\omega$ approaches $\omega_n$ $v_{group} \to 0$ and in principle much narrower line widths may be achieved. In any case we can expect to detect clear peaks well within our spectrum analyser's IF bandwidths and narrower than typical coupling loop frequency tuning ranges.

For reasons of noise suppression (to be discussed below) the pickup should be sensitive to TE modes only and to the $H_z$ magnetic field in particular. This dictates loop pickups whose planes face the z axis. Figure 2 illustrates a schematic arrangement. For practical reasons I have limited the pickup frequency range to lie between 2 and 4 GHz. Noise sources grow inversely with frequency and the mode "thicket" proliferates at higher frequencies. Also, 2-4 GHz is a standard octave for readily available components.

The available modes are:

| Mode | Cutoff ($2\pi\omega_n$) |
|---|---|
| $TE_{11}$ | 1.15 GHz |
| $TE_{21}$ | 1.91 GHz |
| $TE_{01}$ | 2.40 GHz |
| $TE_{31}$ | 2.63 GHz |
| $TE_{12}$ | 3.34 GHz |

Of these $TE_{21}$ and $TE_{31}$ have broad zero's in field about the axis and will poorly couple to our axial beam. $TE_{01}$ goes to zero at R = 0 but only linearly, while its cutoff is at a good frequency point to expect very narrow line widths ($\omega_c/2\pi$ = 2.8 GHz $\Rightarrow \omega/2\pi = \omega_c/2\pi\gamma \simeq$ 2.4 GHz). $TE_{11}$ and $TE_{12}$ are similar and couple maximally to an axial excitation. Diametrically opposite pickups (Fig. 2) must be coupled in phase for $TE_{01}$ detection but $\pi$ out of phase for $TE_{11}$ and $TE_{12}$. This property of $TE_{11}/TE_{12}$ pickup will also surpress noise pickup (see below). The quantity $P(\omega, L)$ of Eq. (9) is defined such that:

$$\begin{array}{l}\text{Energy radiated in } d\omega \\ \text{during one electron passage}\end{array} = P(\omega, L)\, d\omega \, .$$

Therefore the signal we expect to "receive" is:

$$\text{Power} = \frac{I}{q_e} \; P(\omega,\, L)\; d\omega \tag{13}$$

$$= 30 \text{ Ampere} \quad 1.55 \times 10^{-20} \text{ Joule/Amp/s} \cdot 20 \text{ MHz} \cdot |I(\omega)|^2$$

$$= 1.4 \times 10^{-11} \text{ watt} \quad |I(\omega)|^2 \tag{14}$$

which is computed on the basis of a 30 Ampere beam current, a transverse energy of 1 eV, $2\pi\omega_c = 2.8$ GHz, and $TE_{01}$ mode pickup. Notice that a compromise between sensitivity and line-width must be chosen, since as $\omega \to \omega_n$ power $\propto k_n$ <u>and</u> linewidth $\propto k_n$. Therefore the selection of $\omega_c$ (and thus the solenoidal magnetic field) and mode observed will be dictated by noise levels encountered.

## Background & Noise Considerations

FET "front end" amplifiers with 2.0 db noise figures are readily available for operation at room temperature (although 2.0 db corresponds to the noise level from a pure resistance source held at 180° K!). This will set a noise level to be compared to eq. (14) of

$$P_n = KT\ \Delta\nu = k(180^{\circ}K)\ 20\ \text{MHz} = 6 \times 10^{-14}\ \text{watt}$$

Tests performed with the drift tubes we intend to use as pickup guides indicate that a given mode can be coupled to with ~ 20% efficiency over wide bandwidths (several hundred mega Hertz). With this, a signal to noise ratio of at least 10:1 is expected in a real system.

Even if the electrons were not spiralling at all, their longitudinal trajectories represent currents which will excite TM modes through the $E_z$ component of electric field. The analysis for computing the power excited is the same as that leading to eq. (10) except that no condition of resonance similar to eq. (1) is satisfied (i.e., the $\omega_c z/\beta c$ phase term is now absent). For a given (TM) mode the ratio of transverse cyclotron generated power to this longitudinal power is:

$$\frac{P_c}{P_L} = \frac{E_\perp}{E_{Beam}} \cdot \frac{|I_c(\omega)|^2}{|I_L(\omega)|^2}$$

Since we are far from the center of the diffraction pattern ($I(\omega)$) for $P_L$ the peaks of $I_L$ are down by a factor ~ $(\frac{\omega L}{c})^2$.

Of course we could always adjust $\omega_c$ to make this ratio $\infty$, but the worst case is:

$$\frac{P_c}{P_L} = \frac{1 \text{ ev}}{120 \text{ Kev}} \cdot 1.4 \times 10^5 = 1.1 \qquad (15)$$

which is based on 1 ev transverse energy at 2.8 GHz (ratio increasing as $\omega^2$). To the extent that we construct a "perfect" Hz loop (no coupling to E or $B_\theta$ fields) the ratio will be $\infty$.

Any electrical pickup next to a beam will see Schottky noise (actually the $P_c$ & $P_L$ signals are also Schottky "noise" in the sense that in the limit of a beam of particles where $q \to 0$ such that I = constant then <u>both</u> $P_c \to 0$ and $P_L \to 0$), which we may consider from the Weisäker-Williams picture.[2] Each electron induces a pulse on the loop which has a certain Fourier spectrum. The more distant a pickup is from the beam the longer this pulse and hence lower in frequency will be the upper cutoff in its Fourier components. Thus by having as squat a loop as possible and by operating at high enough frequency we should be able to minimize interference from the overlapping Schottky spectrum. The relative powers are:

$$\frac{P_c}{P_s} = \frac{1}{4\pi} \frac{E_\perp}{M_e C^2} \cdot \frac{b^2}{A} \cdot \frac{c\omega}{k_n} \quad [L^2 \, f_n^{-1}] \quad [\tfrac{\pi}{2} \, x \, e^{-2x}]^{-1}$$

where $\quad b$ = impact parameter

$A$ = loop area

$x = \omega b/\gamma\beta c$

numerically,:

$$\frac{P_c}{P_s} = \begin{matrix} 0.46 & (4\ \text{GHz}) \\ 0.0068 & (2\ \text{GHz}) \end{matrix} \qquad (16)$$

This analysis assumes the same efficiency for both cyclotron and Schottky pickup. However, longitudinally moving electrons produce no $H_z$ field so that our loop design should have a minimal coupling to Schottky noise. Monitoring either the $TE_{11}$ or $TE_{12}$ mode allows the illustrated (Fig. (2)) use of two diametrically opposite and out of phase loops. In the limit of a vanishing radius axial beam the Schottky pickup on each loop would cancel. Thus we expect background much suppressed from (16).

I emphasize that the backgrounds $P_s$ and $P_L$ are not fluctuations; they are smooth pedestals of power on top of which the cyclotron peaks are to be observed. The fluctuations in these background powers are very much smaller (down by $N^{-1/2}$; $N \simeq 10^{12}$). Thus, in principle, we can see signal peaks much smaller than background, provided these peaks can be unambiguously identified and provided the spectrum analyser has sufficient dynamic range.

## Limits of Resolution

An unneutralized electron beam has evaluation in $\beta$ radially. With $\delta E_{||}/E_{||} \sim 1\%$ (1200 V space charge potential drop) we expect:

$$\delta\beta/\beta = \frac{\delta E_{||}}{E_{||}} \frac{\gamma - 1}{\gamma^3 \beta^2} \sim 0.5\%$$

Using Eq. (3) this gives

$$\frac{\delta\omega}{\omega} = \frac{\beta\sqrt{1-\left(\frac{\omega_n}{\omega_c}\right)^2}}{1 \pm \beta\sqrt{1-\left(\frac{\omega_n}{\omega_c}\right)^2}} \cdot \frac{\delta\beta}{\beta}$$

which gives for $\omega_c >> \omega_n$:

$$\frac{\delta\omega}{\omega} \simeq \frac{\delta\beta}{\beta}$$

This gives a 15 MHz smearing of the central diffraction peak. Since we expect central peak widths which may be tuned even narrower than this (maximally 20 MHz wide) there is a distinct possibility of using this technique to monitor the neutralization fraction.

The diamagnetic twist of the electron beam is another component of transverse circular motion. The angular azimuthal frequency is:[3]

$$\dot{\theta} = \frac{1}{2}\,\omega_c\left(1 \pm \sqrt{1-2\,\frac{\omega_p^2}{\gamma^2\omega_c^2}}\right)$$

where $\omega_p$ = plasma frequency.

At beam radius, electrons have a transverse twist energy:

$$E_T = \frac{1}{2}\,m(\dot{\theta}R)^2 = \frac{1}{2}\,\frac{\dot{\theta}^2}{C^2}\,R^2 \cdot (M_c C^2)$$

at $\omega_c = (2\pi)$ 2.8 GHz this is:

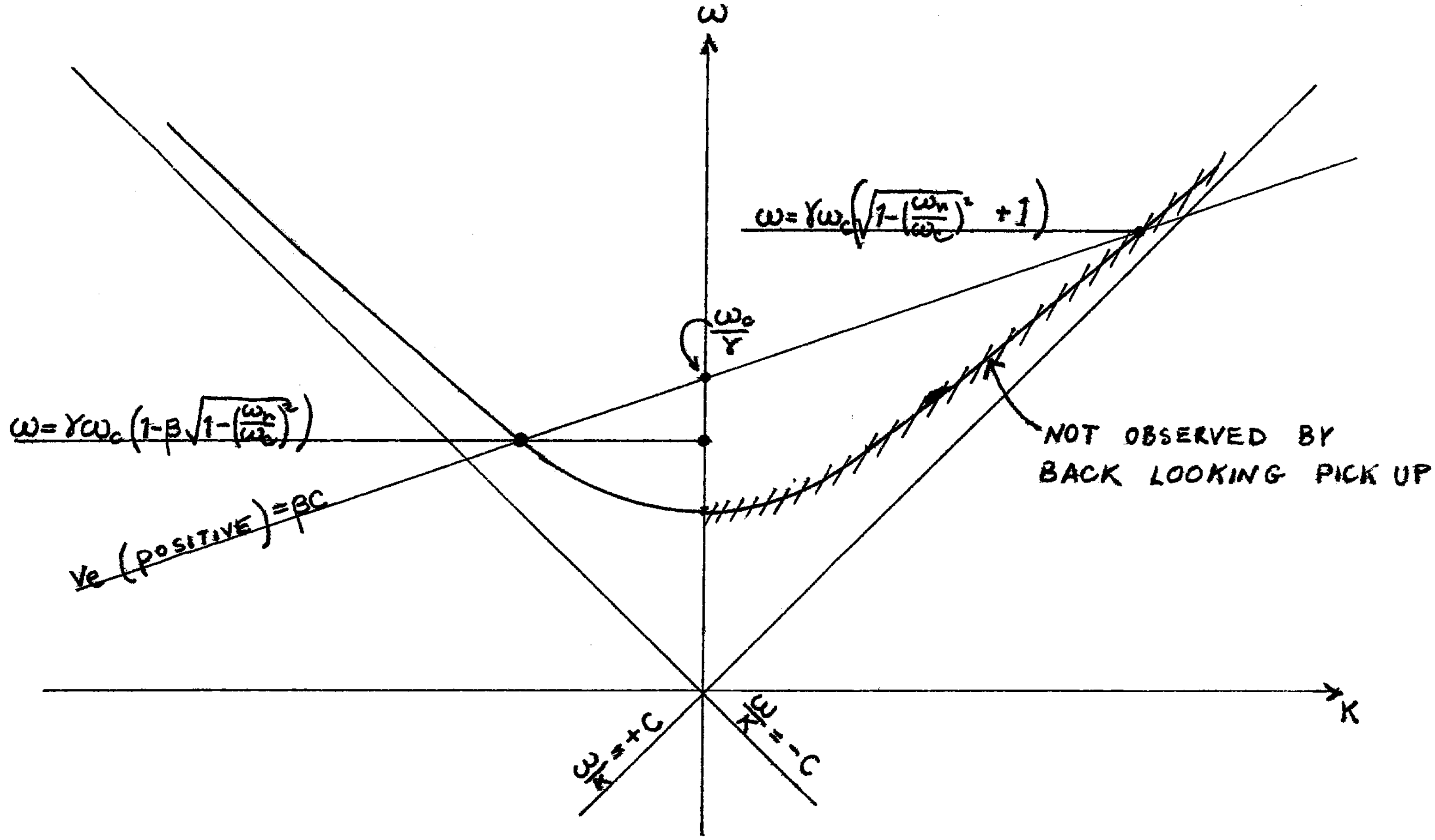
$$E_T = \frac{1}{2}\,\frac{(8\text{ MHz})^2}{C^2}\,(2.5\text{ cm})^2\;\;5 \times 10^5\text{ ev} \simeq 5\text{ ev}$$

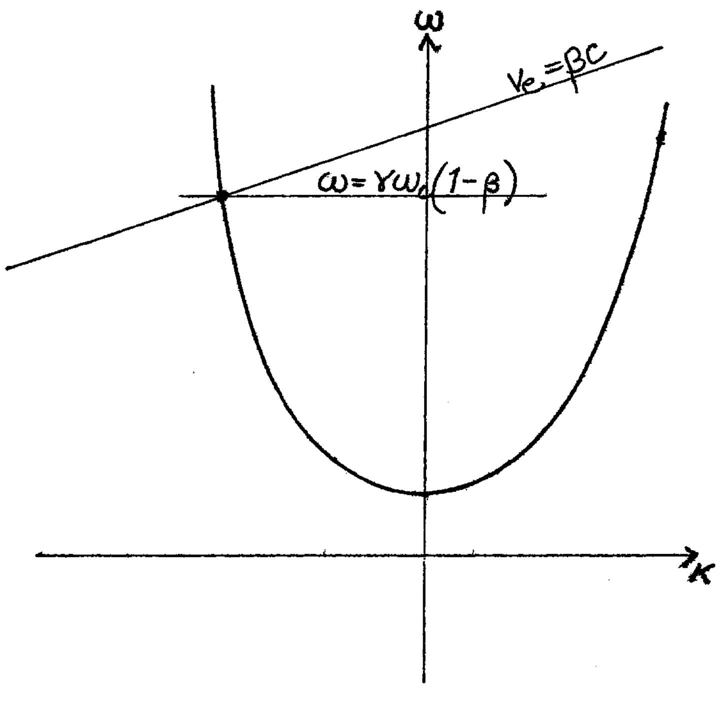
This 8 MHz twist should be observable as sideband lines ($\omega \pm 2\pi(8\text{ MHz})$) from our pickups. The resolution of these is perhaps marginal.

References

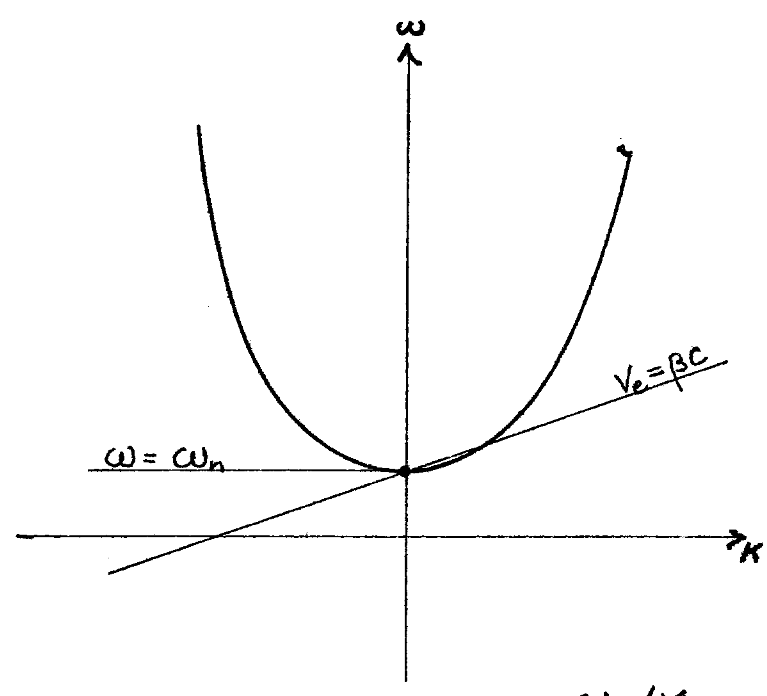
1. M. Collin, Theory of Guided Waves, McGraw Hill, 1961.
2. J. D. Jackson, Classical Electrodynamics, 1st ed., W. J. Wiley, 1962, pp. 520-523.
3. F. T. Cole and F. E. Mills, MHD Study of Equilibrium of Interacting Proton and Electron Beams in Electron Cooling, (unpublished), 1976.

# FIGURE 1



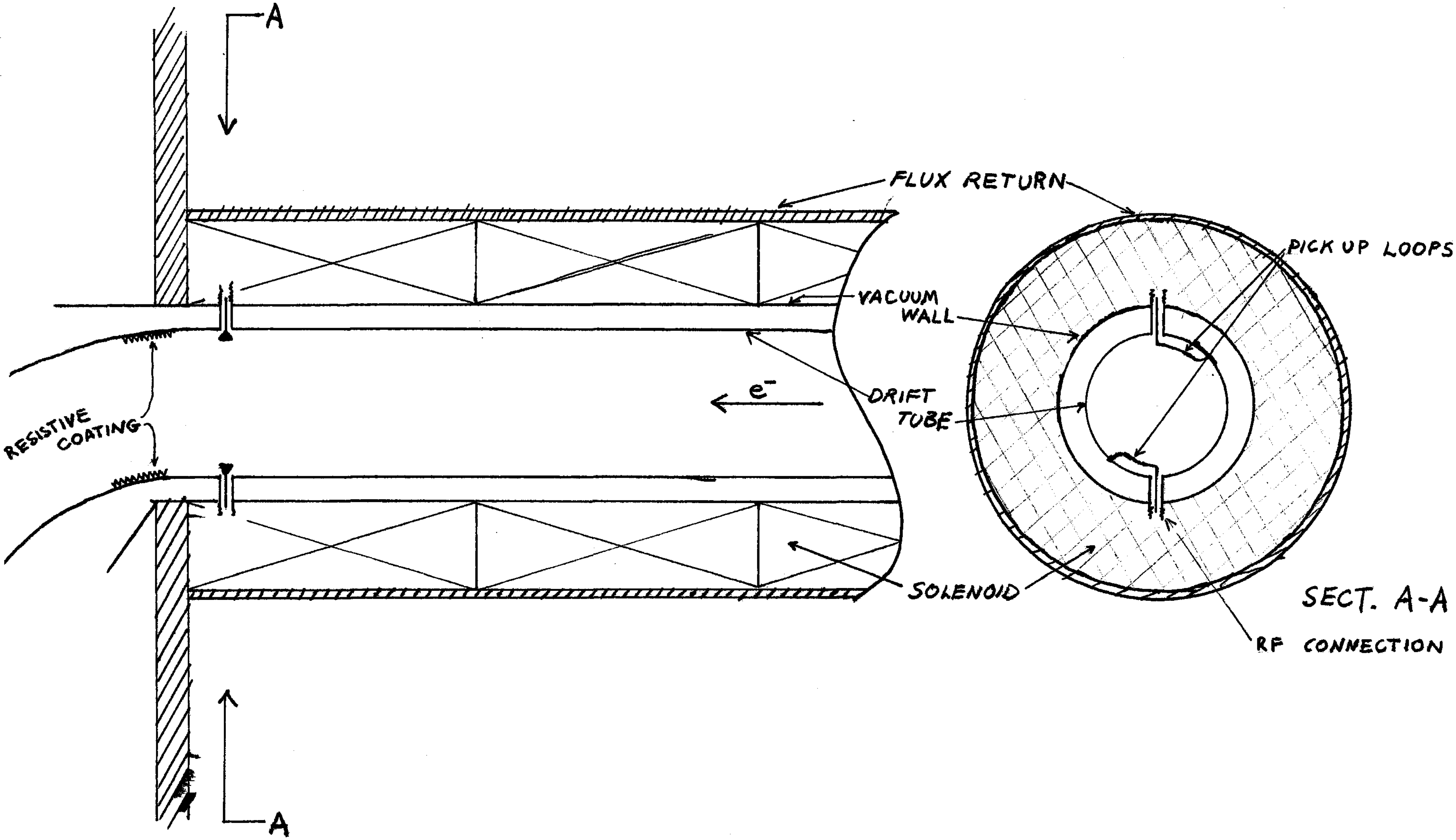




LIMITING CASE: $\omega_c \gg \omega_n$




LIMITING CASE: $\omega_n = \omega_c/\gamma$

FIGURE 2


A
A
FLUX RETURN
PICK UP LOOPS
VACUUM WALL
$e^-$
DRIFT TUBE
RESISTIVE COATING
SOLENOID
SECT. A-A
RF CONNECTION