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# DERIVATION OF SCALE VIOLATION FORMULA for DEEPLY INELASTIC MUON SCATTERING

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The formula which I propose for fitting $\nu W_2$ including the E98,398 observed scale violations comes from combining aspects of one dimensional phase space for the momentum carriers (quarks and gluons) as originally derived by Bjorken & Paschos[1], generalized vector dominance (GVD) as applied by Schildknecht[2] to the domain of small $Q^2$ inelastic scattering, and a phenomenological approach to the scaling violation which owes a debt to the QCD formulation of Buras & Gaemers[3]. I have discussed my fits with all of the above theorists (except Gaemers) and they were interested and intrigued by the connections among their respective works. A number of other theorists have seen this work and were interested in varying degrees.

I take as my model a Heisenberg picture in which the proton can be viewed as having a probability $P_i$ of being in various initial states when it scatters an incident muon (virtual photon). The scattering itself is assumed to be only from spin 1/2 quarks and to be elastic. (Presumably, radiative corrections should be applied to the quark lines, but never mind that for the moment, since the quarks are never observed anyhow.) The quarks have the valence electric charges (-1/3, 2/3, 2/3) plus any number of sea quark anti-quark pairs with (±1/3, ±2/3) charges taken with equal probability.

Each of the initial proton states is further taken to be a state in which i quarks and j gluons are distributed in momentum according to a simple one dimensional phase space distribution as first considered by Bjorken and Paschos[1]. I depart from them by including a boson propagator factor for sea quark, anti-quark pairs. This factor can be thought of as an expression for the uncertainty principle applied to the sea quarks in a virtual state. Schildknecht has justified this approach with a proper calculation in which the form chosen was derived from the basic principles of GVD and dispersion theory. The general expression for the momentum carrier particle density and the momentum density as a function of x becomes, respectively:

$$\text{i)}\quad \frac{dn}{dx} = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j)\,\frac{dn_{ij}}{dx} .$$

$$\text{ii)}\quad \frac{1}{P_0}\frac{dP}{dx} = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j)\; x\frac{dn_{ij}}{dx} \equiv x\frac{dn}{dx} .$$

P(i,j) is the probability of the proton being found in a state with i quarks and j gluons. Note that the following normalizations must hold:

$$\text{iii)}\quad \int_0^1 \frac{dn}{dx}\,dx = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j)\int_0^1 \frac{dn_{ij}}{dx}\,dx = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j)(i+j) = \langle i+j\rangle$$

$$\text{iv)}\quad \int_0^1 x\frac{dn}{dx}\,dx = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j)\int_0^1 \frac{x\,dn_{ij}}{dx}\,dx = \sum_{i=3}^{\infty}\sum_{j=0}^{\infty} P(i,j) = 1 .$$

The value of iii) must give the average total number of momentum carriers and iv) must express conservation of linear momentum. We are interested in the lowest values of i and in finding the appropriate values for j. We also determine that the average value of j is logarithmically varying with $Q^2$. This result is responsible for the scale violation we observe in this model, is physically reasonable (the number of gluons ovserved should increase as $Q^2$ increases and the interaction four-volume shrinks), and is found in a comparable form in the QCD formulae derived by Buras and Gaemers[3]. They find a $\log(\log(Q^2))$ behavior while I find a $\lg(Q^2)^{1/4}$ behavior. Both are extremely slowly varying functions of $Q^2$, of course.

$$x)\quad \overline{\frac{dn_{ij}}{dx}} = (\langle j\rangle + i)(\langle j\rangle + i - 1)(1-x)^{\langle j\rangle + i - 2}.$$

$$xi)\quad \overline{\frac{dn_{ij}}{dx}} = (G_i + i)(G_i + i - 1)(1-x)^{G_i + i - 2}.$$

Notice that equation xi) integrates to the value $(G_i + i)$, exactly equal to the total number of momentum carriers in the state. Also note that:

$$xii)\quad \int_0^1 x\,\overline{\frac{dn_{ij}}{dx}}\,dx = 1 = \frac{1}{P_0}\int_0^1 \frac{dP}{dx}\,dx.$$

That is to say, the total momentum carried by the $(G_i + i)$ momentum carriers exactly equals the proton's momentum seen in the infinite momentum frame (as is necessary for the Feynman picture of x to be reconciled with the Bjorken definition. With these observations in mind, we combine equations vi), viii) and xi) to obtain:

$$xiii)\quad \nu W_2(x,Q^2) = \sum_{i=3}^{\infty} P(i,\langle j\rangle)\; x\,\overline{\frac{dn_{ij}}{dx}}\cdot\langle q_i^2\rangle.$$

$$xiv)\quad \overline{\frac{dn_{ij}}{dx}} = (G_i + i)(G_i + i - 1)(1-x)^{G_i + i - 2}.$$

$$xv)\quad \langle q_i^2\rangle \equiv \frac{\sum_{k=3}^{i} q_k^2}{(G_i + i)}.$$

This is the easy part. Now, we must discover how to describe $P(i,\langle j\rangle)$. Hopefully, we can get away with only a few values for i before $P\rightarrow 0$. We start with the value i=3 (valence quarks). In this case:

$$xvi)\quad \langle q_3^2\rangle = \frac{1/9 + 4/9 + 4/9}{G_i + 3} = \frac{1}{G_i + 3}.$$

We further guess (!) that $P(3,\langle j\rangle)$ is <u>constant</u>. This gives:

$$xvii)\quad (\nu W_2)_3 = C_3(G_3 + 2)\,x\,(1-x)^{G_3 + 1}.$$

Next, we must relate $x\frac{dn}{dx}$ to the nuclear structure function $\nu W_2$ (also known as $F_2$). The relationship as shown by Bjorken & Paschos is:

$$\text{v)} \quad \nu W_2(x) = \langle q^2 \rangle x\frac{dn}{dx} = F_2(x).$$

where $\langle q^2 \rangle$ is the average quark charge squared for the scatterers measured in units of the charge of the electron. We have, therefore:

$$\text{vi)} \quad F_2(x) = \langle q^2 \rangle \sum_{i=3}^{\infty} \sum_{j=0}^{\infty} P(i,j)\, x\frac{dn_{ij}}{dx} = \sum_{i=3}^{\infty} \sum_{j=0}^{\infty} q_{ij}^2\, P(i,j)\, x\frac{dn_{ij}}{dx}.$$

$$\text{vii)} \quad q_{ij}^2 \equiv \frac{\sum_{k=3}^{i} q_k^2}{(i+j)}. \quad \text{(remember, the gluon chg. is zero!)}$$

Our problem now is to determine the appropriate description for $\frac{dn_{ij}}{dx}$ for various values of i and j. Since j is not directly observable, we will average over j for particular values of i:

$$\text{viii)} \quad \sum_{j=0}^{\infty} P(i,j)\, x\frac{dn_{ij}}{dx} \cong P(i,\langle j \rangle) \times \frac{\overline{dn}_{ij}}{dx}.$$

The average value of j can be expected to increase with $Q^2$ on general physical grounds as more and more gluons virtually emitted from quark lines are seen as $Q^2$ increases. Experimentally, I find that $\langle j \rangle$ varies approximately as $\chi \log (Q^2 + M_o^2/M_o2)$. Buras and Gaemers parameterize the $Q^2$ variation as log $(\log [Q^2/\Lambda^2]/\log [Q_o^2/\Lambda^2])$. This is unsatisfactory as it blows up as $Q^2 \to 0$. They have no prescription for going to zero. I chose a form that goes smoothly to $Q^2=0$, but the functional form $\chi \log (Q^2 + M_o^2/M_o^2)$ is not determined from theory and is therefore not required. I chose it by looking at our scale violation data. We get with this ansatz:

$$\text{ix)} \quad \langle j \rangle_i = G_{io} + \chi \log\left[\frac{Q^2 + m_o^2}{m_o^2}\right] \equiv G_i(Q^2).$$

clearly there are $G_{io}$ gluons accompanying a state of i quarks as seen by a real photon ($Q^2=0$).

Now, we can write the phase space for a state containing i quarks and $\langle j \rangle$ gluons as:

The next appropriate value for i is 5, since there can be three valence quarks plus one sea quark - antiquark pair due to quantum fluctuations (or GVD if you prefer). The probability P(5,<j>), however, depends on various kinematic factors since the sea quarks appear only as a fluctuation. On the basis of special relativity, the Heisenberg Uncertainty Principle and dimensional analysis, we guess that:

$$\text{xviii)}\quad P(5,\langle j\rangle) = C_5 \frac{s}{Q^2+m_0^2} .$$

$$\text{xix)}\quad s \equiv (\nu+M)^2 - \vec{q}^{\,2} \equiv 2M\nu - Q^2 .$$

We can rewrite equation xviii) as:

$$\text{xx)}\quad P(5,\langle j\rangle) \cong C_5/(x + m_0^2/2M\nu) = \frac{C_5}{x}\left(\frac{Q^2}{Q^2+m_0^2}\right) .$$

From equation xx) we can find:

$$\text{xxi)}\quad (\nu W_2)_5 = C_5\left(\frac{14}{9}\right)(G_5+4)(1-x)^{G_5+3}\left(\frac{Q^2}{Q^2+m_0^2}\right) .$$

since, in the symmetric SU4 (or SU6):

$$\text{xxii)}\quad \langle q_5^2\rangle = \frac{1/9 + 4/9 + 4/9 + 1/9 + 4/9}{G_5+5} = \frac{14}{9}\left(\frac{1}{G_5+5}\right) .$$

Now, if the proton had unit probability to appear in this 5-quark state we would demand:

$$\text{xxiii)}\quad 1 = \frac{1}{\langle q_i^2\rangle}\int_0^1 (\nu W_2)_5\, dx = C_5 (G_5+4)(G_5+5)\int_0^1 (1-x)^{G_5+3} \frac{Q^2}{Q^2+m_0^2}\, dx .$$

which at $Q^2 >> M_O{}^2$ becomes:

$$\text{xxiv)}\quad 1 = C_5 (G_5+1) \Rightarrow C_5 = \frac{1}{G_5+5} .$$

Thus, for $Q^2 >> M_o{}^2$, we have:

$$\text{xxv)}\quad (\nu W_2)_5 \quad \left(\frac{14}{9}\right)\frac{(G_5+4)}{(G_5+5)}(1-x)^{G_5+3}\left(\frac{Q^2}{Q^2+m_o^2}\right).$$

For values of $Q^2 \lesssim M_o{}^2$, the prescription is not clear at all. One way to proceed would be to rewrite the propagator factor in terms of X and S again and observe that the integral is convergent for <u>all</u>, spacelike values of $Q^2$ (including $Q^2=0$). This would give:

$$\text{xxvi)}\quad \frac{1}{(G_5+5)} \leq C_5 \leq \frac{m_o^2}{2M\nu} \quad \text{as} \quad \infty \leq Q^2 \leq 0.$$

This is equivalent to saying that $C_5$ is constant only for $Q^2 >> M_o{}^2$. At $Q^2=0$, to normalize the momentum to unity, the expression would be:

$$\text{xxvii)}\quad \lim_{Q^2\to 0}(\nu W_2)_5 = \left(\frac{14}{9}\right)(G_5+4)\left(\frac{m_o^2}{2M\nu}\right)(1-x)^{G_5+3}\frac{Q^2}{Q^2+m_o^2}.$$

This gives a number which is orders of magnitude smaller than the real value of $\sigma_{incl}$ ($Q^2=0$). A much closer estimate is received by merely letting $C_5$ be constant and evaluating it for $Q^2 >> M_o{}^2$. As R. Wilson and B.Gordan[4] have pointed out, this gives about 50μb as the extrapolated value of $\sigma_T$ at $Q^2=0$. The true real photon cross section is more like 118μb. They regard this as a serious flaw in the model. To me it's a miracle that it comes as close as it does given the considerations noted above. For this reason, I do not think it is appropriate to force the fit to be constrained to the optical point. Clearly it is interesting that it comes close and this circumstance demands further analysis of why, but it is not a simple question and is not a reason to abandon the fit and its interpretation for $Q^2 >> M_o{}^2$.

The analysis could be continued for i = 7, 9, ---- etc., but I prefer to fit only the first two terms as they nearly saturate the momentum integral. We get, therefore:

xxviii) $\nu W_2 \cong A(G_3+2)x(1-x)^{G_3+1} + B\left(\frac{14}{9}\right)\frac{(G_5+4)}{(G_5+5)}(1-x)^{G_5+3}\frac{Q^2}{Q^2+m_0^2}$.

where;

xxix) A = Probability proton is in 3 quark plus gluon state.

B = Probability proton is in 5 quark plus gluon state.

When this form was fitted by me to the data with R=0.19, I found:

| | $Q^2 > 2$ | $Q^2 > 0.2$ |
|---|---|---|
| A= | $0.555 \pm .067$ | $0.528 \pm .071$ |
| B= | $0.356 \pm .013$ | $0.344 \pm .007$ |
| $G_3$= | $0.408 \log\left(\frac{Q^2+.758}{.758}\right) + 1.13$ | $0.417 \log\left(\frac{Q^2+.605}{.605}\right) + 0.959$ |
| $G_5$= | $0.418 \log\left(\frac{Q^2+.758}{.758}\right) + 3.27$ | $0.417 \log\left(\frac{Q^2+.605}{.605}\right) + 2.40$ |
| $M_o^2$= | $.758 \pm .16$ | $.605 \pm .050$ |
| $X^2$= | 44/40 D.F. | 110/70 D.F. |

When I tried fitting to the combined SLAC (Riordan)/E-398 data, I found similar values provided. I let the overall SLAC normalization float with E-398. I have not tried varying the value or functional form of R so far, but B. Gordan (Harvard) has done so and finds rather good fits for the Field & Feynman form:

xxx) $R = \frac{R_0(1-x)}{Q^2}$. $R_0 \sim 1.20$

There is no reason to believe R=constant, so I am interested in investigating this R question. Meanwhile, I hope the basis for the fit has been clarified (and its questionable aspects clearly admitted).

## REFERENCES

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