Fermilab

TM-749
0431.010

EXPECTED BUNCH LENGTH AND MOMENTUM SPREAD OF THE BEAM IN THE MAIN RING WITH CEA CAVITIES

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October 24, 1977

I.

One of the major findings from a series of beam storage studies in the main ring is the lengthening of bunches.[1] The full bunch length is typically $\sim$ 2 ns at the beginning of the storage but this grows to $\sim$ 5 ns during the first few seconds or, sometimes, during the first few minutes. If the initial length is made longer, $\sim$ 3.5 ns, using the bunch spreader, the subsequent growth is much less than that. For colliding beam experiments with the main ring beam, one naturally wants to keep the bunch length as small as possible for a shorter length of the interaction region and for a higher luminosity.

At a relatively high intensity ($\sim 1.8\times10^{13}$), it has not yet been established if the lengthening is caused predominantly by coupled-bunch instabilities or by single-bunch instabilities. At a lower intensity ($\sim 0.8\times10^{13}$), it has been observed that a bunch does not grow if it is isolated. In the actual observation of this phenomenon, fifteen bunches were eliminated on both sides of two bunches and these two bunches maintained their original length for $\sim$ ten minutes.[2] The range of the field responsible for the bunch lengthening has not been determined but it is believed to cover a few bunches.

Coupled-bunch instabilities are clearly indicated as the major cause of the lengthening, at least at the lower intensity, although one cannot exclude the possibility of single-bunch instabilities at higher intensities.

The surest way to eliminate the coupled-bunch instability is to remove the object(s) that are responsible for creating the longitudinal field. Unfortunately, this is often imparactical and the common cure is to introduce a large spread in the synchrotron oscillation frequency, either from bunch-to-bunch or within each bunch.[3] An example of the former is the booster with a small amount of rf voltage at the harmonic number 83 added to the main voltage of harmonic number 84.[4] Another example is the CERN SPS in which the rf voltage is modulated by a few percent at the revolution frequency.[5] The latest example of the latter is the higher harmonic cavity in the CERN ISR, commonly called the Landau cavity, with its harmonic number three or four times that of the other cavities.[6] (The name is derived from the Landau damping of instabilities. The higher harmonic cavity introduces a large spread in the oscillation frequency, thereby creating a strong damping effect.) Several people have suggested that the old CEA cavities be used in the main ring for the similar purpose. These cavities were originally operated at 475 MHz which is very close to nine times the main ring rf frequency of 53.1 MHz. The purpose of this note is to estimate the expected bunch length and the momentum spread of the beam when the CEA cavities are driven out-of-phase relative to the main rf cavities and their amplitude is one-nineth of the main cavity total voltage, the mode most

effective in increasing the spread in frequency. Threshold currents, with and without CEA cavities, for the microwave instability[7] and for the coupled-bunch dipole mode instability are also discussed, the latter using Sacherer's criteria.[8]

II.

For the stored beam at high energies, equations of phase motion are[9]

$$d\phi/dt = A\,y \tag{1}$$

$$dy/dt = -\,B\,[\,\sin\phi + k\,\sin(n\phi + \delta)] \tag{2}$$

$$A = (ch/R)^2(\eta/E), \quad B = (eV/2\pi h) \tag{3}$$

where $(\phi,\ y \equiv \Delta E/\omega_{rf})$ are the usual canonical coordinates of the longitudinal phase space. The higher harmonic cavity with the harmonic number nh and the amplitude kV can be either in-phase ($\delta = 0$) or out-of-phase ($\delta = \pi$), or it may be used to compensate for a possible decelerating voltage ($\delta = \pi/2$) with a very short range (fraction of the bunch length).[10] Other symbols in Eqs. (1) - (3) are more or less standard (see ref. 9, p. 39). The corresponding Hamiltonian is

$$H = (1/2)Ay^2 - B\,[\cos\phi + (k/n)\cos(n\phi + \delta)]. \tag{4}$$

The angular frequency of the phase oscillation is, when the amplitude is ϕ_o,

$$\begin{aligned}\omega_s &= (\pi/2)\ [\textstyle\int_0^{\phi_o} d\phi/(d\phi/dt)]^{-1} \\ &= (\pi/2)A\ [\textstyle\int_0^{\phi_o} d\phi/y(\phi)]^{-1}.\end{aligned} \tag{5}$$

This is shown in Fig. 1 for $k = 0$ (no higher harmonic cavity), 1/18

and 1/9 when $n = 9$. The abscissa is the ratio of the beam area to the bucket area with $k = 0$ and it is independent of the voltage V. With the choice $\delta = \pi$ and $k = 1/n$, the right hand side of Eq. (2) vanishes in the lowest order in ϕ. There is no restoring force and the frequency is zero at the origin. For $k = 0$, the frequency can be expressed in terms of a complete elliptic integral of the first kind[11]

$$\omega_s = \omega_s(\phi_o=0)(\pi/2)/K(\sin^2\phi_o/2) \tag{6}$$

$$K(m) = (\pi/2)(1 + m/4 + 9m^2/64 + \cdots\cdots). \tag{7}$$

For a typical case in the main ring, V = 1.5 MV and the beam area = 0.25 eV s, the frequency spread is only 1% without the higher harmonic cavity whereas it is almost 100% with the optimum choice of amplitude and phase of the higher harmonic cavity.

Assume that the "boundary" of the bunch is specified by

$$y = y_B(\phi), \tag{8}$$

that is, max.$|y| \equiv y_m = |y_B(\phi=0)|$, max.$|\phi| \equiv \phi_m$ and $y_B(\pm\phi_m)=0$. The value of the Hamiltonian on the boundary is

$$H_B = (1/2)Ay_B^2(\phi) - B\,[\cos\phi + (k/n)\cos(n\phi + \delta)]. \tag{9}$$

It is often convenient to take the elliptic distribution in phase space,

$$\psi(\phi, y) \propto \sqrt{H_B - H(\phi)} \propto \sqrt{y_B^2(\phi) - y^2(\phi)} \tag{10}$$

since the resulting distribution in phase has the same ϕ-dependence

as the external potential,

$$\lambda(\phi) = 2\int_{-y_B}^{y_B} dy\ \psi(\phi,y) \propto \int_{-y_B}^{y_B} dy \sqrt{y_B^2 - y^2} \propto y_B^2(\phi)$$

$$= 2H_B/A + (2B/A)[\cos\phi + (k/n)\cos(n\phi+\delta)]. \qquad (11)$$

The first term of the last line is independent of ϕ and the second term is proportional to the external potential. Note that, in the lowest order in ϕ, $\lambda(\phi)$ is quadratic in ϕ for $k = 0$ and quartic for $k = 1/n$ and $\delta = \pi$. If $\lambda(\phi)$ is normalized such that it gives the line density of particles,

$$\lambda(\phi) = (N/2R)[f(\phi) - f(\phi_m)]/D(\phi_m) \qquad (12)$$

where N is the total number of particles in the ring (N/h particles in each bunch) and*

$$f(\phi) = \cos\phi \qquad \text{for } k=0$$

$$= \cos\phi - n^{-2}\cos(n\phi) \qquad \text{for } k\neq 0 \qquad (13)$$

$$D(\phi_m) = \sin\phi_m - \phi_m\cos\phi_m \qquad \text{for } k=0$$

$$= \sin\phi_m - \phi_m\cos\phi_m - n^{-3}\sin(n\phi_m) + n^{-2}\phi_m\cos(n\phi_m) \qquad \text{for } k\neq 0 \qquad (14)$$

For a wall considered to be perfectly conducting with a distributed inductance dL/ds per unit length, the beam-induced voltage per turn is[12]

$$U(\phi) = ehc\ |Z/n|\ \lambda(\phi) \qquad (15)$$

where $|Z/n|$ is the impedance $|Z| = \omega L$ divided by the mode number $n = \omega/\omega_{rev}$. There is an additional contribution arising from

*From now on, only the choice $k=1/n$ and $\delta=\pi$ will be considered.

the capacitance between the beam and the wall but this is entirely negligible at high energies ($1/\gamma^2$ dependence). Because of the induced voltage, one must modify the Hamiltonian, Eq. (4), by adding the term

$$\Delta H = (e/2\pi h)\ U(\phi)$$

$$= (1/2)eI_o|Z/n|\ [f(\phi) - f(\phi_m)]/D(\phi_m) \tag{16}$$

where $I_o = (eNc/2\pi R)$ is the total current in the ring. It is easy to see that, with the elliptic distribution, the net effect of the induced voltage is simply to change B to B^*,

$$B^*/B = 1 - (eI_o/2)|Z/n|/[BD(\phi_m)] \tag{17}$$

and

$$H = (1/2)Ay^2 - B^*f(\phi) - (B-B^*)f(\phi_m) \tag{18}$$

which is identical to Eq. (4) in form except for the constant term. From Eq. (3), one also sees that the change is equivalent to the reduction of the rf voltage from V to V^*,

$$V^* = V\ (B^*/B). \tag{19}$$

The Hamiltonian takes the value H_B on the boundary of the bunch

$$H_B = H(\phi, y_B(\phi)\ ) = H(\phi=\phi_m,\ y=0) \tag{20}$$

and this gives the equation of the beam boundary

$$y_B(\phi) = \pm\sqrt{2B^*/A}\ \sqrt{f(\phi) - f(\phi_m)}\ . \tag{21}$$

The phase space area of the beam is

$$S = 4 \int_0^{\phi_m} y_B(\phi)\, d\phi \tag{22}$$

and the maximum value of y is

$$y_m = y_B(\phi=0) = \sqrt{2B^*/A}\ \sqrt{f(0) - f(\phi_m)}. \tag{23}$$

Eqs. (21) - (23) are exact for both k=0 and k≠0 as long as the phase space distribution is elliptic and, together with Eqs. (13), (14) and (17), they give the total bunch length $2\phi_m$, the total momentum spread

$$(\Delta p/p)_{total} = 2(hc/RE)y_m$$

and the voltage reduction ratio V^*/V for a given set of V, $I_o|Z/n|$ and S.. Results are shown in Figs. 2 and 3. The ratio V^*/V is almost independent of k and results for k=0 shown in Fig. 2 are applicable to k≠0 as well. Note that the bucket area reduction for a finite value of $I_o|Z/n|$ is proportional to $(V^*/V)^{1/2}$.

The total momentum spread of the beam is always reduced by the presence of the Landau cavity, although the difference is not significant. The bunch length is generally increased when the phase space area of the beam is small but the Landau cavity tends to reduce the bunch length when the beam area is relatively large. This is not surprising since the voltage of the Landau cavity is added to (instead of subtracted from) the main voltage beyond $\pm 20^o$. If the harmonic multiplier is three, for example, this will not happen until the phase spread is beyond $\pm 60^o$. The bunch length will then be increased by the Landau cavity for the same range of S and $I_o|Z/n|$ used in Fig. 2.

III.

It has been observed in the ISR[7] that the bunch length as a function of the current follows Eq. (21) up to 100 mA but it grows much faster beyond that. The bunch area calculated from the measured bunch length stays constant up to this threshold current but it increases linearly with the current thereafter. At the same time, they have observed very high frequency signals, the mode number of which ranging from ∿1,000 to ∿6,000. The same phenomenon occurs when the rf voltage is reduced adiabatically or during debunching of the beam. The similar growth of high frequency signals has been detected in the SPS[5] and in the main ring.[13] The frequency range observed in the SPS is from a few hundred MHz to 1.6 GHz (mode number up to 37,000). This type of instability is called the "microwave" instability and the cause of this is believed to be high frequency impedances arising primarily from discontinuities in the vacuum chamber cross section.

The abnormal lengthening of the bunch in the ISR beyond the threshold current can be explained if one uses the coasting beam stability criterion[14] together with the overshoot relation of Dory.[15] The coasting beam criterion is

$$|Z/n|_{MW} \leqslant F(A/e)\ [\bar{y}^2(\phi)/I(\phi)]_{local} \tag{24}$$

where A is given in Eq. (3), $|Z/n|_{MW}$ is simply a measure of the wall impedance for the microwave instability, F is the form factor of the momentum distribution, $\bar{y}(\phi)$ is FWHM of $y(\phi)$ and $I(\phi)$ is the instantaneous current. The quantity within the square bracket must be evaluated at each value of ϕ so that it may not be constant in a

bunch. One can show that for the elliptic distribution,

$$\bar{y}(\phi) = \sqrt{3}|y_B(\phi)|. \tag{25}$$

At the same time,

$$I(\phi) \propto \lambda(\phi) \propto y_B^2(\phi) \tag{26}$$

so that $\bar{y}^2(\phi)/I(\phi)$ is independent of ϕ. This is another convenient property of the elliptic distribution. The form factor F is of the order of unity for commonly used distribution[16] and $F = 0.6 \sim 0.7$ for the elliptic distribution. In obtaining the results in this note, $F = 0.64$ given in ref. 7 has been used for all cases. The usual justification for using the coasting beam criterion in discussing the bunched beam instability is that the growth time is much shorter than the phase oscillation period and the concept of the phase oscillation loses its physical significance. Dory's overshoot relation states that, if the initial value of $\bar{y}^2/I$ is too small to satisfy the condition (24), the momentum spread of the beam will grow to the final value such that

$$(\bar{y}^2/I)_{initial} \cdot (\bar{y}^2/I)_{final} = [(\bar{y}^2/I)_{thr.}]^2 \tag{27}$$

where $(\bar{y}^2/I)_{thr}$ is obtained by taking the equal sign in Eq. (24). If there is microwave instability, the beam boundary is no longer determined by Eqs. (21) and (22). One must calculate $y_B(\phi)$ from $(\bar{y}^2/I)_{final}$ and the bunch length and the phase space area (no longer conserved) must be recalculated from Eqs. (21) and (22), respectively.

In order for a bunch with ϕ_m and y_m to be below the microwave instability, the following condition must be satisfied,

$$eI_o|Z/n|_{MW} \lesssim [6FB^* D(\phi_m)/\pi]. \tag{28}$$

This condition is rather insensitive to the ratio

$$r = |Z/n|_{MW}/|Z/n|$$

for $r \gtrsim 1$. The presence of the Landau cavity changes the threshold value of $I_o|Z/n|_{MW}$ by at most ~10% and results shown in Fig. 4 are valid for both k=0 and k≠0. In plotting the threshold current, $I_o|Z/n|_{MW}$ is converted to the equivalent value of $N|Z/n|_{MW}$ using the relation

$$I_o \text{ in amp} = 0.0764\ N(\text{in } 10^{13}).$$

The value of $|Z/n|_{MW}$ for the SPS is believed to be 10 ~ 30 ohms[5] and there are no obvious reasons to expect higher values than this for the main ring. With 30 ohms, one finds from Fig. 4 that the threshold value of N is 2.5×10^{13} if the beam area is 0.2 eV s. It seems unlikely that the microwave instability will affect the bunched beam at high energies in the main ring as long as $N < 3\times10^{13}$ and $S > 0.2$ eV s. This is not in contradiction with the microwave instability observed during the debunching of 8 GeV beam.[13] One can show that the quantity $\bar{y}^2/I$ continuously decreases as the beam gets debunched,

$$(\bar{y}^2/I)_t = (\bar{y}^2/I)_{t=0} / (1 + A^2 (y_m/\phi_m)^2_{t=0}\ t)^{1/2} \tag{29}$$

and sooner or later the microwave instability will set in according to the criterion (24).

IV.

It is clear from Fig. 1 that the Landau cavity will introduce a large spread in the oscillation frequency within a bunch, thereby providing a strong damping effect. In order to make a quantitative estimate of this effect, one must generally solve a linearized Vlasov equation and find the dispersion relation for a specific mode (dipole, quadrupole, etc.). Recently, Frank Sacherer has made such an estimate[8] for the dipole mode taking only the beam-induced voltage of the form (15). He used the Boltzman distribution

$$\psi(\phi, y) \propto e^{-\alpha H} \tag{30}$$

since his analysis was intended for electron bunches. Furthermore, he took only the lowest order terms in ϕ,

$$\begin{aligned} H &= (1/2)Ay^2 + (1/2)B\phi^2 && \text{for } k=0 \\ &= (1/2)Ay^2 + (1/24)(n^2-1)B\phi^4 && \text{for } k\neq 0. \end{aligned} \tag{31}$$

The phase oscillation frequency and ϕ as a function of time for this Hamiltonian are given in the appendix. In view of these assumptions, his stability criteria may not be reliable for large values of ϕ_m.[17] Nevertheless, it is instructive to estimate the threshold values of $I_o|Z/n|$ based on his conditions. Sacherer states that the stability conditions for the dipole mode are

$$I_o|Z/n| < (V^*/48\pi h)\phi_m^5 \qquad \text{for } k=0 \tag{32}$$

$$< (0.151\ V^*/h)\phi_m^5 \qquad \text{for } k\neq 0. \tag{33}$$

In deriving Eq. (32), the spread in phase oscillation frequency is

taken to be

$$|\omega_s(\phi_m) - \omega_s(0)| = \omega_s(0)[1 - (\phi_m^2/16)]. \qquad (34)$$

Threshold values of $N|Z/n|$ are shown in Fig. 5 when V = 1.5 MV. The value of $|Z/n|$ for the main ring is not known but a recent study[2] has indicated a rather low value, less than ten ohms. This should be compared to the measured value of 20 ohms for the ISR. For ten ohms and without Landau cavity, the threshold value of N varies from 1.5×10^{12} (for S=0.2 eV s) to 6×10^{12} (for S=0.35 eV s). With Landau cavity, the corresponding values of N are 4×10^{13} and 1.3×10^{14}, respectively. These numbers may very well be only semi-quantitative but there is little doubt that the Landau cavity will increase the threshold currents for the coupled-bunch instability by an order of magnitude or more.

Appendix Small-Amplitude Phase Oscillation with Landau Cavities

The Hamiltonian for a small-amplitude oscillation is

$$H = (1/2)Ay^2 + (1/4)B_n\phi^4 \qquad \text{(A. 1)}$$

where A is given in Eq. (3), the harmonic number of Landau cavity is nh and

$$B_n = (eV/12\pi h)(n^2 - 1).$$

Consider an oscillation with

$$\text{max. } y = y_o \quad \text{and} \quad \text{max. } \phi = \phi_o \ .$$

The value of the Hamiltonian for this motion is

$$H_o = (1/2)Ay_o^2 = (1/4)B_n\phi_o^4 \qquad \text{(A. 2)}$$

and the motion in phase space is specified by

$$y(\phi) = \pm\sqrt{(2H_o/A) - (B_n/2A)\phi^4} \ . \qquad \text{(A. 3)}$$

From equations of motion

$$\dot{\phi} = Ay \quad \text{and} \quad \dot{y} = -B_n\phi^3$$

and (A. 2), one finds

$$\dot{\phi}_o \equiv \max(\dot{\phi}) = Ay_o = \sqrt{2AH_o} \qquad \text{(A. 4)}$$

$$\phi_o = (4H_o/B_n)^{1/4} \ . \qquad \text{(A. 5)}$$

A. Phase oscillation frequency

From Eq. (5) and (A. 3), the angular frequency of oscillation with the amplitude ϕ_o is

$$\omega_s(\phi_o) = (\pi/2)A\{\int_o^{\phi_o} d\phi/y(\phi)\}^{-1}$$

$$= (\pi/2)\{\sqrt{AB_n}\ /K(1/2)\}\ \phi_o \qquad \text{(A. 6)*}$$

where K(1/2) is a complete elliptic integral of the first kind

$$K(m) = \int_o^{\pi/2}(1-m\ \sin^2\theta)^{-1/2}\ d\theta\ .$$

The frequency is a linear function of the amplitude.

B. Phase motion

From equations of motion,

$$\ddot{\phi} = -\ AB_n\ \phi^3. \qquad \text{(A. 7)}$$

General solutions of this differential equation can be expressed in terms of Jacobian elliptic functions[18] cn(u|m), sn(u|m) and dn(u|m). For example, if the initial conditions are

$$\phi(t=0) = \phi_o \quad \text{and} \quad \dot{\phi}(t=0) = 0,$$

the solution is

$$\phi(t) = \phi_o\ \mathrm{cn}(\sqrt{AB_n}\ \ \phi_o t|1/2)$$

$$= \phi_o\ \mathrm{cn}(\tfrac{2}{\pi}K\omega_s(\phi_o)\ t|1/2) \qquad \text{(A. 8)}$$

where K = K(1/2). For the initial conditions

$$\phi(t=0) = 0 \quad \text{and} \quad \dot{\phi}(t=0) = \dot{\phi}_o\ ,$$

the solution is

$$\phi(t) = \phi_o(1/\sqrt{2})\ \frac{\mathrm{sn}(\sqrt{AB_n}\ \phi_o t|1/2)}{\mathrm{dn}(\sqrt{AB_n}\ \phi_o t|1/2)} \qquad \text{(A. 9)}$$

where ϕ_o is related to $\dot{\phi}_o$ by (A. 4) and (A. 5).

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* $\int_o^1 dx\ (1-x^4)^{-1/2} = K(1/2)/\sqrt{2}$.

Jacobian elliptic functions can be expanded in terms of trigonometric functions.[19] For example, with $K = K(1/2)$,

$$\mathrm{cn}(u|1/2) = (2\sqrt{2}\,\pi/K) \sum_{0}^{\infty} \frac{e^{-\pi(n+1/2)}}{1+e^{-\pi(2n+1)}} \cos(2n+1)v \qquad \text{(A. 10)}$$

where $v = (\pi/2K)\,u$. Using (A. 10), one can rewrite (A. 8) as

$$\phi(t) = \phi_o\{0.9550 \cos(\omega_s t) + 0.0431 \cos(3\omega_s t) + 0.0019 \cos(5\omega_s t) + \ldots\} \qquad \text{(A. 11)}$$

It is somewhat surprising to see that the motion is still predominantly sinusoidal with the frequency $\omega_s(\phi_o)$ and the higher harmonic contents are less than 5%.

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16. Tom Collins has pointed out that $\bar{y}$ should be a measure of the length of falling-off region of the distribution. Therefore, one should not use FWHM when the distribution has a flat central part.

17. The difference in the distribution $\psi(\phi,y)$ may not be important. See A. G. Ruggiero, IEEE NS-24 (1977), 1205.

18. Ref. 11, p. 567.

19. Ref. 11, section 16.23, p. 575.

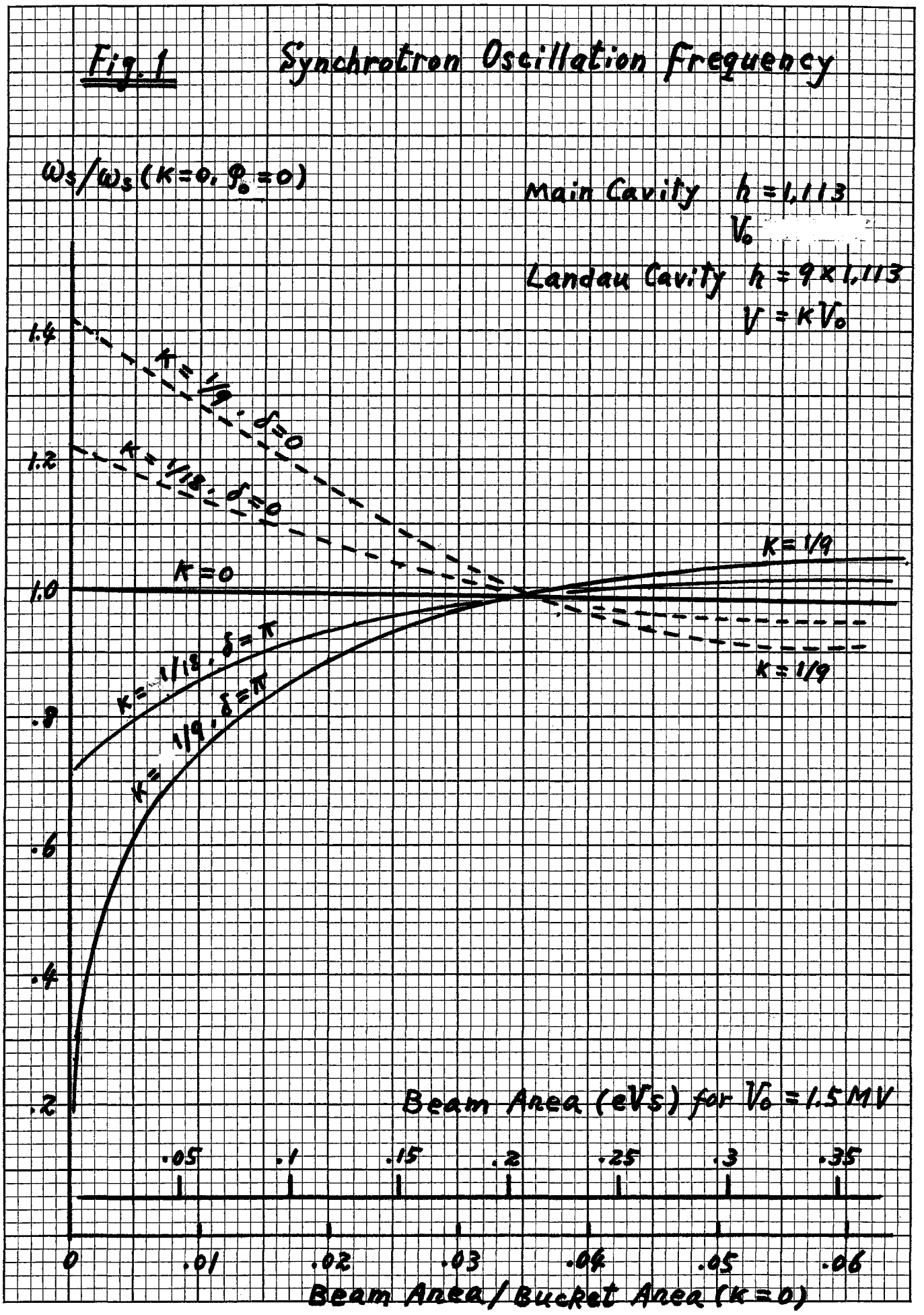
Fig. 1 Synchrotron Oscillation Frequency
ωs/ωs(K=0, φ0=0)
Main Cavity h=1,113
V0
Landau Cavity h=9×1,113
V=KV0
1.4
1.2
1.0
.8
.6
.4
.2
K=1/9, δ=0
K=1/18, δ=0
K=0
K=1/9
K=1/18, δ=π
K=1/9, δ=π
K=1/9
Beam Area (eVs) for V0=1.5MV
.05
.1
.15
.2
.25
.3
.35
0
.01
.02
.03
.04
.05
.06
Beam Area/Bucket Area (K=0)

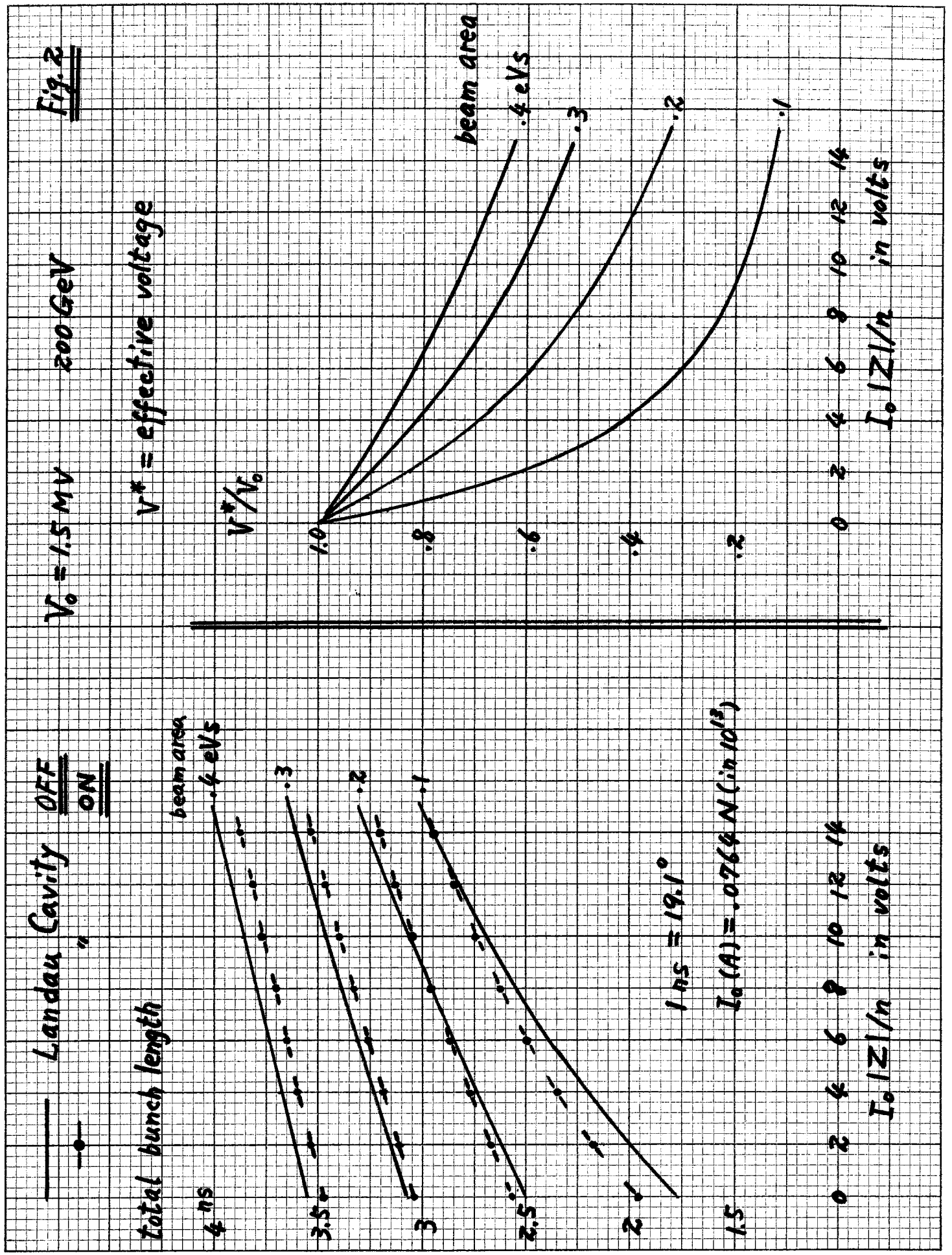
Fig. 2
Landau Cavity OFF
ON
$V_0 = 1.5$ MV
200 GeV
$V^* =$ effective voltage
V^*/V_0
beam area
.4 eVs
.3
.2
.1
1.0
.8
.6
.4
.2
0
2
4
6
8
10
12
14
$I_0|Z|/n$ in volts
Total bunch length
4 ns
3.5
3
2.5
2
1.5
1 ns = 19.1°
I_0 (A) = .0764 N (in 10^{13})

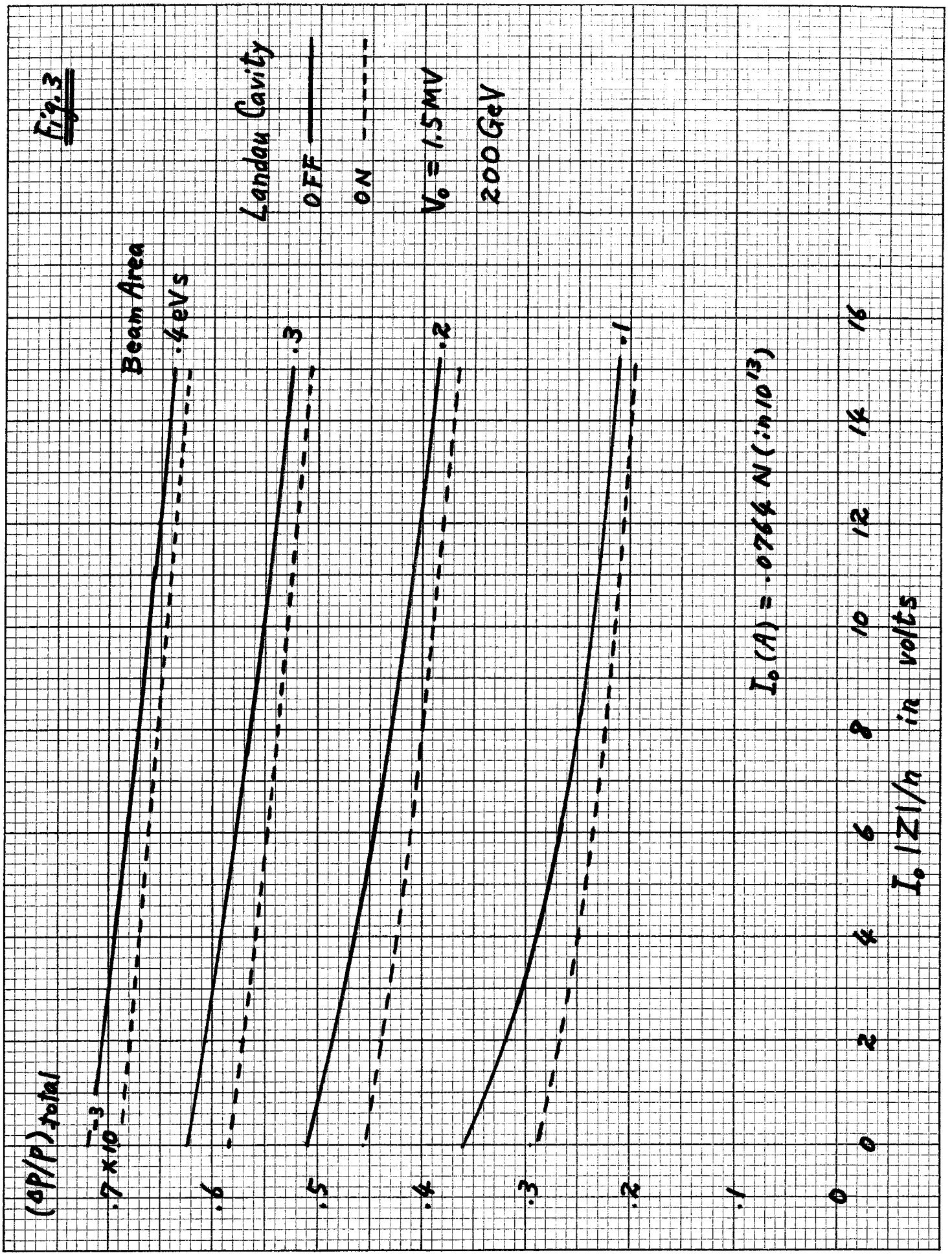
Fig. 3
Beam Area
.4 eVs
Landau Cavity
OFF
ON
$V_0 = 1.5$ MV
200 GeV
.3
.2
.1
I_0 (A) = .0764 N (in 10^{13})
$(\Delta P/P)_{total}$
$.7 \times 10^{-3}$
.6
.5
.4
.3
.2
.1
0
0
2
4
6
8
10
12
14
16
$I_0 |Z|/n$ in volts

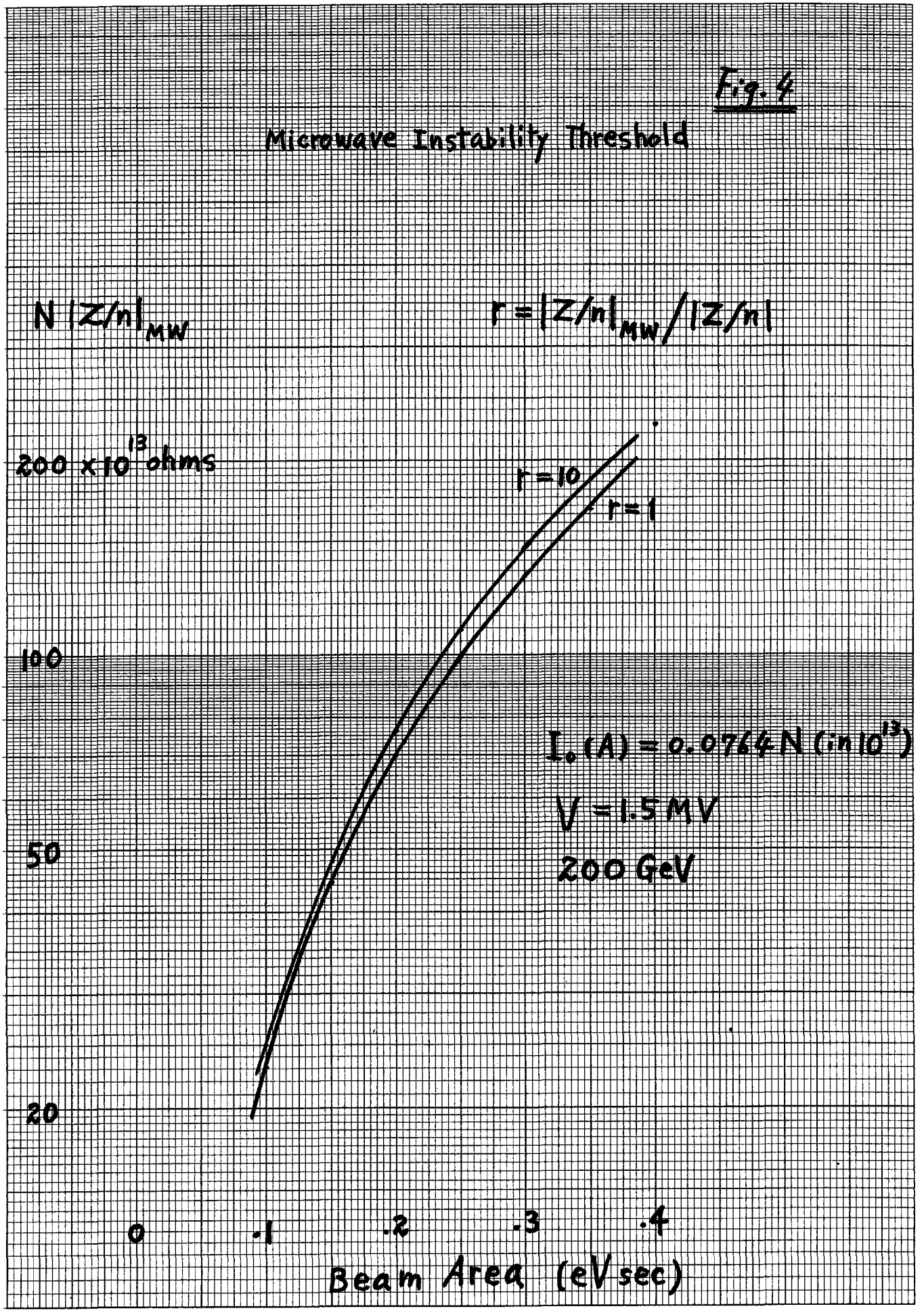
Fig. 4
Microwave Instability Threshold
N |Z/n|MW
r = |Z/n|MW/|Z/n|
200 x 10^13 ohms
r = 10
r = 1
100
I₀(A) = 0.0764 N (in 10^13)
V = 1.5 MV
200 GeV
50
20
0
.1
.2
.3
.4
Beam Area (eV sec)

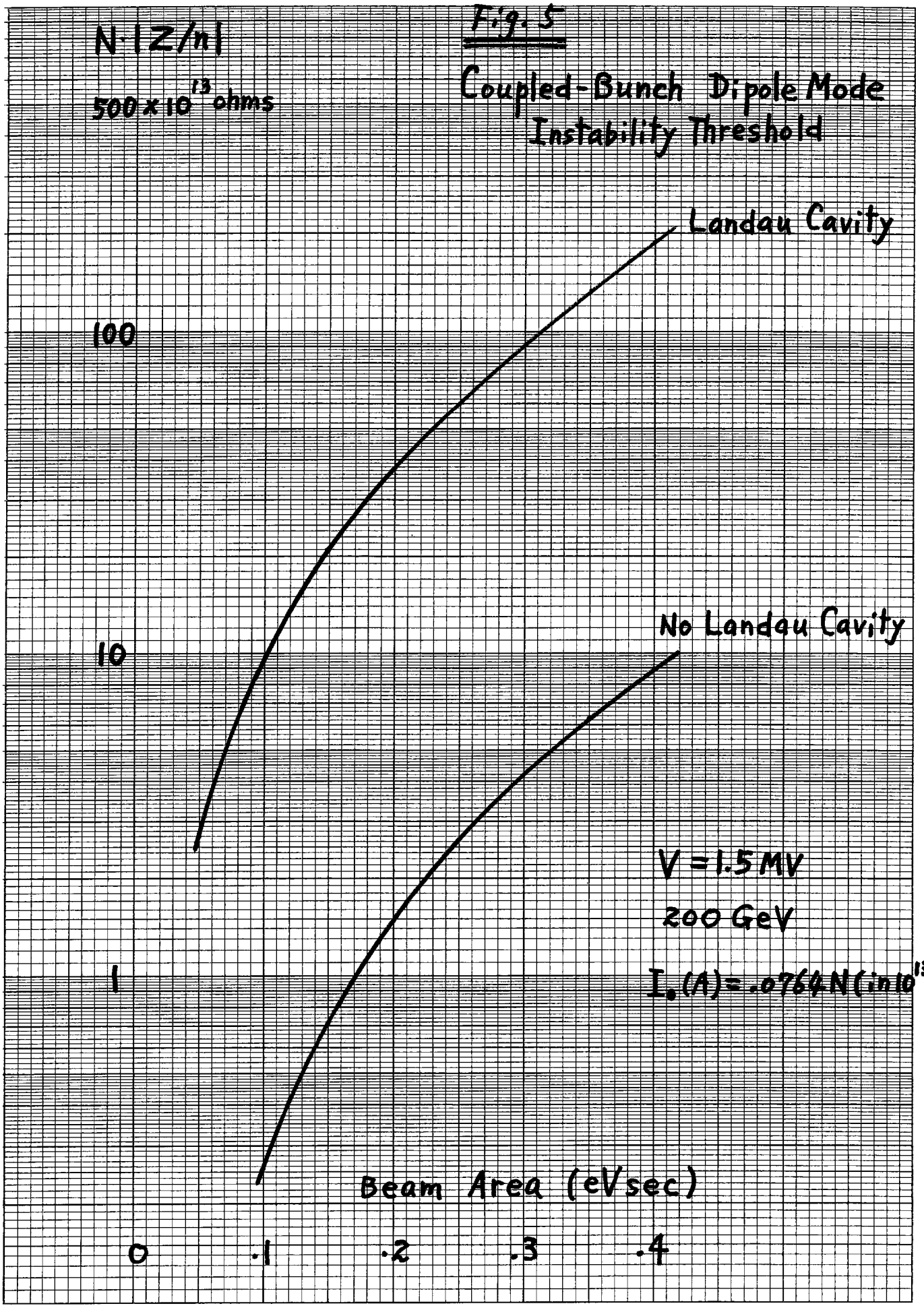

Fig. 5

Coupled-Bunch Dipole Mode Instability Threshold