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RF Stacking and Other RF Considerations for The Energy Doubler

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It is planned to get the Energy Doubler (ED) to intersect with the Main Ring (MR) for p-p and p-$\bar{p}$ colliding beam experiments. Both beams would be bunched, and, in the approximation the lattice β-function does not change very much across the bunch length, the luminosity is given by the formula

$$L = \frac{I_{MR} \cdot I_{ED}/2\pi e^2 f}{\sqrt{(a_{MR}^2+a_{ED}^2)}\sqrt{(b_{MR}^2+b_{ED}^2)+(\sigma_{MR}^2+\sigma_{ED}^2)\theta^2}} \tag{1}$$

where I is the beam current, f the frequency of encounter of the bunches, 2θ the total crossing angle, and a, b, σ respectively the bunch width, height and length standard deviations at the crossing point.

High luminosity values can be achieved by either increasing the beam current or reducing the bunch size. We shall consider here the first possibility. The beam current can be increased with multiturn injection in the MR and ED. Multiturn injection in the MR has already been discussed in another paper[1]. At most, the current in the MR can be doubled with two turn stacking. Multiturn injection in the ED can be accomplished by either stacking in the betatron phase space or by stacking in the momentum phase space (RF stacking). The latter looks more advantageous and we shall describe it here.

The working assumption is that the ED lattice is merely a replica of the MR lattice so that all the lattice functions are translated identically from the MR to the ED. The stacking geometry is shown in Fig. 1. This corresponds to the location 6.324 m downstream the station A17 where clockwise injection in the ED is supposed to occur. At that location the dispersion is 5.0 m and $\beta_H = 76.5$ m. For the reversed injection in the ED, we assume it occurs at F17 at the same corresponding point with the same beta and dispersion values.

Define the momentum aperture as that range over which the machine betatron tunes change only by a tolerable amount. We assume $\Delta p/p = \pm 4 \times 10^{-3}$. This, at the location shown in Fig. 1, corresponds to 4 cm, more than 50% of the ED magnet physical aperture. This is just an assumption, more than reasonable, so far really not yet proven. The momentum aperture of the ring will depend not only on the quality of the magnet field but also on the cancellation of the natural chromaticity on both planes.

The beam is injected with a partial aperture kicker (on the innerside of the ring according to Fig. 1). Each pulse is bunched and captured by standing RF buckets. Between pulses a shutter is lowered between the injection area and the stacking region. The shutter has the function to shield the stacked beam from the kicker field which deflects the freshly injected pulse. Here we take 10 mm for the field shielding. Once a pulse is injected and captured the kicker is turned off, the shutter removed and the pulse accelerated at constant guide field toward the outer side of the aperture. There the RF is turned off and the beam let to debunch on top of the previous stack. Subsequent pulses are released at the same location; this will have the effect to displace the previous pulses by an amount which corresponds to the bucket area (Liouville theorem). Also particles will be sca--red to form a low-energy tail every time they are crossed by the unstable fixed points of the RF buckets. This effect is measured by the stacking efficiency parameter η which is the ratio of the ideal momentum width to the actual momentum width[2].

The RF stacking cycles repeat as long as there is room for new beam. At the end the beam is captured again, displaced toward the center of the vacuum chamber and accelerated.

Beam Size and RF Matching

A beam measurement at 8 GeV out of the Booster gave a betatron emittance of $1.5\ \pi\ 10^{-6}$m. This includes 95% of the beam for a MR current corresponding to $2.0\text{x}10^{13}$ppp at large energy. At 100 GeV the equivalent emittance is $0.132\,\pi\,10^{-6}$m assuming no beam dilution occurs during the capture and the acceleration in the MR, and the transfer to the ED. At the location of the kicker (β_H = 76.5 m) the contribution of this betatron emittance to the beam size is ±3.2 mm. Another contribution to the total width comes from the momentum spread in the beam. This depends on the bunch area S and on the RF voltage in the MR and ED. Unfortunately the bunch area is not well defined and depends largely on the beam stability (which gets worse with increasing the intensity) and on whether the MR spreader is activated or not. It is believed S can be as large as 0.3 eV·s, assuming it includes 95% of the bunch particles.[3]

Since the beam in the MR is bunched at the harmonic h = 1113 (RF frequency: 53.1 MHz), it is convenient to operate the RF stacking in the ED at the same MR frequency. Indeed empty buckets would cause further dilution to the stack and are to be avoided. Also any other scheme which involves beam debunching and rebunching is to be avoided because it also causes ultimately bunch dilution. One requirement is that the MR buckets (stationary, namely no acceleration, at 100 GeV) are to be matched to the ED bucke (also stationary at the same energy) standing by to capture injected pulses (at the right hand side of Fig. 1). This is accomplished by taking the same RF voltage in the MR and in the ED.

At the present the lowest voltage that can be achieved in the MR is 1 MV/turn.

Probably this can be reduced in the beam future by a factor 2 or 3 before the system becomes too sensitive to multipactoring. A further reduction of the RF voltage is possible by paraphasing the RF cavities in two groups. But it is not clear what is the lowest voltage attainable since this depends a big deal on beam loading. Indeed we remind that the voltage across the gap of a cavity is the vector resultant of the input current and beam current. Probably a quantity like 24 kV/turn should be possible.

For beam energies $E \gtrsim 100$ GeV in the MR and ED the following formulae apply ($h = 1113$)

A = bucket area = $0.34\alpha(\phi_s)\sqrt{VE}$ eV·s

Δ = bucket height $(\frac{\Delta p}{p}) = \pm 0.014\ \beta(\phi_s)\sqrt{V/E}$

where E is in GeV and V in MV. α and β are two functions[4] of the synchronous phase ϕ_s. For stationary buckets, $\phi_s = 0$ and

$$\alpha = \beta = 1.$$

If the bunch S is defined to include 95% of the beam and the particles have a bigaussian distribution in the phase space, the following relations apply for the rms bunch length σ and momentum spread δ

$$\sigma = 140\sqrt{S/\sqrt{EV}} \quad \text{cm}$$

$$\delta = 0.011\sqrt{S\sqrt{V/E^3}}$$

where again S is in eV·s, E in GeV and V in MV. In the most pessimistic case one can expect $\delta = 1\text{x}10^{-4}$.

In Fig. 1 the beam is supposed to be injected at an off momentum value of $\Delta p/p = -3.5\text{x}10^{-3}$, namely 17.5 mm inside from the aperture center. If the beam has a kinetic energy of exactly 100 GeV the ED magnet field has to be set so that the central orbit corresponds to an off momentum value of $\Delta p/p = +3.5\text{x}10^{-2}$ compared to the beam momentum. Also in Fig. 1 the field shielding is 10 mm thick and has the median plane located at 7.5 mm inside. At the right hand side of the shield there is enough room, as shown, for the injected pulse. The rectangle is the contribution from momentum spread and the circles from the betatron emittance.

RF Stacking

A phenomenological formula for the stacking efficiency is the following[2]

$$\eta = \left[1+\frac{2\sin\phi_s}{3\alpha(\phi_s)\sqrt{n}}\right]^{-1}$$

where n is number of pulses in the stack and ϕ_s the synchronous phase with which pulses are accelerated to the stack. Higher stacking efficiency is obtained after several pulses and at low phase.

Once a pulse has been captured at the injection orbit, the stationary bucket is connected to a moving bucket that tightly fits the beam bunch. Eventually the final bucket area equals the bunch area to avoid too much dilution in the stack. The adiabaticity of the process is measured by the synchrotron frequency which in our case is simply given by

$$f_s = 1 \text{ kHz} \sqrt{\frac{V}{E} \cos\phi_s}$$

where again V is in MV and E in GeV. T ransition should occur in a time which is comparable to one phase oscillation period. If V = 25 kV, this can be about 50 msec, reasonably small.

If the beam area is known one can then calculate ϕ_s by letting the moving bucket area to be equal to the bunch area. Some values are shown below for V = 25 kV.

S	α	β	Δ	ϕ_s
0.05 eV·s	0.093	0.282	$\pm 0.62\text{x}10^{-4}$	54°
0.10	0.186	0.427	0.95	42
0.15	0.279	0.531	1.18	34
0.20	0.372	0.626	1.39	27
0.25	0.465	0.709	1.57	21
0.30	0.558	0.778	1.72	16

The energy gain per turn is obviously $\delta E = eV\sin\phi_s$. The energy range of acceleration depends on the value where subsequent pulses are released. As shown in Fig. 1, we take the release point which corresponds to an off momentum value $\Delta p/p = +2.5\text{x}10^{-3}$ to insure a safety margin to the left for subsequent recapture of the stack. Thus the displacement interval is from $\Delta p/p = -3.5\text{x}10^{-3}$ to $\Delta p/p = +2.5\text{x}10^{-3}$ which corresponds to an acceleration of 0.6 GeV at 100 GeV. The required accelerating frequency variation to accomplish the displacement is 876 Hz and the accuracy for the RF turn-off of some tens of Hz.

The time required for the displacement of each pulse is obviously

$$\tau = \frac{T\Delta E}{eV\sin\phi_s}$$

where ΔE = 600 MeV and T = 20 μs is the revolution period. Taking again V = 25 kV

$$\tau = \frac{0.48 \text{ sec}}{\sin\phi_s} .$$

For instance if S = 0.3 eV·s, one has ϕ_s = 16° and τ = 1.74 sec which would be adequate for a 2-second MR cycle at 100 GeV.

At the top of the stack, the RF voltage should be turned off adiabatically so that the beam would present the smallest momentum spread. This is quite a difficult operation and here we simply assume the RF is abruptly turned off. After that the beam will debunch and its final momentum width is the same of the moving bucket.

As we said earlier, each pulse will have the effect to displace the previous stack by an amount which corresponds to the width of a pulse adiabatically debunched. In our case this amount is

$$(\Delta p/p)_a = 0.531\times10^{-3}\ S_{eV\cdot s} .$$

In conclusion after n pulses the width of the stack is

$$(\Delta p/p)_n = \left[2\Delta+(n-1)(\Delta p/p)_a\right] /\eta .$$

We list below some numbers for n = 10 and 20 turns stacking

S	ϕ_s	$\eta_{n=10}$	$\eta_{n=20}$	$(\Delta p/p)_{n=10}$	$(\Delta p/p)_{n=20}$
0.05 eV·s	54°	.353	.435	1.03×10^{-3}	1.44×10^{-3}
0.10	42	.569	.651	1.17	1.84
0.15	34	.703	.770	1.36	2.27
0.20	27	.795	.846	1.55	2.71
0.25	21	.860	.897	1.75	3.16
0.30	16	.906	.931	1.96	3.62

Thus it seems there is enough room for 10 turns RF stacking also with an original bunch area of 0.3 eV·sec. The case shown in Fig.1 corresponds in fact to this case. Observe that, because we assumed the same RF voltage, one has larger stacking efficiency for larger bunch area, though at the cost of lower stacking speed. If the bunch area is small, say 0.1 eV·s, there is actually room for more pulses, up to at least 20.

In the following we shall assume that $(\Delta p/p)_n$ corresponds to 95% of the beam. At the RF frequency of 53.1 MHz the beam area is

$$S_{stack} = 1.88 \cdot (\Delta p/p)_n \text{ eV}\cdot\text{s}$$

where $(\Delta p/p)_n$ is the full width of the stack in 10^{-3} units.

RF Capture and Acceleration of the Stack

Once the stacking has been accomplished the beam has to be captured and accelerated. The RF voltage will be again turned on adiabatically to insure efficient capture of the beam with the least dilution. There are two requirements to be satisfied: (i) the final bucket area has to equal at least the beam area and (ii) the turning on of the RF has to occur in a period of time which is comparable to the phase oscillation period.

For what concerns the first requirement one has

$$S_{stack} = \frac{\beta^2 E}{f_{RF}} \left(\frac{\Delta p}{p}\right)_n$$

$$= \alpha \frac{8\beta}{\pi f_{RF}} \sqrt{\frac{eEV/h}{2\pi\left(\frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}\right)}} \ .$$

This relation says that the voltage required increases linearly with the harmonic number h, namely with the RF frequency. Thus it is convenient to capture the beam at moderately low frequency if one wants to avoid exceedingly high voltages. This is also consistent with the requirement for higher luminosity as one can see from Eq. (1). On the other side if all the protons are to be effectively used and the MR is operated at 53.1 MHz, then it is also convenient to bunch the beam stacked in the ED at the same frequency. We shall assume here that this is the case.

The amount of voltage required to capture the beam is

$$V = 0.3\left(\frac{\Delta p}{p}\right)_n^2 \quad \text{MV/turn}$$

with $(\Delta p/p)_n$ in 10^{-3} units. For instance $(\Delta p/p)_n = 2\text{x}10^{-3}$ would require at least 1.2 MV/turn. The phase oscillation frequency is in our case

$$f_s = 55.3\left(\frac{\Delta p}{p}\right)_n \text{ Hz}, \quad \left(\left(\frac{\Delta p}{p}\right) \text{ in } 10^{-3}\right)$$

so that to capture again $2\text{x}10^{-3}$, the RF should be turned on in about 10 msec. The corresponding bucket height is

$$\Delta p/p = \pm 0.78\text{x}10^{-3}\left(\frac{\Delta p}{p}\right)_n \text{ in } 10^{-3}$$

small enough to be accommodated within the assumed momentum aperture.

Observe that the center of the beam at the moment of capture does not correspond to the aperture center as one can see for instance in Fig. 1. Thus the RF has to be turned on at the frequency value which corresponds to the beam central momentum.

After the capture the beam is accelerated to the final energ (1000 GeV). For this purpose the synchronous phase ϕ_s is raised from its original value, $\phi_s = 0$, to a final value which depends on the acceleration rate. At the same time also the RF voltage is increased by an amount $1/\alpha(\phi_s)$ to compensate for the bucket area reduction. Some numbers are shown below for the case the stack width is ±0.1%.

ϕ_s	α	V	$\Delta E = V\sin\phi_s$	Time for Acceleration to 1000 GeV
0°	1.0	1.20 MV	-	-
2	0.918	1.31	45.7 kV	394 sec
4	0.854	1.41	93.4	193
6	0.797	1.51	157.8	114
8	0.745	1.61	224.1	80
10	0.696	1.72	298.7	60

During this operation the bucket width increases at most by 3% and the phase oscillation frequency by 19% (for $\phi_s = 10°$). The transition to a moving bucket should be programmed over about 10 milliseconds.

Care has to be taken to ramp the guide field faster at the beginning to displace the beam back to the center of the aperture.

At the present in the MR with an intensity of 0.2 A and an acceleration rate of 100 GeV/sec, the power to the beam is 0.4 MW and about 0.2 MW to the RF system itself. Thus the total power consumption is about 0.6 MW and the available input power is around 1 MW. The power requirement for the ED depends on the beam intensity. The power to the beam for a 2 A current is at most 0.6 MW, and, considering the power which is dissipated in the cavities, the total requirement should not exceed 1 MW.

At the top energy of 1000 GeV, a peak voltage of 1 MV/turn supplies stationary buckets of area 10.8 eV·s which should be more than adequate to contain the high intensity beam. For the bunch size we have the following formulae (standard deviations)

$$\sigma = 34\sqrt{(\Delta p/p)_n} \text{ cm (length)}$$

$$\delta = 8.5\text{x}10^{-5}\sqrt{(\Delta p/p)_n} \text{ (momentum spread)}$$

where $(\Delta p/p)_n$ is the momentum width of the stack at 100 GeV in 10^{-3} units.

References

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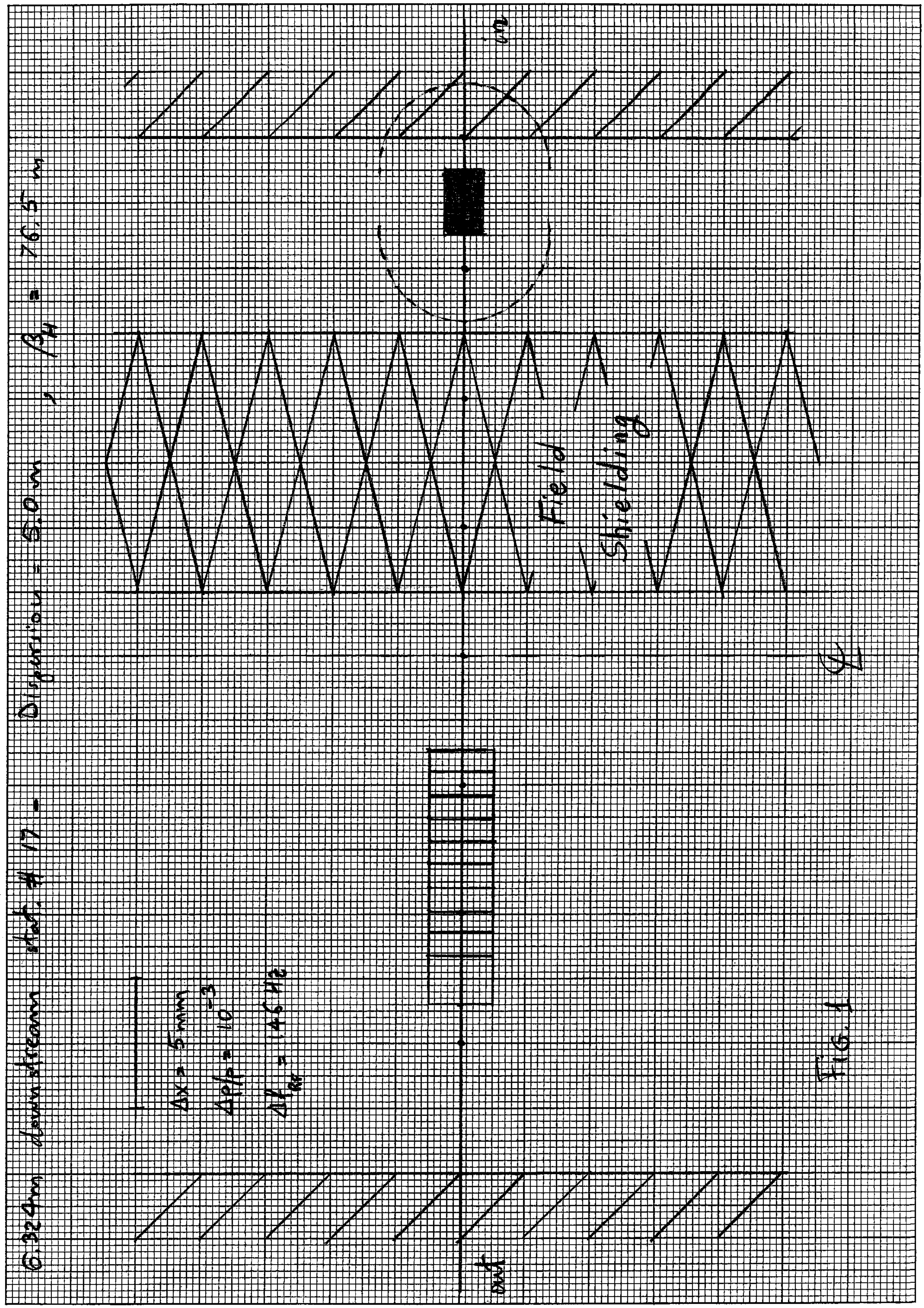
6.324m downstream stat. #17 - Dispersion = 5.0m , β_H = 76.5 m
in
Field
Shielding
Δx = 5 mm
Δp/p = 10^-3
Δf_rf = 146 Hz
out

Fig. 1