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Clearing Electrodes and Other Considerations Related to the Electron Production in the Energy Doubler

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Introduction

An intense charged beam circulating in a particle accelerator or storage ring produces pairs of free negative ions and electrons by interacting with the residual gas in the vacuum chamber. If the beam is made of protons, the negative ions would be repulsed against the wall and the electrons attracted toward the beam. Because of the mass difference, the negative ions have a much lower velocity than the electrons and they take some time before they hit the wall. On the other side, if the beam is bunched, electrons can escape between two bunches because of their higher speed. If the main beam is unbunched and intense enough, electrons can be trapped forever. As a consequence there are two effects that can make the main beam unstable. In the first effect, the electron space charge can neutralize the electric field produced by the proton beam itself, but at the same time the electrons have too small velocity and are not capable of neutralizing the magnetic field associated to the main beam current.[1] Henceforth the proton beam is now affected by its own magnetic field, the effect of which was otherwise cancelled by the beam electric field. This can cause a modification of the ring lattice and the enhancement of one or more machine resonances. This effect is measured in terms of the betatron tune shifts due to the electron space charge. To keep this shift reasonably small one has to limit the number of electrons in the beam.

The second effect is an instability that can be enhanced by a coherent oscillation of the main beam; the electron beam, though trapped by the potential barrier, can nevertheless respond by starting similar oscillations. Since the two beams interact each other electo-magnetically, once the two oscillations are in plase, one can positively feedback the other causing an amplitude growth and eventually beam enlargement and loss.[2] This effect can be controlled not only by reducing the number of electrons, but also with either a betatron tune spread in the beam[2] (Landau damping) or with an external rf damper.

In this paper we shall always assume a debunched, high-intensity beam for the Energy Doubler and consider only the effect of the electrons. We remind, nevertheless, that there are experimental evidences of this kind of instability also for bunched beams,[3,4] though it is not yet clear whether electrons or negative ions are responsible for it.[3,5]

We shall give a look to the electron production and the related effects in the Energy Doubler. To limit the number of electrons one requires installation of clearing electrode plates as it was done for the ISR.[6] The Energy Doubler performance as a storage ring for a high-intense beam depends a great deal on the quality of the vacuum, the deficiency of which can be compensated only with the installation of clearing electrodes.

Rate of Electron Production

Denoting with n_e the total number of electrons at any instant, we have for the rate of production

$$dn_e/dt = N_p \beta c \sigma \delta$$

where N_p is the total number of protons in the ring, βc the proton beam velocity, σ the cross-section for ionization on the residual case which we take to be

$$\sigma = 1.2 \times 10^{-18} \ Z \ cm^2$$

with Z the atomic number of the residual gas, and δ is the residual gas density, that is the number of neutrals per unit of volume. Here we assume that the residual gas is made of only one component with atomic number Z, mass number A and molecular weight M. In the approximation the gas is perfect and isolated we have

$$\delta = \frac{MP}{Am_p RT}$$

where $m_p = 1.6725 \times 10^{-24}$g is the mass of a proton, P is the pressure, T the temperature in absolute degrees (^{0}K) and R = 0.0821 liter x atm x $^0K^{-1}$ per mole.

A is the number of nucleons per neutral and M = A grams. Then at 4.5 ^{0}K

$$\delta = 2.13 \times 10^{18} P \ cm^{-3}$$

with P in mmHg. Expressing N_p in terms of beam current I, finally we obtain

$$dn_e/dt = 1.0 \times 10^{25} \text{ ZIP electrons/second} \tag{1}$$

where I is in Amps. For instance: Z = 2 (helium), I = 5 Amp, $P = 10^{-12}$ mmHg (at 4.5°K) gives

$$dn_e/dt = 10^{14} \text{ electrons second.}$$

It is convenient to consider a sector of the machine which has a fraction α of the total circumference. If the sector is "cold", namely at 4.5°K, denoting with N_e the total number of electrons in the sector we have for the local production from (1)

$$dN_e/dt = 1.0 \times 10^{25} \text{ ZIP}\alpha \text{ electrons/second.} \tag{2a}$$

The long and medium straight-sections of the Energy Doubler will be "warm", namely at room temperature (~300°K), there the electron production rate is higher by about the square root of the temperatures ratio

$$\sqrt{\frac{800^{\circ}K}{4.5^{\circ}K}} \sim 8$$

In the long and medium straight sections then we take

$$dN_e/dt = 8.0 \times 10^{25} \text{ ZIP}\alpha \text{ electrons/second.} \tag{2b}$$

where α = 0.0080 for a long straight section and α = 0.0024 for a medium one.

The average energy of the electrons is few times the ionization potential, that is a few tens of eV, say ~50 eV. In the following we shall assume uniform distribution of the electron velocity and position in each machine sector.

The Beam Potential Barrier

Let us take an upright ellipse for the beam cross-section. The ellipse semi-axis a(horizontal,x) and b(vertical,y) will change from location to location around the ring. Let us also take uniform distribution of the beam charge within the ellipse. If we ignore the effect of the vacuum chamber wall, the potential function is[7]

$$V = \frac{2\lambda}{a+b}\left(\frac{x^2}{a} + \frac{y^2}{b}\right), \text{ inside the beam}$$

and

$$V = \lambda\left\{1 + 2\log\left(\frac{2\sqrt{x^2+y^2}}{a+b}\right)\right\}, \text{ far away from the beam}$$

with λ the charge per unit length. The voltage distribution looks like as shown in Fig. 1

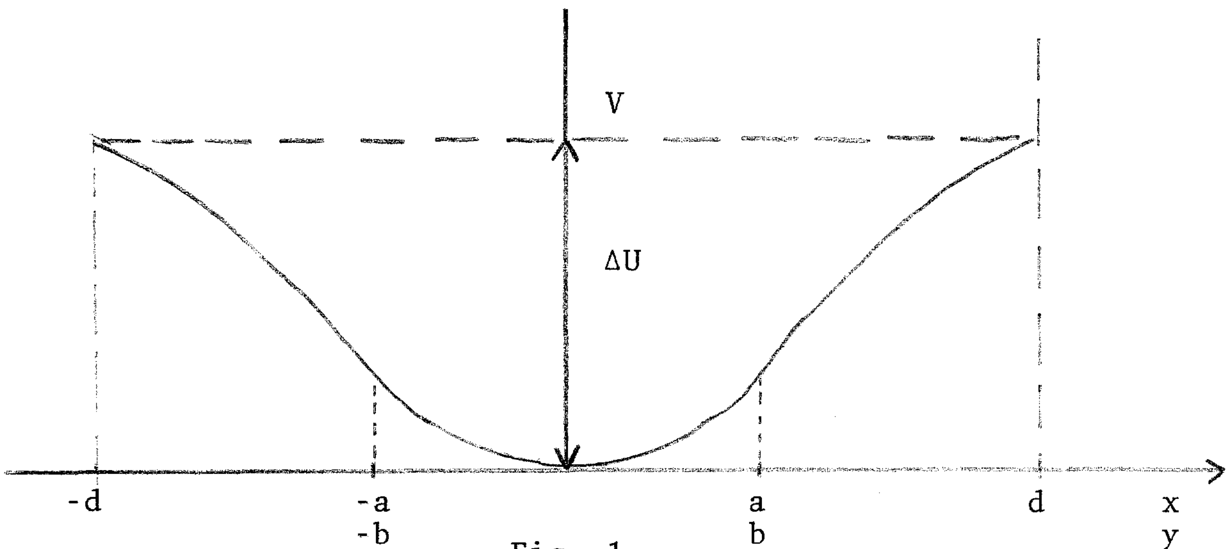

Fig. 1

where 2d is the pipe size in either direction. Figure 1 gives also an idea of the depth ΔU of the potential barrier. Let

$$g = 1 + 2\log\left(\frac{2d}{a+b}\right) \tag{3}$$

then in terms of the beam current I (in Amps) and for the Energy Doubler

$$\Delta U = 30 \text{ gI volts.} \tag{4}$$

Energy Doubler Beam Size

The beam size depends on how the multiturn stacking is performed in the Energy Doubler and on the quality of the beam coming out of the Main Ring. Here we assume that a high-intense beam is obtained by rf stacking several pulses each of 10^{13} protons and round. If one takes a betatron emittance

$$\varepsilon_\beta = \frac{10^{-5}}{\beta\gamma} \pi 10^{-6} \text{m}$$

and a longitudinal emittance

$$\varepsilon_s = 0.1 \text{ eV}\cdot\text{s/bunch}$$

we have with with a stacking efficiency of 0.5 and an rf harmonic number of 1113

$$\begin{aligned} b &= \sqrt{10\beta_V/\gamma\beta} \text{ mm} \\ a &= \left(\sqrt{10\beta_H/\gamma\beta} + 74.5 \; IX_p/\gamma\beta\right) \text{ mm} \end{aligned} \tag{5}$$

where γ is the ratio of the particle energy to the rest energy and β_V, β_H and X_p are the beta and dispersion functions in meters. The beam current I is given in Amps. One should consider eq.(5) as a resonable example for the size of the Energy Doubler beam and not as definitive.

When numbers are inserted in (5) and in (3) one finds that the factor g ranges between 4 and 9 for any energy, location of the ring and beam currents below 5 Amps. Small current corresponds to a large g and vice versa. This means from (4) that the beam potential barrier has a depth of one or more hundreds of volts, that is deep enough to capture practically all the electrons produced.

The Clearing Electrodes

When an electric field ξ is applied across the aperture the minimum of the potential distribution shown in Fig. 1 can be moved out the vacuum chamber range, provided that

$$|\xi| > \frac{4\lambda}{a+b}$$

and this applies on either plane. It is convenient to locate clearing electrode plates in locations where (a+b) is large. Also, to reduce the voltage difference across and for more efficient distribution it is advisable to make use of wide horizontal plates. In the limit of large beam current, $\lambda \to I$ and $(a+b) \to I$, and the electric field required does not depend on the current. We have

$$|\xi| \to 15\gamma/X_p \quad \text{V/cm}$$

(X_p in meters). At a location where X_p = 3m and for a 1000 GeV beam one needs a voltage difference of 25 kV between two plates separated by 5 cm.

The plate length l has to be large enough to remove an average electron produced on the axis of the vacuum chamber with zero transverse velocity. Using classical equations one obtains

$$l > v_e \sqrt{\frac{2dm_e}{e\xi}}$$

where m_e is the mass at rest of the electron and v_e its longitudinal velocity. If we take for the plate separation 2d = 5 cm and a voltage difference of 25 kV, we have in terms of the electron kinetic energy T_e (in eV)

$$1 > 0.045 \sqrt{T_e} \text{ cm}$$

A one-inch plate should be capable of clearing electrons with energies up to 3 keV. This length is reasonably small and probably can also be accommodated between superconducting magnets.

Interference with Magnetic Field

So far we have described the motion of an electron in a drift-space. Inside a magnet the motion is greatly influenced by the magnetic field B. Within a dipole the trajectory equations are

$$x = \hat{x}\cos(\omega t+\mu) + x_o$$

$$z = \hat{z}\sin(\omega t+\mu) + ut + z_o$$

where z is the longitudinal coordinate. The motion on the vertical plane is not influenced by the external field. The following relations hold

$$\omega^2 = \frac{e|\xi|}{m_e} + \left(\frac{eB}{m_e c}\right)^2$$

$$\hat{z} = \frac{eB}{m_e \omega} \frac{\hat{x}}{c}$$

and

$$u = \frac{\xi}{B} c x_o$$

The other parameters $\hat{x}$, μ, x_o and z_o are determined with the initial conditions. The space charge contribution to the angular rotation frequency ω can be neglected. An electron makes spirals with radius ranging from a fraction of a millimeter to a couple of millimeters. There is fortunately a longitudinal drift velocity u which depends on where the electron is created. The maximum value occurs at the proton beam edge ($x_o = a$) and is (in the limit of large current)

$$u = \frac{3.85 \times 10^5}{\beta X_p} \text{ m/s } (X_p \text{ in meters}) \qquad (6)$$

In terms of kinetic energy, this corresponds to

$$T_{drift} = \frac{0.4}{\beta^2 X_p{}^2} \text{ eV } (X_p \text{ in meters})$$

much smaller than the average energy the electrons are produced with. Nevertheless one can assume the velocity (6) is enough for the electrons to escape a bending field.

A problem occurs at the edge of the magnet because of their fringe field. Because of the field longitudinal component, electrons can be trapped there. The condition for the particles not to be trapped is that the vertical bending radius, which depends on the magnitude of the longitudinal field, and typically is few to about ten centimeters, is larger than the field decaying range. Since this is not a well clear situation, it is advisable to install clearing electrode plates at the edges of the magnets in the similar way it was done for the ISR.

Finally observe also that the electrons can be accelerated by a longitudinal electric field produced by the main beam and due to the variations of the beam and vacuum chamber cross-section. Because of these variations, the electrons have a tendency to be trapped in regions where the vacuum chamber is wider (and one can install there clearing plates) or where the beam is narrower.

Sweeping Rate of Electrons

Let us consider a sector of the machine between two consecutive cleaning electrodes. Let L be the length of the sector and take (6) for the average drift velocity. The time τ to cross the sector then is $\tau = L/u$. The average sweeping rate due to the clearing action of the plates is

$$\frac{dN_e}{dt} = - N_e/\tau$$

when this is combined to either (2a) or (2b) we obtain the equation for the number of electrons in the sector.

The equilibrium number of electrons is

$$N_\infty = 10^{25} ZIP\alpha\tau$$

in the cold sections and eight times larger in the warm ones.

Introduce the neutralization coefficient η as the ratio of the equilibrium number of electrons to the number of protons in the sector. If we denote the average value of X_p in the sector with $\overline{X}_p$ we have, for a "cold" sector

$$\eta = 1.25 \times 10^9 Z\ P\alpha\overline{X}_p \qquad (7)$$

where $\overline{X}_p$ is in meters and P in -mHg.

For instance, by placing electrode plates energy focussing quad ($\alpha = 0.01$) and taking $P = 10^{-12}$ mmHg at 4.5°K, we obtain $\eta = 7 \times 10^{-5}$ in the "cold" sectors. In the "warm" long straight sections the neutralization is about an order of magnitude larger.

The Electron Frequency

On the vertical plane this is given by

$$f_e = \frac{2Ir_p c}{\pi e \overline{b(b+a)}} . \tag{8}$$

r_p is the proton classical radius. In the limit of large current

$$f_e = 24.6\,\gamma^{3/4} \text{ kHz}.$$

For instance at 1000 GeV, f_e = 4.4 MHz. The corresponding frequency on the horizontal plane is smaller.

The frequency (7) defined, as obvious, as the one the electrons would respond with in the proton beam potential barrier, can be used as diagnostic tool to recognize the presence of electrons in the beam and henceforth their number. Observe nonetheless that because of the variation of the beam and vacuum chamber cross-section, and because of the non-linearities of the potential barrier, there is a spread around the frequency (8).

Betatron-Tune Shift

The formula for 100% beam neutralization is[2]

$$\Delta = \frac{2r_p R^2}{\gamma\nu c\, \overline{b(a+b)}}\ \frac{I}{e}$$

where R is here the machine radius and ν the betatron tune. In the limit of large current and for ν = 19.4, one has

$$\Delta = 0.7\sqrt{\gamma}$$

If a total tune-shift has to be kept at a level of 10^{-3} then an average beam neutralization not exceeding 5×10^{-5} is allowed at 1000 GeV. This corresponds to a pressure somewhat larger than 10^{-12} mmHg at 4.5°K, as one can derive from (7), if the plates are located at least every quadrupole.

The tune-shift on the horizontal plane is smaller.

Electron-Proton Coherent Instability

This occurs when a coherent oscillation is enchanced between the electron beam and the proton beam. If ω denotes the angular collective frequency, in absence of coupling this is $\omega = \Omega(n-\nu) = \omega_o$, with Ω the angular revolution frequency and n the spatial harmonic number. Because of the space charge, though, there is a shift

$$\Delta\omega = \frac{\Omega\Delta\eta\omega_o^{\ 2}}{\omega_o^{\ 2}-\omega_e^{\ 2}+2i\omega_o/\tau} \tag{9}$$

which is complex. When the imaginary part is positive there is instability. In eq. (9), τ is an average value over sectors of L/u and $\omega_e = 2\pi f_e$ with f_e given by (8). A resonance occurs when $\omega_o \approx \omega_e$ namely at the electron frequency. For instance at 1000 GeV, this corresponds to $n-\nu \approx 92$. At the resonance the shift is

$$|\Delta\omega|_r = \frac{1}{2}\,\Omega\Delta\eta\omega_o\tau$$

The stability condition is:

spread in $|(n-\nu)\Omega| > |\Delta\omega|_r$.

(a) Landau damping from momentum spread. One has stability when

$$\Delta p/p > \frac{1}{2}\,\Omega\Delta\eta\tau\gamma_T^{\ 2} \tag{10}$$

where $\gamma_T = 18.75$ is the ratio of the transition energy to the rest energy. On the other side consistently with (5) we have

$$\frac{\Delta p}{p} = \frac{0.15}{\beta\gamma}\,I \tag{11}$$

with I in Amps. Combining (10) and (11) one gets a condition for the neutralization constant

$$\eta \lesssim 10^{-7} \frac{I}{\alpha\gamma^{3/2}}$$

which would give in any case a too low figure for η.

(b) Landau camping from tune-spread. The stability condition is

$$\Delta\nu > \frac{1}{2} \Delta\nu\omega_o\tau$$

This also can be reversed in terms of η

$$\eta < \frac{2.5\Delta\nu}{L\gamma^{5/4}}$$

where, here, L is the average separation between electrodes in meters. A spread of 0.01 and L = 30m require $\eta \approx 4 \times 10^{-7}$, a too small number to achieve in practice.

(c) Growth time. This is simply the inverse of the shift $|\Delta\omega_r|$. In the case plates are installed every quad (L = 28m) and P = 10^{-12} mmHg at 4.5°K, we have

$$t_e = \frac{7.6 \text{ msec}}{\gamma^{5/4}}$$

which is a very short time. An rf damper can probably be built for the frequency range of interest (1-10 MHz). The question is whether it can be that fast to damp the electron-proton instability.

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