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On the Calculation of Rose Shims

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Introduction

Cyclotron magnets have been shimmed for many years using the calculations of Rose[1] as a guide. His main result is only given in graphical form which makes reconstruction of the calculation difficult. In an attempt to improve on the calculations by introducing conditions for an overshoot of the magnetic field the original basis of the Rose calculation is produced. In addition, first order corrections are made to account for the finite radius of curvature of the magnet.

Rose Calculation

It is assumed that the radius of curvature of either edge of the magnet is so great that the solutions of the following equation are adequate to describe the field.

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0, \tag{1}$$

where V is the magnetic scalar potential, x is the horizontal or radial coordinate and y is the vertical coordinate.

Transformation Theory

Since any function of the complex variable $z=x+iy$ is a solution of Eq. (1) one may use complex variable transformation theory to simplify the boundary value problem. It is first assumed that the two edges of the magnet are sufficiently far apart that the influence of one edge is not felt at the other edge. In this case the semi-infinite geometry of Fig. 1 applies.

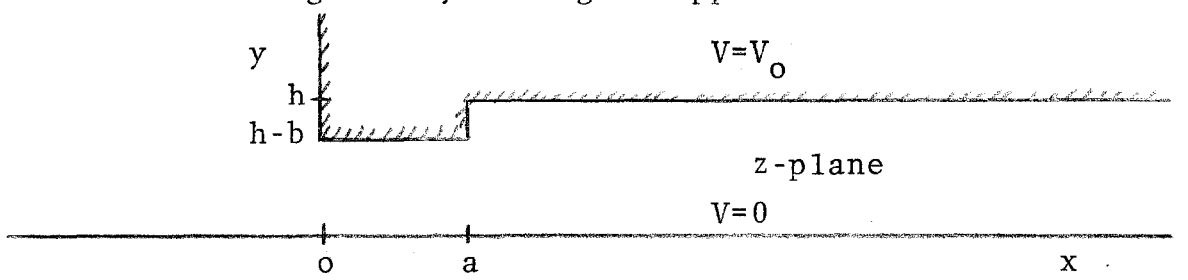

Fig. 1. Geometry of the z-plane.

A Schwarz-Christoffel transformation[2] is used to transform the boundaries to the real axis of the t-plane.

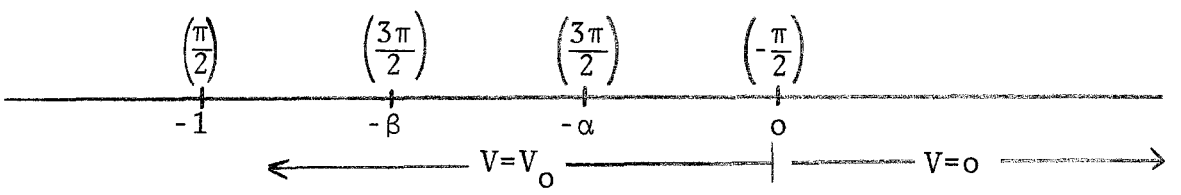

Fig. 2. Geometry of the t-plane.

$$z = C_1 \int t^{-\frac{3}{2}}(\alpha+t)^{\frac{1}{2}}(\beta+t)^{\frac{1}{2}}(1+t)^{-\frac{1}{2}}dt + C_2 \tag{2}$$

It is shown in Appendix A that the integration in Eq. (2) yields

$$z = 2C_1\left\{ -\sqrt{\beta(1-\alpha)}\left[F(\eta,k)-E(\eta,k)+v\sqrt{\frac{1-k^2v^2}{1-v^2}}\right] + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\,\Pi\left(\nu,\ \frac{1}{1-\alpha},\ k\right)\right\} + C_2 \tag{3}$$

where

$$k^2 = \frac{\beta-\alpha}{\beta(1-\alpha)}, \tag{4}$$

$$\sin\nu = u = \sqrt{\frac{(1-\alpha)t}{t+\alpha}}, \tag{5}$$

$$\sin\eta = v = \sqrt{\frac{\beta(1+t)}{t+\beta}}, \tag{6}$$

and F, E, Π are incomplete elliptic integrals of the first, second, and third kinds respectively.

Evaluation of Constants

For large $|t|$ $z \to C_1 \ln t$. But for large negative real t, $z = x+ih$. Hence

$$C_1 = \frac{h}{\pi}. \tag{7}$$

For $t = -1$, $z = a+ih$:

$$v = o,\ \eta = o;\ u = 1,\ \nu = \frac{\pi}{2} \tag{8}$$

Hence

$$a+ih = 2C_1 \frac{\alpha}{\sqrt{\beta(1-\alpha)}} \Pi\left(\frac{\pi}{2}, \frac{1}{1-\alpha}, k\right) + C_2. \tag{9}$$

For $t = -\alpha$, $z = i(h-b)$:

$$v = \frac{1}{k},\ \eta = \frac{\pi}{2} - i\,\mathrm{ch}^{-1}\frac{1}{k};\ u = \infty,\ \nu = \frac{\pi}{2} + i\infty \tag{10}$$

Hence

$$i(h-b) = 2C_1\left\{ -\sqrt{\beta(1-\alpha)}\left[F\left(\frac{\pi}{2}-i\,\mathrm{ch}^{-1}\frac{1}{k};\ k\right) - E\left(\frac{\pi}{2}-i\,\mathrm{ch}^{-1}\frac{1}{k}, k\right)\right] + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\Pi\left(\frac{\pi}{2}+i\infty, \frac{1}{1-\alpha}, k\right)\right\} + C_2. \tag{11}$$

For $t=-\beta$, $z=a+i(h-b)$:

$$v = \infty,\ \eta = \frac{\pi}{2} - i\psi;\ u = \frac{1}{k},\ \nu = \frac{\pi}{2} + i\,\mathrm{ch}^{-1}\frac{1}{k}, \tag{12}$$

where ψ is to be taken in the limit $\psi=\infty$ after evaluation of the expressions. Note that the choice of a negative imaginary part for the angle η comes from choosing a physical result for b vs. a rather than an unphysical solution given by using the positive imaginary part.

Hence

$$a+i(h-b) = 2C_1\left\{-\sqrt{\beta(1-\alpha)}\left[F\left(\frac{\pi}{2}-i\psi,k\right) - E\left(\frac{\pi}{2}-i\psi,k\right) + kv\right] + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\,\Pi\left(\frac{\pi}{2}+i\,\mathrm{ch}^{-1}\frac{1}{k},\ \frac{1}{1-\alpha},k\right)\right\} + C_2. \tag{13}$$

Further reduction of the elliptic integrals is outlined in Appendix B so that Eqs. (9,11,13) become

$$a+ih = 2C_1\ \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\,\Pi\left(\frac{1}{1-\alpha},k\right) + C_2, \tag{14}$$

$$i(h-b) = 2C_1\left\{-\sqrt{\beta(1-\alpha)}\left[K(k) - E(k) - i\,E'(k)\right] + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\left[\Pi\left(\frac{1}{1-\alpha},k\right) + \Pi\left(\frac{\beta-\alpha}{\beta},k\right) - K(k) - i\,\frac{\beta-\alpha}{\alpha}\,\Pi\left(\frac{1-\beta}{1-\alpha},k'\right)\right]\right\} + C_2, \tag{15}$$

and

$$a+i(h-b) = 2C_1\left\{+i\,\sqrt{\beta(1-\alpha)}\ E'(k) + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\left[\Pi\left(\frac{1}{1-\alpha},k\right) - i\,\frac{\beta-\alpha}{\alpha}\Pi\left(\frac{1-\beta}{1-\alpha},k'\right)\right]\right\} + C_2 \tag{16}$$

Solving Eqs. (14-16) for the constants one has

$$a = \frac{2h}{\pi}\left\{ \sqrt{\beta(1-\alpha)}\left[K(k)-E(k) \right] + \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\left[-\Pi\left(\frac{\beta-\alpha}{\beta},k\right) + K(k) \right]\right\} \tag{17}$$

$$b = \frac{2h}{\pi}\left\{ -\sqrt{\beta(1-\alpha)}\, E'(k) + \frac{\beta-\alpha}{\sqrt{\beta(1-\alpha)}}\, \Pi\left(\frac{1-\beta}{1-\alpha},k'\right)\right\} \tag{18}$$

$$\mathrm{Re}C_2 = \frac{2h}{\pi}\left\{ \sqrt{\beta(1-\alpha)}\left[K(k)-E(k) \right] - \frac{\alpha}{\sqrt{\beta(1-\alpha)}}\left[\Pi\left(\frac{1}{1-\alpha},k\right) + \Pi\left(\frac{\beta-\alpha}{\beta},k\right) - K(k) \right]\right\} \tag{19}$$

$$\mathrm{Im}\, C_2 = h \tag{20}$$

Potential and Field

The source that excites the equipotentials shown in Fig. 2 is

$$W = \frac{V_o}{\pi}\ln t, \tag{21}$$

where the magnetic scalar potential is the imaginary part of W. Hence the magnetic field $H=H_x+iH_y$ is given by

$$H^* = i\,\frac{dW}{dz} = i\,\frac{V_o}{\pi}\,t^{-1}\,\frac{dt}{dz} = i\,\frac{V_o}{h}\sqrt{\frac{t(1+t)}{(\alpha+t)(\beta+t)}}\,. \tag{22}$$

Since this expression approaches a uniform field for large $|t|$, let

$$s = \frac{1}{t} \tag{23}$$

and expand in a series in s,

$$H^* = i\,\frac{V_o}{h}\left\{ 1+\frac{1}{2}(1-\alpha-\beta)s + \text{----} \right\}. \tag{24}$$

Thus the final condition which will be imposed is to set (Rose shim condition)

$$\alpha+\beta=1 \tag{25}$$

thereby causing the field to deviate from a constant field according to s^2 instead of s. Note that for small $|s|$ or large $|t|$

$$s = e^{-\frac{\pi z}{h}}. \tag{26}$$

If on the other hand one does not impose the condition in Eq. (25) but asks for a certain factor R of overshoot in the median plane field versus horizontal distance, it is possible to extend the "good field region" somewhat. To accomplish this note that Eq. (22) gives

$$\frac{dH^*}{dz} = \frac{dt}{dz}\cdot\frac{dH^*}{dt} = i\,\frac{\pi}{2}\,\frac{V_o}{h^2}\,t\left\{\frac{\alpha\beta+2\alpha\beta t+(\alpha+\beta-1)t^2}{(\alpha+t)^2(\beta+t)^2}\right\}. \tag{27}$$

Clearly the condition at the peak of the overshoot is

$$(1-\alpha-\beta)t^2 - 2\alpha\beta t - \alpha\beta = 0. \tag{28}$$

From Eq. (22) one has for real positive t

$$R^2 = \frac{HH^*}{\left(\frac{V_o}{h}\right)^2} = \frac{t(1+t)}{(\alpha+t)(\beta+t)}\ . \tag{29}$$

By choosing the simultaneous solution of Eqs. (28-29) for positive t one finds implicitly the relation between α and β for a given R. Since, in practice R is slightly greater than 1, t must be large. Hence an approximate relation between α and β is

$$t = \frac{2\alpha\beta}{1-\alpha-\beta} = \frac{1-(\alpha+\beta)R^2}{R^2-1}\ . \tag{30}$$

Numerical Evaluation

Equations (17), (18), and (25) enable one to find a as a function of b as indicated by Rose. If on the other hand one uses the implicit relation between α and β given by Eqs. (28-29), then the shim sizes for a predetermined overshoot are obtained. Finally a first order correction for finite curvature of the shims as in a cyclotron magnet may be found by noting that

$$r = R_1 e^{u}, \quad z = R_1 v \tag{31}$$

permits Laplace's equation in cylindrical coordinate to become

$$\frac{\partial^2 V}{\partial u^2} + \frac{\partial^2 V}{\partial v^2} = 0 \tag{32}$$

for small u. Similarly Eq. (32) obtains if

$$r = R_2 e^{-u}, \quad z = R_2 v \tag{33}$$

Hence, for the smaller radius R_1 the solution for a becomes

$$a = R_1 \left(e^{\frac{"a"}{R_1}} - 1 \right) \tag{34}$$

For the larger radius R_2 the solution for the shim width becomes

$$a = R_2 \left(1 - e^{-\frac{"a"}{R_2}} \right), \tag{35}$$

where "a" is the solution obtained for the shim width when the radius of curvature R_1 or R_2 is infinite.

The code ROSE contains all of these considerations and an example is given for the H-minus bending magnet.

References

[1]M. E. Rose, Magnetic Field Corrections in the Cyclotron, PR 53, p 715 (1938).

[2]W. R. Smythe, "Static and Dynamic Electricity," Third Edition, McGraw Hill Book Co., New York, p 82 (1968).

[3]I. S. Gradshteyn and I. W. Ryzhik, "Table of Integrals Series and Products," Fourth Edition, Academic Press, New York (1965).

[4]M. Abramowitz and I. A. Stegun, Editors, "Handbook of Mathematical Functions," U. S. Government Printing Office, Washington, DC (1964).

Appendix A. Integration to Obtain z(t).

From Eq. (2) one must evaluate

$$I = \int t^{-\frac{3}{2}}(\alpha+t)^{\frac{1}{2}}(\beta+t)^{\frac{1}{2}}(1+t)^{-\frac{1}{2}}dt. \tag{A1}$$

Note that this may be written as

$$I = \int \frac{1}{t}\cdot\frac{\alpha\beta+(\alpha+\beta)t+t^2}{\sqrt{t(t+\alpha)(t+\beta)(t+1)}}\,dt. \tag{A2}$$

Hence

$$I = \alpha\beta I_1 + (\alpha+\beta)I_2 + I_3, \tag{A3}$$

where

$$I_1 = \int \frac{1}{t\sqrt{t(t+\alpha)(t+\beta)(t+1)}}\,dt, \tag{A4}$$

$$I_2 = \int \frac{1}{\sqrt{t(t+\alpha)(t+\beta)(t+1)}}\,dt, \tag{A5}$$

and

$$I_3 = \int \frac{t}{\sqrt{t(t+\alpha)(t+\beta)(t+1)}}\,dt. \tag{A6}$$

Evaluate I_2 first since less substitution is involved. Let, as in Eq. (5)

$$t = \frac{\alpha u^2}{1-\alpha-u^2} \tag{A7}$$

and, for convenience let

$$R = t(t+\alpha)(t+\beta)(t+1) \tag{A8}$$

Then

$$R = \alpha^2\beta(1-\alpha)^3\,\frac{u^2(1-u^2)(1-k^2u^2)}{(1-\alpha-u^2)^4}\,. \tag{A9}$$

where k is given by Eq. (4).

Also

$$dt = 2\alpha(1-\alpha)\ \frac{u}{(1-\alpha-u^2)^2}du \tag{A10}$$

Thus

$$I_2 = \int \frac{dt}{\sqrt{R}} = \frac{2}{\sqrt{\beta(1-\alpha)}} \int \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\ du \tag{A11}$$

or

$$I_2 = \frac{2}{\sqrt{\beta(1-\alpha)}}\ F(\nu,k). \tag{A12}$$

Next evaluate I_3 using the substitution (A7)

$$I_3 = \int \frac{t}{\sqrt{R}}\ dt = \frac{2}{\sqrt{\beta(1-\alpha)}} \int \frac{\alpha u^2}{1-\alpha-u^2} \cdot \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\ du \tag{A13}$$

or

$$I_3 = \frac{2}{\sqrt{\beta(1-\alpha)}} \int \left[\frac{\alpha(1-\alpha)}{1-\alpha-u^2} - \alpha\right] \cdot \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\ du \tag{A14}$$

or

$$I_3 = \frac{2}{\sqrt{\beta(1-\alpha)}} \left\{ \int \frac{1}{1-\frac{1}{1-\alpha}u^2} \cdot \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\ du - \int \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\ du \right\}. \tag{A15}$$

By definition then

$$I_3 = \frac{2\alpha}{\sqrt{\beta(1-\alpha)}} \left\{ \Pi(\nu, \frac{1}{1-\alpha}, k) - F(\nu,k) \right\} \tag{A16}$$

Finally for I_1 first use the substitution (A7)

$$I_1 = \int \frac{1}{t\sqrt{R}}\,dt = \frac{2}{\sqrt{\beta(1-\alpha)}}\int \frac{1-\alpha-u^2}{\alpha u^2}\cdot\frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\,du \qquad \text{(A17)}$$

or

$$I_1 = \frac{2}{\alpha}\sqrt{\frac{1-\alpha}{\beta}}\int \frac{1}{u^2\sqrt{(1-u^2)(1-k^2u^2)}}\,du$$

$$\frac{2}{\alpha\sqrt{\beta(1-\alpha)}}\int \frac{1}{\sqrt{(1-u^2)(1-k^2u^2)}}\,du \qquad \text{(A18)}$$

In the first of these integrals substitute

$$u^2 = \frac{1-v^2}{1-k^2v^2} \qquad \text{(A19)}$$

Then

$$v^2 = \frac{1-u^2}{1-k^2u^2} = \frac{\beta(t+1)}{t+\beta} \qquad \text{(A20)}$$

as in Eq. (6).

Thus

$$\frac{du}{u^2\sqrt{(1-u^2)(1-k^2u^2)}} = -\frac{1}{1-v^2}\cdot\sqrt{\frac{1-k^2v^2}{1-v^2}}\,dv \qquad \text{(A21)}$$

$$= \left\{\frac{1}{1-k^2v^2} - 1 + \left[1 + (1-k^2)\frac{v^2}{(1-v^2)(1-k^2v^2)}\right]\right\}\cdot\sqrt{\frac{1-k^2v^2}{1-v^2}}\,dv \qquad \text{(A22)}$$

Equation (A18) then becomes

$$I_1 = -\frac{2}{\alpha}\sqrt{\frac{1-\alpha}{\beta}}\left\{F(\eta,k) - E(\eta,k) + v\sqrt{\frac{1-k^2v^2}{1-v^2}}\right\}$$

$$-\frac{2}{\alpha\sqrt{\beta(1-\alpha)}}F(\nu,k). \qquad \text{(A23)}$$

In total using Eq. (A3)

$$I = -2\sqrt{\beta(1-\alpha)}\left\{ F(\eta,k) - E(\eta,k) + v\sqrt{\frac{1-k^2v^2}{1-v^2}} \right\}$$

$$+ \frac{2\alpha}{\sqrt{\beta(1-\alpha)}}\,\Pi\left(\nu, \frac{1}{1-\alpha}, k\right) \qquad \text{(A24)}$$

Appendix B. Reduction of Specific Integrals

Let

$$P = \sqrt{(1-x^2)(1-k^2x^2)} \tag{B1}$$

$$Q = \sqrt{\frac{1-k^2x^2}{1-x^2}} \tag{B2}$$

Then

$$F\left(\frac{\pi}{2}\pm i\,\mathrm{ch}^{-1}\frac{1}{k},k\right) = \int_0^{\frac{1}{k}} \frac{dx}{P} = \int_0^1 \frac{dx}{P} \pm \frac{i}{k}\int_1^{\frac{1}{k}} \frac{dx}{\sqrt{(x^2-1)\left(\frac{1}{k^2}-x^2\right)}} \tag{B3}$$

From the table of integrals[3] by Gradshteyn and Ryzhik (use GR for short) GR 3.152(10) then gives

$$F\left(\frac{\pi}{2}\pm i\,\mathrm{ch}^{-}\frac{1}{k},k\right) = K(k)\pm iK'(k). \tag{B4}$$

$$E\left(\frac{\pi}{2}\pm i\,\mathrm{ch}^{-1}\frac{1}{k},k\right) = \int_0^{\frac{1}{k}} Q\,dx = \int_0^1 Q\,dx \pm ik\int_1^{\frac{1}{k}} \sqrt{\frac{\frac{1}{k^2}-x^2}{x^2-1}}\,dx, \tag{B5}$$

which by GR 3.169(16) gives

$$E\left(\frac{\pi}{2}\pm i\,\mathrm{ch}^{-1}\frac{1}{k},k\right) = E(k)\pm i\left[K'(k)-E'(k)\right]. \tag{B6}$$

$$F\left(\frac{\pi}{2}+i\infty,k\right) = \int_0^{\infty} \frac{dx}{P} = \int_0^{\frac{1}{k}} \frac{dx}{P} + \frac{ii}{k}\int_{\frac{1}{k}}^{\infty} \frac{1}{\sqrt{(x^2-1)\left(x^2-\frac{1}{k^2}\right)}}\,dx, \tag{B7}$$

which by Eq. (B4) and GR 3.152(12) gives

$$F\left(\frac{\pi}{2}\pm i\infty,k\right) = \pm iK'(k). \tag{B8}$$

For $v=\sin\left(\frac{\pi}{2}+i\psi\right) = \mathrm{ch}\psi$ (v is assumed to be large)

$$E\left(\frac{\pi}{2}+i\psi,k\right) = \int_0^{v} Q\,dx = \int_0^{\frac{1}{k}} Q\,dx + k\frac{i}{i}\int_{\frac{1}{k}}^{v} \sqrt{\frac{x^2-\frac{1}{k^2}}{x^2-1}}\,dx, \tag{B9}$$

which by Eq. (B6) and GR 3.169(18) gives

$$E\left(\frac{\pi}{2}\pm i\psi,k\right) = E(k)\pm i\left[K'(k)-E'(k)\right] - E\left(\frac{\pi}{2}-\frac{1}{kv},k\right) + kv \qquad \text{(B10)}$$

where the last two terms are approximate for large v.

For integrals of the third kind note that

$$\Pi\left(\frac{\pi}{2},\ \frac{1}{1-\alpha},\ k\right) = \Pi\left(\frac{1}{1-\alpha},\ k\right) = \int_0^1 \frac{1}{\left(1-\frac{1}{1-\alpha}x^2\right)P}\,dx. \qquad \text{(B11)}$$

Then

$$\Pi\left(\frac{\pi}{2}+i\,\mathrm{ch}^{-1}\frac{1}{k},\ \frac{1}{1-\alpha},\ k\right) = \Pi\left(\frac{1}{1-\alpha},k\right) + \int_1^{\frac{1}{k}} \frac{1}{\left(1-\frac{1}{1-\alpha}x^2\right)P}\,dx$$

$$= \Pi\ \frac{1}{1-\alpha},k + i\left(\frac{1-\alpha}{k}\right)\cdot$$

$$\cdot\int_1^{\frac{1}{k}} \frac{1}{\left(1-\alpha-x^2\right)\sqrt{\left(x^2-1\right)\left(\frac{1}{k^2}-x^2\right)}}\,dx, \qquad \text{(B12)}$$

which by GR 3.157(9) becomes

$$\Pi\left(\frac{\pi}{2}+i\,\mathrm{ch}^{-1}\frac{1}{k},\frac{1}{1-\alpha},k\right) = \Pi\left(\frac{1}{1-\alpha},k\right) - \frac{i}{\alpha}\,\Pi\left(-\frac{1-\beta}{\beta},k'\right) + iK'(k). \qquad \text{(B13)}$$

$$\Pi\left(\frac{\pi}{2}+i\infty,\frac{1}{1-\alpha},k\right) = \int_0^\infty \frac{1}{\left(1-\frac{1}{1-\alpha}x^2\right)P}\,dx$$

$$= \Pi\left(\sin^{-1}\frac{1}{k},\frac{1}{1-\alpha},k\right) - ii\left(\frac{1-\alpha}{k}\right)\cdot$$

$$\cdot\int_{\frac{1}{k}}^\infty \frac{1}{\left(x^2-1+\alpha\right)\sqrt{\left(x^2-1\right)\left(x\ -\frac{1}{k^2}\right)}}\,dx, \qquad \text{(B14)}$$

which by Eq. (B13) and GR 3.157(12) becomes

$$\Pi\left(\frac{\pi}{2}+i\infty,\frac{1}{1-\alpha},k\right) = \Pi\left(\frac{1}{1-\alpha},k\right) + \Pi\left(\frac{\beta-\alpha}{\beta},k\right) - K(k)$$

$$- \frac{i}{\alpha}\,\Pi\left(-\frac{1-\beta}{\beta},k'\right) + iK'(k). \qquad \text{(B15)}$$

However, from Handbook of Mathematical Functions,[4] by Abramowitz and Stegun (AS for short) AS 17.7.17 gives

$$\Pi\left(-\frac{1-\beta}{\beta},k'\right) = (\beta-\alpha)\left(\frac{1-\beta}{1-\alpha},k'\right) + \alpha K'(k). \tag{B16}$$

Hence Eq. (B15) becomes

$$\Pi\left(\frac{\pi}{2}+i\infty,\frac{1}{1-\alpha},k\right) = \Pi\left(\frac{1}{1-\alpha},k\right) + \Pi\left(\frac{\beta-\alpha}{\beta},k\right) - K(k)$$
$$-i\frac{\beta-\alpha}{\alpha}\,\Pi\left(\frac{1-\beta}{1-\alpha},k'\right) \tag{B17}$$

Finally, to evaluate the integrals of the third kind note that, since $k'^2<\frac{1-\beta}{1-\alpha}<1$, AS 17.7.14 gives

$$\Pi\left(\frac{1-\beta}{1-\alpha},k'\right) = K'(k) + \frac{\pi}{2}\,\frac{\sqrt{\beta(1-\alpha)}}{\beta-\alpha}\left[1-\Lambda_o\left(\sin^{-1}\sqrt{\beta},\ k'\right)\right], \tag{B18}$$

where from AS 17.4.39

$$\Lambda_o\left(\sin^{-1}\sqrt{\beta},k'\right) = \frac{2}{\pi}\left\{K'(k)E\left(\sin^{-1}\sqrt{\beta},k\right) - \left[K'(k)-E'(k)\right]F\left(\sin^{-1}\sqrt{\beta},k\right)\right\}. \tag{B19}$$

In addition, since $o<\frac{\beta-\alpha}{\beta}<k^2$, AS 17.7.6 gives

$$\Pi\left(\frac{\beta-\alpha}{\beta},k\right) = K(k) + \frac{\sqrt{\beta(1-\alpha)}}{\alpha}\,K(k)Z\left(\sin^{-1}\sqrt{1-\alpha},\ k\right), \tag{B20}$$

where from AS 17.4.27 this becomes

$$\Pi\left(\frac{\beta-\alpha}{\beta},k\right) = K(k) + \frac{\sqrt{\beta(1-\alpha)}}{\alpha}\left[K(k)E\left(\sin^{-1}\sqrt{1-\alpha},\ k\right)-E(k)F\left(\sin^{-1}\sqrt{1-\alpha},\ k\right)\right] \tag{B21}$$

Equations (17) and (18) become

$$a = \frac{2h}{\pi}\left\{\sqrt{\beta(1-\alpha)}\left[K(k)-E(k)\right]-K(k)E\left(\sin^{-1}\sqrt{1-\alpha},\ k\right)+E(k)F\left(\sin^{-1}\sqrt{1-\alpha},\ k\right)\right\}, \tag{B22}$$

$$b = \frac{2h}{\pi}\left\{-\sqrt{\beta(1-\alpha)}\,E'(k) + \frac{\beta-\alpha}{\sqrt{\beta(1-\alpha)}}\,K'(k) + \frac{\pi}{2}\right.$$
$$\left.-\left[K'(k)E\left(\sin^{-1}\sqrt{\beta},\ k\right)-\left[K'(k)-E'(k)\right]F\left(\sin^{-1}\sqrt{\beta},\ k\right)\right]\right\} \tag{B23}$$

ROSE SHIMS FOR H-MINUS BENDING MAGNET ROSE

MAGNET HALF GAP(IN)	=	1.5000	INNER RADIUS(IN)	=	3.50000	OUTER RADIUS(IN)	= 12.50000
INITIAL ALPHA	=	.05000	FINAL ALPHA	=	.30000	OVERSHOOT FACTOR	= 1.00020

T AT BMAX	ALPHA	BETA	A(IN)	B(IN)	ARIN(IN)	AROUT(IN)
10.8462	.05000	.94093	.0446	.0438	1.2172	1.0022
11.3431	.05500	.93553	.9938	.0478	1.1493	.9553
11.8133	.06000	.93015	.9473	.0517	1.0878	.9123
12.2603	.06500	.92480	.9043	.0556	1.0319	.8724
12.6870	.07000	.91946	.8644	.0594	.9805	.8352
13.0945	.07500	.91413	.8272	.0633	.9332	.8004
13.4845	.08000	.90882	.7924	.0671	.8892	.7678
13.8587	.08500	.90352	.7596	.0709	.8483	.7370
14.2188	.09000	.89823	.7286	.0747	.8100	.7078
14.5656	.09500	.89295	.6993	.0784	.7741	.6801
14.9000	.10000	.88769	.6716	.0822	.7403	.6538
15.2223	.10500	.88243	.6452	.0859	.7084	.6288
15.5342	.11000	.87718	.6200	.0896	.6783	.6049
15.8359	.11500	.87194	.5960	.0932	.6497	.5820
16.1279	.12000	.86670	.5730	.0969	.6226	.5601
16.4108	.12500	.86148	.5510	.1005	.5967	.5390
16.6851	.13000	.85626	.5299	.1041	.5721	.5188
16.9507	.13500	.85104	.5097	.1077	.5487	.4994
17.2089	.14000	.84584	.4902	.1113	.5262	.4807
17.4597	.14500	.84064	.4715	.1149	.5048	.4627
17.7032	.15000	.83544	.4535	.1184	.4842	.4454
17.9399	.15500	.83025	.4362	.1219	.4645	.4286
18.1700	.16000	.82507	.4194	.1254	.4456	.4125
18.3938	.16500	.81989	.4033	.1289	.4274	.3968
18.6114	.17000	.81472	.3877	.1323	.4100	.3817
18.8231	.17500	.80955	.3726	.1358	.3932	.3671
19.0292	.18000	.80438	.3581	.1392	.3770	.3530
19.2297	.18500	.79922	.3440	.1426	.3614	.3393
19.4249	.19000	.79407	.3304	.1459	.3465	.3260
19.6149	.19500	.78891	.3172	.1493	.3320	.3132
19.7999	.20000	.78377	.3044	.1526	.3180	.3007
19.9801	.20500	.77862	.2921	.1559	.3046	.2887
20.1549	.21000	.77348	.2801	.1592	.2916	.2770
20.3256	.21500	.76835	.2685	.1625	.2791	.2656
20.4918	.22000	.76321	.2573	.1657	.2670	.2546
20.6536	.22500	.75808	.2464	.1689	.2553	.2440
20.8112	.23000	.75296	.2358	.1721	.2440	.2336
20.9645	.23500	.74783	.2256	.1753	.2330	.2236
21.1137	.24000	.74272	.2157	.1784	.2225	.2138
21.2589	.24500	.73760	.2061	.1815	.2123	.2044
21.4002	.25000	.73249	.1968	.1846	.2024	.1952
21.5377	.25500	.72738	.1877	.1876	.1929	.1863
21.6713	.26000	.72227	.1790	.1907	.1836	.1777
21.8013	.26500	.71717	.1705	.1937	.1747	.1694
21.9276	.27000	.71206	.1623	.1966	.1661	.1613
22.0504	.27500	.70697	.1543	.1996	.1578	.1534
22.1696	.28000	.70187	.1466	.2025	.1498	.1458
22.2854	.28500	.69678	.1392	.2054	.1420	.1384
22.3978	.29000	.69169	.1320	.2082	.1345	.1313
22.5068	.29500	.68660	.1250	.2111	.1272	.1244
22.6126	.30000	.68152	.1182	.2139	.1202	.1177